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Low-Complexity Factor Graph Receivers for Spectrally Efficient MIMO-IDMA

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***Abstract*—Interleave-division multiple access (IDMA) has recently been introduced as an attractive alternative to CDMA. IDMA employs user-specific interleavers combined with low-rate channel coding for user separation. In this paper, we consider a MIMO-IDMA system with increased spectral efficiency due to the use of higher-order symbol constellations. Based on a factor graph framework and the sum-product algorithm, we develop an iterative turbo multiuser receiver. Gaussian approximations for certain messages propagated through the factor graph lead to a complexity that scales only linearly with the number of users. To further reduce complexity, we introduce a selective message update scheme. Numerical simulations demonstrate the performance of the proposed receiver algorithms.**

I. Introduction

Code-division multiple access (CDMA) is widely used in multiuser communications due to its many attractive properties [1], [2]. Recently, *interleave-division multiple access* (IDMA) has been proposed as an alternative to CDMA [3]. With IDMA, user separation is obtained via user-specific interleavers combined with low-rate channel coding. Like CDMA, IDMA offers diversity against fading and allows a mitigation of inter-cell interference [3]. However, IDMA has some important advantages over CDMA: it allows the use of multiuser detectors that are significantly less complex than those required for CDMA; it can outperform coded CDMA when iterative (turbo) receivers are used [3]; and it can be integrated into a *multiple-input multiple-output* (MIMO) system more easily than CDMA [4]. Most IDMA systems proposed so far use BPSK modulation to avoid excessive receiver complexity. An exception is [5], where results for a (SISO-)IDMA system with QPSK are provided.

In this paper, we consider a MIMO-IDMA system that employs higher-order modulation for increased spectral efficiency. We use a factor graph framework [6], [7] to develop a corresponding iterative receiver. Straightforward application of the sum-product algorithm [6] would result in a complexity that is exponential in the number of users. Based on a Gaussian approximation for certain messages propagated through the factor graph, we derive an efficient multiuser detector whose complexity is only linear in the number of users. To further reduce complexity, we propose a variant of the sum-product algorithm with selective message updates. This results in a multiuser detector that does not update messages corresponding to reliable bits whose log-likelihood ratio (LLR) magnitude is above a threshold. With this receiver, complexity can easily be traded against performance by adjusting the LLR threshold.

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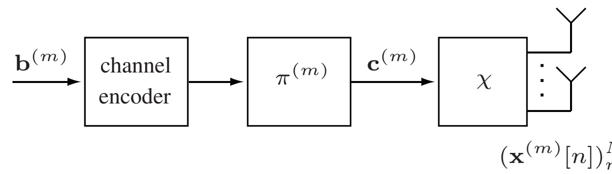

Fig. 1. Block diagram of the MIMO-IDMA transmitter for the mth user.

This paper is organized as follows. In Section II, we describe the MIMO-IDMA system. The factor graph and the messages on which the iterative receiver is based are derived in Section III. In Section IV, we develop the low-complexity multiuser detector and the selective message update. Finally, simulation results demonstrating the performance of the proposed receiver algorithms are presented in Section V.

II. MIMO-IDMA System

We consider an uplink multiple-access scenario with M users, each of which employs M_T transmit antennas for spatial multiplexing [8]. The base station has M_R receive antennas. Assuming flat fading, and considering the equivalent complex baseband after symbol-rate sampling, the length-M_R receive vector at symbol time n is given by

$$\mathbf{r}[n] = \sum_{m=1}^{M} \mathbf{H}^{(m)}[n]\,\mathbf{x}^{(m)}[n] + \mathbf{w}[n] \qquad (1)$$
$$= \sum_{m=1}^{M}\sum_{i=1}^{M_\mathrm{T}} \mathbf{h}_i^{(m)}[n]\,x_i^{(m)}[n] + \mathbf{w}[n]\,, \quad n = 1,\ldots,N.$$

Here, $\mathbf{x}^{(m)}[n] = \big(x_1^{(m)}[n] \cdots x_{M_\mathrm{T}}^{(m)}[n]\big)^T$ is the transmit vector of the mth user, $\mathbf{H}^{(m)}[n] = \big(\mathbf{h}_1^{(m)}[n] \cdots \mathbf{h}_{M_\mathrm{T}}^{(m)}[n]\big)$ is the $M_\mathrm{R} \times M_\mathrm{T}$ MIMO channel matrix from the mth user to the base station, $\mathbf{w}[n] = (w_1[n] \cdots w_{M_\mathrm{R}}[n])^T \sim \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$ is i.i.d. complex Gaussian noise, and N is the number of symbols per block.

A MIMO-IDMA transmitter is shown in Fig. 1; it extends the BPSK-based MIMO-IDMA transmitter of [4] to higher-order modulation alphabets. The length-K sequence of information bits of the mth user, $\mathbf{b}^{(m)} = \big(b_1^{(m)} \cdots b_K^{(m)}\big)^T$, is encoded into a length-L sequence of code bits, with rate $R = K/L < 1$. The code is a concatenation of a terminated convolutional code and a low-rate repetition code. The code bit sequence is passed through a user-specific interleaver $\pi^{(m)}$, yielding the bit sequence $\mathbf{c}^{(m)} = \big(c_1^{(m)} \cdots c_L^{(m)}\big)^T = \mathcal{C}_m(\mathbf{b}^{(m)})$. Here, the one-to-one function $\mathcal{C}_m(\cdot)$ denotes the combined effect of channel coding and interleaving. Different users employ identical codes but different interleavers. The repetition code together with the user-specific interleaver replaces the spreading employed in CDMA systems.

The complex transmit symbol $x_i^{(m)}[n]$ on the ith antenna of the mth user at time n is obtained by mapping a group of B successive interleaved bits $\mathbf{c}_i^{(m)}[n] = \big(c_{l(n,i)+1}^{(m)} \cdots c_{l(n,i)+B}^{(m)}\big)^T$, with $l(n,i) = \big[(n-1)M_\mathrm{T} + i - 1\big]\, B$, to a symbol from an alphabet $\mathcal{S}$ of size $|\mathcal{S}| = 2^B$. This will be denoted as $x_i^{(m)}[n] = \chi\big(\mathbf{c}_i^{(m)}[n]\big) \in \mathcal{S}$ with the one-to-one symbol mapping χ. We will refer to $\mathbf{c}_i^{(m)}[n]$ as the symbol label associated to $x_i^{(m)}[n]$. The transmit symbol vector of the mth user at time n will be similarly written as $\mathbf{x}^{(m)}[n] = \chi\big(\mathbf{c}^{(m)}[n]\big)$ where $\mathbf{c}^{(m)}[n] = \big(\mathbf{c}_1^{(m)T}[n] \cdots \mathbf{c}_{M_\mathrm{T}}^{(m)T}[n]\big)^T$. Note that the number N of symbol vectors per user and the number L of code bits are related as $N = L/(M_\mathrm{T} B)$.

III. Factor Graph and Messages

We next derive a factor graph for an iterative MIMO-IDMA receiver. This will be used in Section IV as a basis for developing a low-complexity MIMO multiuser detector.

A. Derivation of the Factor Graph

The proposed MIMO-IDMA receiver is based on the optimal (maximum a posteriori) bit detector [1], [9]

$$\hat{b}_k^{(m)} = \arg \max_{b_k^{(m)} \in \{0,1\}} p(b_k^{(m)}|\mathbf{r})\,. \tag{2}$$

Here, $b_k^{(m)}$ is the kth information bit of the mth user, $\mathbf{r} = \big(\mathbf{r}^T[1] \cdots \mathbf{r}^T[N]\big)^T$ is the received vector sequence (cf. (1)), and $p(b_k^{(m)}|\mathbf{r})$ denotes the conditional probability of $b_k^{(m)}$ given $\mathbf{r}$. In what follows, let $\mathbf{b} = \big(\mathbf{b}^{(1)T} \cdots \mathbf{b}^{(M)T}\big)^T$ and $\mathbf{c} = \big(\mathbf{c}^{(1)T} \cdots \mathbf{c}^{(M)T}\big)^T$ denote the vectors containing all information bits and code bits, respectively; furthermore, let $\mathbf{X} = (\mathbf{X}[1] \cdots \mathbf{X}[N])$ with $\mathbf{X}[n] = \big(\mathbf{x}^{(1)}[n] \cdots \mathbf{x}^{(M)}[n]\big)$ be the $M_\mathrm{T} \times NM$ matrix of all transmit vectors $\mathbf{x}^{(m)}[n]$, $n = 1, \ldots, N$, $m = 1, \ldots, M$. Note that there is a one-to-one correspondence between $\mathbf{b}$, $\mathbf{c}$, and $\mathbf{X}$.

To compute $p(b_k^{(m)}|\mathbf{r})$ in (2), we first write it as a marginal of $p(\mathbf{b}|\mathbf{r})$ and apply Bayes' rule (assuming *a priori* equally likely information bit sequences $\mathbf{b}$):

$$p(b_k^{(m)}|\mathbf{r}) = \sum_{\sim b_k^{(m)}} p(\mathbf{b}|\mathbf{r}) \propto \sum_{\sim b_k^{(m)}} p(\mathbf{r}|\mathbf{b})\,, \tag{3}$$

where $\sum_{\sim b_k^{(m)}}$ denotes summation with respect to all components of $\mathbf{b}$ except $b_k^{(m)}$, $p(\mathbf{r}|\mathbf{b})$ is the conditional probability density function of $\mathbf{r}$ given $\mathbf{b}$, and $\propto$ denotes equality up to factors irrelevant to the maximization in (2). With $p(\mathbf{r}|\mathbf{b}) = \sum_{\mathbf{X},\mathbf{c}} p(\mathbf{r}, \mathbf{X}, \mathbf{c}|\mathbf{b}) = \sum_{\mathbf{X},\mathbf{c}} p(\mathbf{r}|\mathbf{X})\, p(\mathbf{X}|\mathbf{c})\, p(\mathbf{c}|\mathbf{b})$, we can write (3) as

$$p(b_k^{(m)}|\mathbf{r}) \propto \sum_{\sim b_k^{(m)}} p(\mathbf{r}|\mathbf{X})\, p(\mathbf{X}|\mathbf{c})\, p(\mathbf{c}|\mathbf{b})\,, \tag{4}$$

where $\sum_{\sim b_k^{(m)}}$ from now on denotes summation with respect to all unknown variables except $b_k^{(m)}$ (in the present case $\mathbf{X}$, $\mathbf{c}$, and all components of $\mathbf{b}$ except $b_k^{(m)}$). Note that $p(\mathbf{r}|\mathbf{X})$ corresponds to the channel (cf. (1)), $p(\mathbf{X}|\mathbf{c})$ describes the modulator (symbol mappings $\mathbf{x}^{(m)}[n] = \chi\big(\mathbf{c}^{(m)}[n]\big)$), and $p(\mathbf{c}|\mathbf{b})$ represents the channel encoder and interleaver (one-to-one correspondences $\mathbf{c}^{(m)} = \mathcal{C}_m(\mathbf{b}^{(m)})$).

There is $p(\mathbf{c}|\mathbf{b}) = 1$ if $\mathbf{c}^{(m)} = \mathcal{C}_m(\mathbf{b}^{(m)})$ for all m and $p(\mathbf{c}|\mathbf{b}) = 0$ otherwise. Using the indicator function $\mathrm{I}\{\cdot\}$, which equals 1 if its argument is true and 0 otherwise, we thus have

$$p(\mathbf{c}|\mathbf{b}) = \prod_{m=1}^{M} \mathrm{I}\big\{\mathbf{c}^{(m)} = \mathcal{C}_m(\mathbf{b}^{(m)})\big\}\,. \tag{5}$$

We note that the code constraint $\mathrm{I}\big\{\mathbf{c}^{(m)} = \mathcal{C}_m(\mathbf{b}^{(m)})\big\}$ can be expressed in a more detailed manner by using the code structure (in particular, a trellis/state representation for the convolutional code) [6], [7], [9]. A similar reasoning yields

$$\begin{aligned} p(\mathbf{X}|\mathbf{c}) &= \prod_{n=1}^{N} \prod_{m=1}^{M} \mathrm{I}\big\{\mathbf{x}^{(m)}[n] = \chi(\mathbf{c}^{(m)}[n])\big\} \\ &= \prod_{n=1}^{N} \prod_{m=1}^{M} \prod_{i=1}^{M_\mathrm{T}} \mathrm{I}\big\{x_i^{(m)}[n] = \chi(\mathbf{c}_i^{(m)}[n])\big\}\,. \end{aligned} \tag{6}$$

Finally, because the receive vectors $\mathbf{r}[n]$ are conditionally independent given the transmit vectors $\mathbf{x}^{(m)}[n]$ (cf. (1)),

$$p(\mathbf{r}|\mathbf{X}) = \prod_{n=1}^{N} p(\mathbf{r}[n]\,|\,\mathbf{X}[n])\,. \tag{7}$$

Here, $p(\mathbf{r}[n]\,|\,\mathbf{X}[n])$ is complex Gaussian with mean $\sum_{m=1}^{M} \mathbf{H}^{(m)}[n]\, \mathbf{x}^{(m)}[n] = \sum_{m=1}^{M} \sum_{i=1}^{M_\mathrm{T}} \mathbf{h}_i^{(m)}[n]\, x_i^{(m)}[n]$ and covariance matrix $\sigma^2 \mathbf{I}$. Inserting the expressions (5)–(7) into (4), we obtain the overall factorization

$$p(b_k^{(m)}|\mathbf{r}) \propto \sum_{\sim b_k^{(m)}} \prod_{n=1}^{N} p(\mathbf{r}[n]\,|\,\mathbf{X}[n]) \prod_{m'=1}^{M} \mathrm{I}\big\{\mathbf{c}^{(m')} = \mathcal{C}_{m'}(\mathbf{b}^{(m')})\big\} \times \prod_{i=1}^{M_\mathrm{T}} \mathrm{I}\big\{x_i^{(m')}[n] = \chi(\mathbf{c}_i^{(m')}[n])\big\},$$

which can be represented by the factor graph [6], [7], [9] shown in Fig. 2. There are factor nodes for the channel, symbol mapper constraints, and code constraints, and variable nodes for the transmit symbols, code bits, and information bits.

B. Sum-Product Algorithm and Messages

For a factor graph without cycles, marginals like (4) and the associated bit decisions (2) can be determined exactly and efficiently using the sum-product algorithm [6]. For a factor

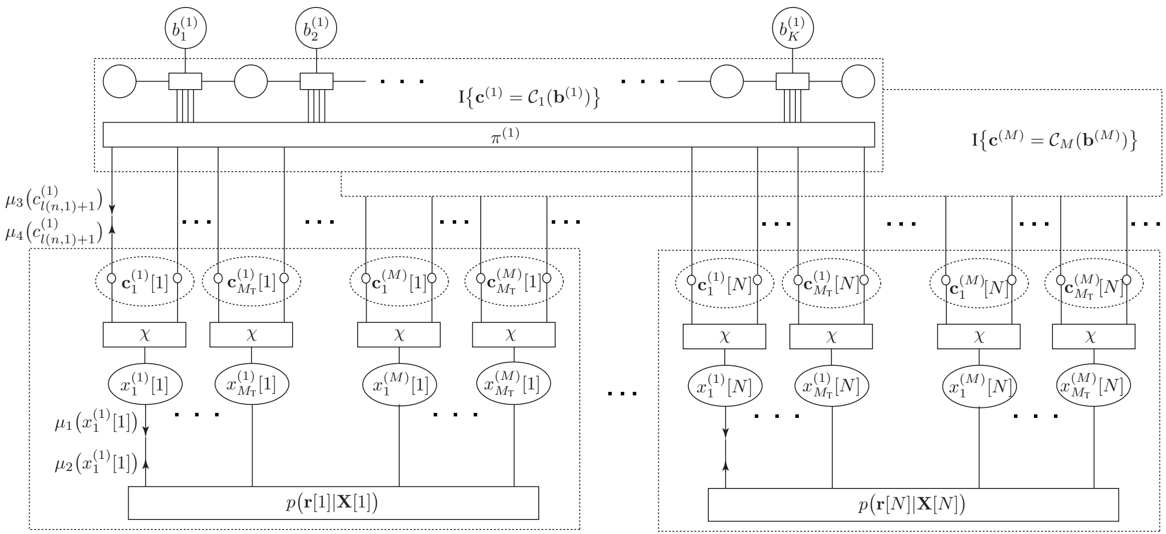

Fig. 2. Factor graph for a MIMO-IDMA system with convolutional encoding and higher-order modulation.

graph with cycles as in Fig. 2, the sum-product algorithm can still be used but it generally becomes iterative, yields only approximate results, and requires suitable message scheduling. In what follows, we calculate the messages to be propagated along the edges of our factor graph according to the update rules of the sum-product algorithm [6]. Because the code bit variable nodes $c_{l(n,i)+j}^{(m)}$ and the transmit symbol variable nodes $x_i^{(m)}[n]$ are connected to only two neighboring factor nodes, they just pass the messages from one neighboring factor node to the other. Thus, we only need to consider the message updates for the factor nodes.

For the code factor nodes $\mathrm{I}\{\mathbf{c}^{(m)} = \mathcal{C}_m(\mathbf{b}^{(m)})\}$ in Fig. 2, the sum-product algorithm amounts to the BCJR algorithm for soft-decoding the convolutional code [6], [10], while the repetition code is soft-decoded by summing the *a priori* LLRs of successive bits (after interleaving). The LLRs produced by the overall soft channel decoder are the sum of extrinsic LLRs and prior LLRs [2]. The extrinsic LLRs, denoted by $\xi_l^{(m)}$, correspond to messages (beliefs) [6], [7], [9]

$$\mu_3(c_l^{(m)}) \;=\; \frac{\exp(\xi_l^{(m)} c_l^{(m)})}{1+\exp(\xi_l^{(m)})}\,, \quad c_l^{(m)} \in \{0,1\} \tag{8}$$

that leave the code factor node $\mathrm{I}\{\mathbf{c}^{(m)} = \mathcal{C}_m(\mathbf{b}^{(m)})\}$ and are propagated to the code variable nodes $\mathbf{c}_i^{(m)}[n]$ and further to the modulator factor node χ.

Again using the sum-product algorithm, the message $\mu_1(x_i^{(m)}[n])$ passed from the modulator factor node χ to the variable node $x_i^{(m)}[n]$ and further to the channel factor node $p(\mathbf{r}[n]|\mathbf{X}[n])$ is obtained from the messages $\mu_3(c_l^{(m)})$ as

$$\begin{aligned}\mu_1(x_i^{(m)}[n]) \;&=\; \sum_{\mathbf{c}_i^{(m)}[n]} \mathrm{I}\{x_i^{(m)}[n] = \chi(\mathbf{c}_i^{(m)}[n])\}\, \bar{\mu}_3(\mathbf{c}_i^{(m)}[n]) \\ &=\; \bar{\mu}_3(\chi^{-1}(x_i^{(m)}[n]))\,,\end{aligned} \tag{9}$$

where $\sum_{\mathbf{c}_i^{(m)}[n]}$ denotes summation over all the 2^B symbol labels and $\bar{\mu}_3(\mathbf{c}_i^{(m)}[n]) = \prod_{j=1}^{B} \mu_3(c_{l(n,i)+j}^{(m)})$.

The message $\mu_2(x_i^{(m)}[n])$ passed from the channel factor node $p(\mathbf{r}[n]|\mathbf{X}[n])$ to the variable node $x_i^{(m)}[n]$ and further to the modulator factor node χ is obtained as

$$\mu_2(x_i^{(m)}[n]) \;=\; \sum_{\sim x_i^{(m)}[n]} p(\mathbf{r}[n]|\mathbf{X}[n]) \prod_{(i',m')\neq(i,m)} \mu_1(x_{i'}^{(m')}[n])\,, \tag{10}$$

where $\sum_{\sim x_i^{(m)}[n]}$ denotes summation with respect to all entries of $\mathbf{X}[n]$ except $x_i^{(m)}[n]$.

Finally, the message $\mu_4(c_l^{(m)})$ passed from the modulator factor node χ to the code variable node $\mathbf{c}_i^{(m)}[n]$ and further to the code factor node $\mathrm{I}\{\mathbf{c}^{(m)} = \mathcal{C}_m(\mathbf{b}^{(m)})\}$ is obtained as

$$\begin{aligned}\mu_4(c_{l(n,i)+j}^{(m)}) \;&=\; \sum_{\sim c_{l(n,i)+j}^{(m)}} \mathrm{I}\{x_i^{(m)}[n] = \chi(\mathbf{c}_i^{(m)}[n])\}\, \mu_2(x_i^{(m)}[n]) \\ &\qquad\qquad \times \prod_{j'\neq j} \mu_3(c_{l(n,i)+j'}^{(m)}) \\ &=\; \sum_{\sim c_{l(n,i)+j}^{(m)}} \mu_2(\chi(\mathbf{c}_i^{(m)}[n])) \prod_{j'\neq j} \mu_3(c_{l(n,i)+j'}^{(m)})\,.\end{aligned} \tag{11}$$

Combining (9) and (10) and inserting the result into (11) yields a message update that takes the code bit beliefs $\mu_3(c_l^{(m)})$ from the channel decoder and yields refined code bit beliefs $\mu_4(c_l^{(m)})$. Hence, these message updates taken together can be thought of as a soft-in/soft-out MIMO multiuser detector.

Since (9) and (11) involve only one antenna of one user, the overall complexity of the sum-product algorithm is dominated by (10). Indeed, the complexity of calculating $\mu_2(x_i^{(m)}[n])$ is exponential in the number of transmit antennas M_{T} and in the number of users M because the sum in (10) involves $|\mathcal{S}|^{M_{\mathrm{T}}M-1}$ terms. For example, $|\mathcal{S}|^{M_{\mathrm{T}}M-1} \approx 2.7\cdot 10^8$ for four users with two transmit antennas and 16-QAM modulation.

IV. Low-Complexity Receiver

We will next derive a multiuser detector whose complexity is only linear in the number of users.

A. Gaussian Approximation

To simplify (10), we approximate the beliefs $\mu_1\big(x_i^{(m)}[n]\big)$ by Gaussian distributions with the same means and variances as those of $\mu_1\big(x_i^{(m)}[n]\big)$, i.e.,

$$\tilde{\mu}_1\big(x_i^{(m)}[n]\big) \;\propto\; \exp\!\bigg(-\,\frac{|x_i^{(m)}[n]-m_i^{(m)}[n]|^2}{\sigma_i^{(m)2}[n]}\bigg).$$

Using (9), the means and variances are obtained as

$$m_i^{(m)}[n] \;=\; \sum_{x_i^{(m)}[n]} x_i^{(m)}[n]\,\bar{\mu}_3\big(\chi^{-1}(x_i^{(m)}[n])\big)\,,$$
$$\sigma_i^{(m)2}[n] \;=\; \sum_{x_i^{(m)}[n]} \big|x_i^{(m)}[n]-m_i^{(m)}[n]\big|^2\,\bar{\mu}_3\big(\chi^{-1}(x_i^{(m)}[n])\big)\,.$$

Replacing in (10) $\mu_1\big(x_i^{(m)}[n]\big)$ with $\tilde{\mu}_1\big(x_i^{(m)}[n]\big)$ and the corresponding summation with an integration allows us to derive the following closed-form approximation to $\mu_2\big(x_i^{(m)}[n]\big)$:

$$\tilde{\mu}_2\big(x_i^{(m)}[n]\big) \;\propto\; \exp\Big(-\big(\mathbf{r}[n]-\mathbf{h}_i^{(m)}[n]x_i^{(m)}[n]-\mathbf{m}_i^{(m)}[n]\big)^H \times\big(\mathbf{C}_i^{(m)}[n]\big)^{-1}\big(\mathbf{r}[n]-\mathbf{h}_i^{(m)}[n]x_i^{(m)}[n]-\mathbf{m}_i^{(m)}[n]\big)\Big), \quad (12)$$

with mean vector

$$\mathbf{m}_i^{(m)}[n] \;=\; \sum_{(i',m')\neq(i,m)} \mathbf{h}_{i'}^{(m')}[n]\,m_{i'}^{(m')}[n]$$

and covariance matrix

$$\mathbf{C}_i^{(m)}[n] \;=\; \sigma^2\mathbf{I} \,+\, \sum_{(i',m')\neq(i,m)} \sigma_{i'}^{(m')2}[n]\,\mathbf{h}_{i'}^{(m')}[n]\,\mathbf{h}_{i'}^{(m')H}[n]$$
$$=\; \mathbf{C}_r[n] \,-\, \sigma_i^{(m)2}[n]\,\mathbf{h}_i^{(m)}[n]\,\mathbf{h}_i^{(m)H}[n]\,. \quad (13)$$

Here,

$$\mathbf{C}_r[n] \;=\; \sigma^2\mathbf{I} + \sum_{i=1}^{M_{\mathrm{T}}}\sum_{m=1}^{M} \sigma_i^{(m)2}[n]\,\mathbf{h}_i^{(m)}[n]\,\mathbf{h}_i^{(m)H}[n]$$

is the current estimate of the covariance matrix of $\mathbf{r}[n]$.

Hence, the (exponentially complex) computation of (10) is replaced with the computation of (12). According to (13), the $M_{\mathrm{T}}M$ covariance matrices $\mathbf{C}_i^{(m)}[n]$ are rank-one updates of $\mathbf{C}_r[n]$. Thus, they can be efficiently inverted via Woodbury's identity [11]. The overall complexity of computing $\tilde{\mu}_2\big(x_i^{(m)}[n]\big)$ can be shown to scale linearly with the number of users and cubically with the number of transmit antennas.

B. Selective Message Updates

To further reduce computational complexity, we propose a selective message update scheme that avoids the computation of updated beliefs for code bits with high reliability. The code bit reliabilities are measured via the posterior LLRs

$$\tilde{\xi}_l^{(m)} = \log\frac{\mu_3(c_l^{(m)}\!=0)\,\mu_4(c_l^{(m)}\!=0)}{\mu_3(c_l^{(m)}\!=1)\,\mu_4(c_l^{(m)}\!=1)}\,.$$

If $|\tilde{\xi}_l^{(m)}|$ exceeds a prescribed threshold, the corresponding message $\mu_4(c_l^{(m)})$ is not updated (i.e., the value from the previous iteration is reused). The idea is that, as the sum-product iterations progress, the code bit reliabilities improve and hence fewer and fewer message updates have to be performed. We note that such a selective message update can be viewed as a specific scheduling [7] of the sum-product algorithm that adapts dynamically to the current bit reliabilities.

The choice of the threshold affects both the number of message updates to be done and the performance of the sum-product algorithm (convergence behavior and final bit error rate). Since the LLRs generally increase with the SNR, the threshold has to be adapted to the SNR. The impact of the LLR threshold on the performance and complexity of the receiver will be studied experimentally in Section V.

C. Overall Receiver Structure

The sum-product algorithm developed above can be interpreted as an iterative turbo receiver structure. The dotted boxes in the lower part of Fig. 2 correspond to a soft-in/soft-out MIMO multiuser detector that exchanges bit reliability information with M parallel single-user soft-in/soft-out channel decoders (the dotted boxes in the upper part of Fig. 2).

The proposed receiver uses parallel message scheduling [7], i.e., the extrinsic LLRs $\xi_l^{(m)}$ for all users are simultaneously updated by the channel decoders, converted to beliefs $\mu_3(c_l^{(m)})$ via (8), and then used by the multiuser detector to calculate refined messages $\mu_4(c_l^{(m)})$ for all users concurrently.

When the sum-product algorithm is terminated after a predefined number of iterations, the signs of the *a posteriori* LLRs of the information bits (computed by the channel decoder) provide the final bit decisions $\hat{b}_k^{(m)}$ approximating (2).

V. Simulation Results

We next illustrate the performance of the proposed MIMO-IDMA receiver.

A. Simulation Setup

We simulated a MIMO-IDMA system with $M=2$ users, $M_{\mathrm{T}}\!=\!2$ antennas per user, and $M_{\mathrm{R}}\!=\!2$ base station antennas. The number of information bits was $K=512$. A terminated rate-$1/2$ convolutional code (code polynomial $[23\ 35]_8$) serially concatenated with a rate-$1/2$ repetition code was used; thus, the overall code rate was $R=1/4$. The interleavers were randomly generated for each data block. The interleaved code bits were mapped to 16-QAM symbols, yielding a total number of $N = K/(RM_{\mathrm{T}}B) = 256$ transmit vectors per user. The sum rate of this system is 4 bits per channel use. The channel matrices $\mathbf{H}^{(m)}[n]$ of size 2×2 were generated independently for each n (fast fading channel), with elements that were i.i.d. Gaussian with zero mean and unit variance.

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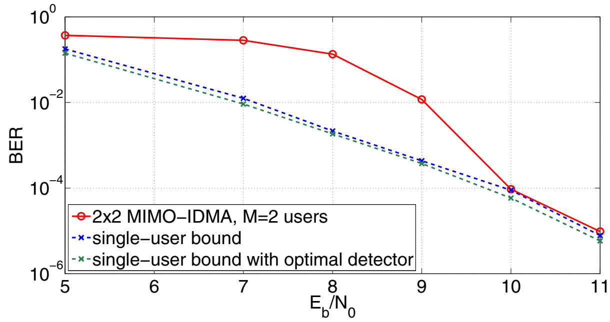

Fig. 3. Average BER versus SNR E_b/N_0 for a 2×2 MIMO-IDMA system with $M=2$ users, after 10 iterations of the low-complexity receiver without selective message updates. The single user bound and the single user bound for the optimal receiver cf. (10) are shown for comparison.

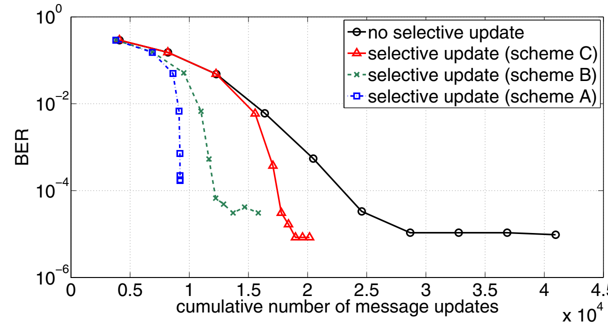

Fig. 4. BER of a 2×2 MIMO-IDMA system with $M=2$ users, at an SNR of 11 dB, versus number of message updates for the same system as in Fig. 3 and different selective message update schemes. The markers on the curves indicate iteration cycles.

B. Results

We first study the performance of the proposed low-complexity iterative receiver algorithm without selective message updates. Fig. 3 shows the bit error rate (BER) after 10 iterations, averaged over the two users, versus the signal-to-noise ratio (SNR) E_b/N_0. It is seen that the receiver features the typical "turbo-behavior," with an SNR of more than 8 dB required for convergence, and a waterfall region above that SNR. For SNR $\geq$ 10 dB, our receiver performs close to the single user bound. We also show the BER of the optimal receiver (cf. (10)) for $M=1$ user. The proposed low-complexity receiver (single user bound) performs almost as well, which justifies the approximations that led to (12).

Next, we consider the low-complexity receiver with selective message updates, at an SNR of $E_b/N_0 = 11$ dB. We compare three different schemes: schemes A and C use a constant LLR threshold of 5 and 30, respectively, while scheme B uses an LLR threshold that increases linearly from 5 (first iteration) to 30 (10th iteration). Scheme B is motivated by the fact that the LLRs tend to increase with the iterations. Fig. 4 shows the BER versus the complexity (number of message updates) for schemes A, B, C and for the receiver without selective message updates. It is seen that the selective message update offers a very favorable performance–complexity tradeoff.

Scheme A exhibits the quickest BER decrease, but saturates at a BER slightly above 10^{-4} and a complexity of about 9000 message updates. The last iterations reduce the BER only slightly but at the same time require only very few message updates since most posterior LLR magnitudes are already larger than 5. The behavior of scheme C initially equals that observed without selective message update. Eventually, however, LLR thresholding sets in and the further BER decrease (down to below 10^{-5}) is achieved with significantly less complexity than without selective update. The behavior of scheme B is intermediate between those of schemes A and C, with quick initial BER decrease and saturation at reasonably low BER.

To achieve a target BER of 10^{-4} (or better), the method without selective update requires six iterations with almost $2.5\cdot10^4$ message updates. Scheme B also requires six iterations but only $1.2\cdot10^4$ message updates, corresponding to computational savings of about 50%.

VI. Conclusion

Based on a factor graph framework and the sum-product algorithm, we have developed a computationally efficient MIMO-IDMA receiver suitable for higher-order modulation. A further reduction of complexity has been achieved by a selective message update scheme that allows an easy complexity-performance tradeoff. The proposed system can be extended in various ways. In particular, the simple convolutional code can be replaced by more sophisticated codes such as LDPC codes [12], which can be optimized for a given receiver. An analytical study of the selective message update scheme is an interesting topic for further research.

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