Semi-blind separation and detection of co-channel signals

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Abstract - **We propose a semi-blind algorithm for smart antennas that, in addition to some structural signal properties, utilises user identifiers that are available in existing cellular systems. Our algorithm estimates, first, the row span of the adequately composed data matrix and then projects the obtained basis vectors iteratively to the finite alphabet (FA) constellation. We call the projection part of our algorithm DILSF (Decoupled Iterative Least-Squares with Subspace Fitting). Assuming two co-channel users in a GSM-like TDMA system and a Rayleigh fading channel with finite angular spread, we obtained a BER<10^{-3} with only 3 antennas, spaced 10λ apart. We also demonstrate that co-channel signals are resolvable even if they are not separable in angle. We gain a factor of up to 70 in computational complexity by employing subspace tracking algorithms and demonstrate the robustness of the method against array imperfections.**

I. Introduction

A traditional class of adaptive antenna systems is based on the direction-of-arrival (DOA) estimation at the base station using algorithms like MUSIC [1], ESPRIT [2] or its more recent extensions [3]. These algorithms estimate the directions of the incoming signal wavefronts and use beamforming techniques to collect all discrete components of the desired signal. Correspondingly the nulls of the antenna pattern are pointed towards the interfering wavefronts. However, in practical environments the incoming wavefronts spread due to reflections in the vicinity of the mobile, which disturbs the plane wave assumption and impairs performance. Additionally, the users must be separable in angle to ensure adequate co-channel interference (CCI) suppression.

Utilisation of temporal reference (TR) algorithms, which are typically based on the well-known training sequence assisted least-square adaptation, is another possibility. They require accurate synchronisation of the incoming signals before weight adaptation.

Recently, the signal processing community has focused on blind source separation and signal detection *without* directional information. With these techniques, the channel response matrix, which maps the transmitted signals to the received array data samples, can be identified without the aid of known bit sequences. Using the combined estimation principle, these methods can yield the estimated symbol sequences directly.

Blind estimation techniques use several structural signal properties: The *fixed symbol rate* allows the factorisation of the received data into the channel response and signal matrices by means of their special structures. Utilisation of the *finite alphabet*, i.e. the limited number of modulation symbols, together with the first property allows solving of the FIR-MIMO (Finite Impulse Response, Multiple Input, Multiple Output) problem. Other useful features enabling blind estimation of the commonly used communication signals are e.g. *cyclostationarity* [4], *constant modulus* [5], *spectral self-coherence* [6] or *higher order statistical properties* [7].

From a mobile communications point of view we see several benefits that motivate the use of blind estimation methods:

- Neither discrete DOAs nor angular separability are required
- Robustness against receiver imperfections - relaxed requirements for array calibration
- No synchronisation requirements for the incoming signals
- Reduced need for overhead information - additional capacity increase

We have previously analysed a performance of the totally blind estimation method [8]. In the present paper we describe a *semi-blind* algorithm based on the same principles, but now employing the user identification fields to initialise the iterative least squares estimation. We have already introduced this idea in [9] but the present paper shows performance in several challenging simulation scenarios.

The reason to focus on the semi-blind method instead of totally blind estimation is twofold. First, in a SDMA (Space Division Multiple Access) system, where several users are served simultaneously in the same traffic channel, some kind of known information must be included in the transmitted signals for the purpose of user identification. This enables to assign the detected symbol sequences to the correct users. Second, the frame structures of all current TDMA-based mobile systems contain training sequences for the channel estimation. It is reasonable to utilise this information already during the estimation process, not only after detection of the symbol vectors.

The paper is organised in the following way: Sec.2. explains the basic idea of the proposed algorithm. The parts of the estimation: joint space-time equalisation and semi-blind DILSF (Decoupled Iterative Least-Squares with Subspace Fitting) algorithm are discussed in Sec.3. and Sec.4., respectively. The subsequent Sec.5. introduces the

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simulation environment and the results. The summary in Sec.6. concludes the paper.

We use the following notation throughout the paper. A^*, $A^\#$ and $\|A\|$ denote the Hermitian transpose, Moore-Penrose pseudo-inverse and the Frobenius norm of the matrix A, respectively. The notation row(A) denotes the row span of A.

II. PRINCIPLE OF THE ALGORITHM

Our semi-blind estimation method yields directly the desired symbol vectors by performing three tasks: joint space-time equalisation, separation, and detection of the multiple oversampled co-channel digital signals. Fig.1. illustrates the principle of the technique.

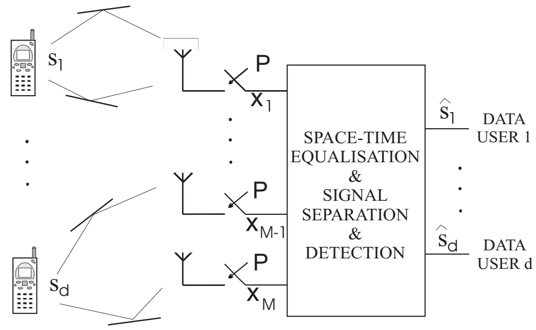

Fig.1. Structure of a semi-blind estimator

The estimation process consists of two parts. First we estimate the basis of the row span for the received data matrix. After that we project these obtained basis vectors to the finite alphabet constellation in our DILSF algorithm. Between these two steps we initialise the FA projections with user identification fields. This leads to fast convergence and an increased robustness of DILSF. During the estimation process we utilise the fixed symbol rate (FSR) of the incoming signals and the finite alphabet (FA) structure of the symbols. Fig. 2. shows the parts of the entire semi-blind estimation process.

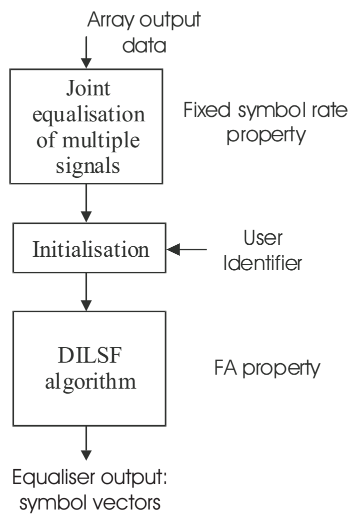

Fig.2. Semi-blind detection

III. JOINT SPACE-TIME EQUALISATION

Joint space-time equalisation based on subspace estimation is the first part of the detection process (first block in Fig.2.). The details can be found in [10],[9] and references therein. Thus we summarise only the basic principle.

An array of M antennas receives d digital signals $s_1(t),...,s_d(t)$ transmitted over the independent channels $h_{ij}(t)$ (connecting each source and antenna element) which have a maximal length of L. Each antenna element is oversampled with a rate P and the data matrix X is constructed by collecting data over N symbol periods. The n-th column of this matrix contains $M{\cdot}P$ samples received during the transmission of the n-th symbol. Our blind estimation problem is to find the factorisation, $X=HS$, for the received data matrix X (size: $MP{\times}N$), where the matrix H (size: $MP{\times}dL$) represents the unknown space-time channel and the matrix S (size: $dL{\times}N$) contains the transmitted symbols.

The estimation principle is based on the equality of row(X) and row(S). This requires the channel response matrix H to have full rank. Because this may cause undue requirements on the number of antenna elements, M, or oversampling rate, P, we create an extended data matrix $\mathcal{X}$ to ensure this full-rank property. This matrix is constructed by left-shifting and stacking the original data blocks m times

$$\mathcal{X} = \begin{bmatrix} \mathbf{x}_0 & \mathbf{x}_1 & \iddots & \mathbf{x}_{N-m} \\ \mathbf{x}_1 & \mathbf{x}_2 & \iddots & \iddots \\ \iddots & \iddots & \iddots & \mathbf{x}_{N-2} \\ \mathbf{x}_{m-1} & \iddots & \iddots & \mathbf{x}_{N-1} \end{bmatrix},$$

where $\mathbf{x}_n$ denotes the n-th column of the original data matrix and the stacking parameter m can be regarded as the length of the equaliser in time-domain. The extended data matrix has a factorisation $\mathcal{X}=\mathcal{H}\mathcal{S}$, where $\mathcal{H}$ (size: $mMP{\times}d(L+m-1)$) and $\mathcal{S}$ (size: $d(L+m-1){\times}N-m+1$) are the block-matrices to be determined during the subsequent steps of the algorithm. The data matrix $\mathcal{X}$ is converted to the real-valued form using a derotation technique [14] which allows to perform subsequent steps by using only real-valued operations.

The first step is to estimate the orthonormal basis of the row span of $\mathcal{X}$, which is equal to that of $\mathcal{S}$ assuming that the full-rank property of the channel matrix is fulfilled. The optimal way to do this is to perform singular value decomposition (SVD) of the data matrix, $\mathcal{X}=U\Sigma V$ [11]. The first $d_{\mathcal{X}}$ rows of V form an orthonormal basis for the row span of $\mathcal{X}$, where the rank of $\mathcal{X}$, $d_{\mathcal{X}}$, satisfies the equation $d_{\mathcal{X}}=d(L+m-1)$. This allows efficient estimation of the number of incoming signals, as shown in [10].

The use of the SVD for subspace estimation yields the optimal results, but it is the computationally most expensive part of the algorithm. In order to relax this complexity we investigated suboptimal but cheaper *adaptive subspace tracking* algorithms. With these techniques the subspace estimate is updated iteratively over the columns or the rows of the matrix instead of considering it as a whole. We compare the performance of the original SVD and estimation using PAST (Projection Approximation Subspace Tracking) and

PASTd (Projection Approximation Subspace Tracking with deflation) [12]. These tracking techniques convert an originally fourth order cost function to the second order problem and minimise it using a least-squares approximation. In case of row span estimation, we have to apply PAST and PASTd on the complex conjugated rows of the $\mathcal{X}$ matrix and continue iterations over all rows. The forgetting factor is set to unity; i.e. it does not play a role in case of row span estimation. The decomposition of $\mathcal{X}$, with size $mMP\times(N-m+1)$, requires $2mMP\cdot[3\cdot(N-m+1)\cdot d_{\mathcal{X}} + d_{\mathcal{X}}^2]$, $2mMP\cdot[4\cdot(N-m+1)\cdot d_{\mathcal{X}}+d_{\mathcal{X}}]$ or $3\cdot(2mMP)^2\cdot(N-m+1)+10\cdot(2mMP)^3$ operations in case of PAST, PASTd and SVD, respectively [11]. In these equations the rank of the matrix $\mathcal{X}$, $d_{\mathcal{X}}$, corresponds to the number of desired eigencomponents. The least squares approximation used destroys the exact orthonormality of the obtained basis vectors. Because the subsequent part of our algorithm requires an orthogonal basis, we have to perform an additional orthonormalisation step once after the last iteration. We used a modified Gram-Schmidt method for this purpose [11].

Introducing n ($1 \leq n \leq L+m-1$) intersections of the shifted versions of the row($\mathcal{X}$), we can reduce the dimensionality of the problem. In case of full intersections, the dimensionality can be decreased to the number of incoming signals, i.e. $\delta=d$. However, as discussed in detail in [10], with ill-defined noisy channels a smaller number of intersections gives better performance. The output of the subspace intersection part is the δ-dimensional basis for row($\mathcal{S}$).

IV. Semi-blind estimation

After the subspace estimation, we have to define which linear combination of these basis vectors gives a finite alphabet structure. For this purpose we can employ an iterative least-squares technique [13], [15]. However, the performance of the projection algorithms is crucially dependent on the accuracy of the initialisation. Especially when GMSK modulation is used, trivial initialisation must be improved before the FA projections. In [14] and [8] the constant modulus property of the signals serves this purpose, by applying the analytical constant modulus algorithm (ACMA) [5]. In contrast, we propose to employ *user identifiers* to initialise the iterative projections. Thus, we can omit the ACMA part even though nonlinear GMSK modulation is used. In addition to the reduced computational burden, this improves the robustness of the estimation.

A. DILSF Algorithm

We call our projection algorithm Decoupled Iterative Least Squares with Subspace Fitting, DILSF. It combines the ideas of the DWILSP (Decoupled Weighted Iterative Least Squares with Projections) [15] and the ILSF (Iterative Least Squares with Subspace Fitting) algorithms [10]. Instead of a simultaneous iteration of all symbol vectors, this approach makes the projections between the FA constellation and the obtained subspace *separately* for each user, which has desirable properties.

The goal of the alternating least square projections is to find symbol vectors s_i (belonging to the FA constellation) and corresponding projection vectors t_i, so that the mean square error $_{s,t;s\in FA}\|s_i\text{-}t_i\cdot Y\|^2$ is minimised for each user. The input matrix Y contains δ orthogonal basis vectors from the preceding part of the algorithm. During the iterations we need only the pseudo-inverse of the input matrix Y, which remains constant over all iterations.

The initialisation of the fitting vectors, $t_i^{(0)}$, by user identification fields ensures convergence to the global minimum. The correct initialisation block Y_{UID} of the input matrix is selected by correlating basis vectors and user identifier. We obtain then the initialisation value, $t_i^{(0)}$, as a least-squares solution using block Y_{UID} and the identifier of user i. We continue iterations until convergence is reached and after that we begin the estimation of the next desired signal. The algorithm converges very fast; in most simulations the correct symbol vectors were obtained already after the first iteration round.

TABLE 1:
SOLVING $min_{s,t;s\in FA}\|s_i\text{-}t_i\cdot Y\|^2$ FOR EACH DESIRED USER i USING DILSF-ALGORITHM

for $i = 1{:}d$
initialise t_i using user identifier: $t_i^{(0)} = s_{UID,i}\cdot Y_{UID}^{\#}$
for $k = 1,2, ...$
i) $s_i^{(k)} = proj_{\Omega}[\, t_i^{(k-1)}\cdot Y]$
ii) $t_i^{(k)} = s_i^{(k)}\cdot Y^{\#}$
* repeat until $(t^{(k)}, s^{(k)}) = (t^{(k-1)}, s^{(k-1)})$
* repeat for all desired signals

Table 1 shows the DILSF algorithm in our special case, where the rows of the input matrix Y form an orthonormal basis. In the general case, if decoupled projections are used without preceding subspace estimation, a prewhitening step can be included in the algorithm, following the idea presented in [15].

V. Simulations

A. Channel Model

We carried out the simulations using the directional Geometry-based Stochastic Channel Model, GSCM [16],[17]. In the model the users are surrounded by local scattering areas. Environments causing a larger delay and angular spread are modelled by additional scattering areas at random positions. Physically these far scatterers correspond to the signal echoes e.g. from high-rise buildings or nearby mountains. The basic principle of our channel model is shown in Fig.3.

Appropriate parameter selection allows simulations of the different propagation environments. In this paper we considered the urban environment with base station antennas above rooftop level. This parameter selection, described detailed in [9], gives an averaged angular spread of about 3 degrees around each nominal direction-of-arrival (DOA).

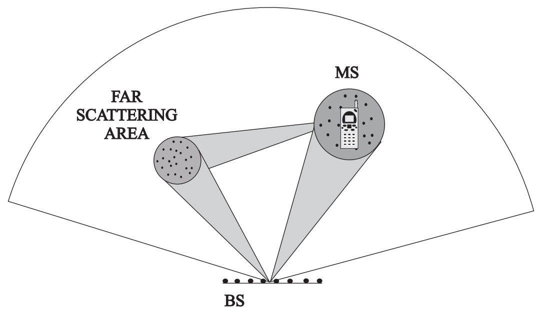

Fig.3. Principle of the channel model

Our main interest in this paper is to consider the BER performance. Thus we average the results over different mobile and scattering area positions, which we assume to be uniformly distributed over the cell area. To guarantee the statistical reliability of our results we randomly selected new mobile, scattering area and scattering point positions for each timeslot.

B. Parameters

Our simulations refer to the GSM radio interface [18] (with minor modifications related to the training sequences [9]) and use a linear approximation of GMSK [19]. Note, however, that the same estimation principle is also applicable to any other system fulfilling the required signal properties.

The results of our simulations are raw bit error rates (BER) as a function of the input SNR. In all simulated cases we assumed two co-channel SDMA users and averaged the BER over both of them. The signal-to-noise ratio values were defined by the received mean power values averaged over a large number of random channel situations. Table 2 shows the used parameter selection of the algorithm.

TABLE 2:
SIMULATION PARAMETERS

N = Number of Snapshots	90
P = Oversampling Rate	2
M = Number of Antennas	2, 3, 4, 6, 8
m = Equaliser Length (stacking factor)	5
n = Number of Intersections	$n = \hat{L}+m-2$

C. BER Performance

Fig. 4 demonstrates the BER performance of the above-described technique when the number of antenna elements in the uniform linear array (ULA) with λ/2 antenna spacing was varied and the singular value decomposition was used for subspace estimation.

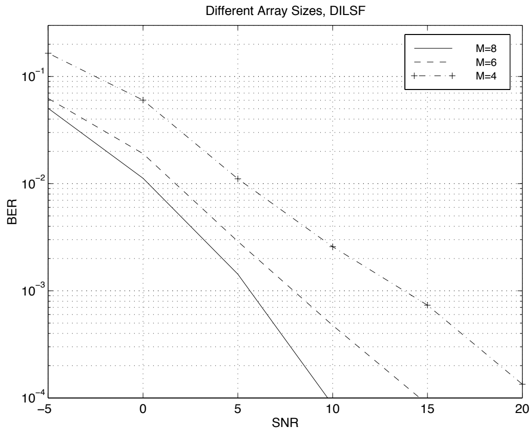

Fig.4. BER with uniform linear array, $\lambda/2$ spacing, M=4, 6, 8

Fig. 5 compares the performance when the number of antenna elements was fixed (M=8) and the required subspace in the space-time equalisation part was estimated using SVD, PAST and PASTd algorithms. The tracking approach worsens the BER performance slightly, but the computational complexity decreases by a factor of 70 with this parameter selection. This reduction factor becomes smaller with less antenna elements, because the complexity of the SVD is strongly dependent on the number of the matrix rows. Additionally, the BER of tracking schemes saturates then at a higher level. This is due to the fact that reduced number of rows leads to a smaller number of iterations with tracking techniques and thus poorer estimation accuracy.

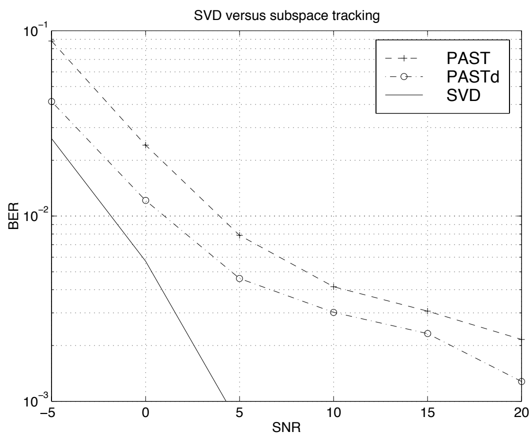

Fig. 5. BER comparison between PAST, PASTd and SVD, λ/2 spacing, M=8

Fig. 6 shows the performance when the number of antenna elements was decreased but the element spacing was increased correspondingly. The inter-element spacing was selected so that the overall length of the array was in the range of 20λ. Thus the element spacing was 20λ, 10λ and 7λ corresponding to the array sizes of M=2, M=3 and M=4 elements, respectively. The larger distance reduces the correlation between fading signals received by the independent elements and thus improves performance compared to the situation considered in Fig. 4.

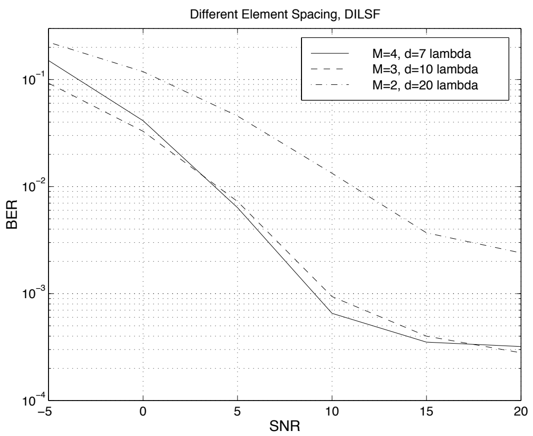

Fig.6. BER with different element spacing, M=2, 3, 4

The performance shown in these figures is very promising especially concerning hardware realisation. Simulations show that already a small number of elements with two times oversampling gives sufficient performance. Increasing the number of independent samples by using an array with more elements improves performance further.

D. Special Simulation Scenarios

Angular separability:

Traditional DOA-estimation algorithms are based on the Vandermonde structure [11] of the multichannel matrix, which is used for the estimation of the propagation angles of the incoming wavefronts. Thus, detection of different signals is possible if they are separable in the angular domain. Our approach is based on the estimation of the unknown channel response matrix, and thus also angularly overlapping signals are separable in case that their channel response matrices are different.

To demonstrate this property we created the following modified channel scenario. Instead of random positioning of the mobiles we placed both users close to one another (same MS-BS distance, DOA difference 2°). Additionally we used the same scattering points for both users (only local scatterers were present). Thus, in this worst case scenario (Fig. 7.) all signal components transmitted by two co-channel users were propagating via the same scattering points before arriving at the base station array.

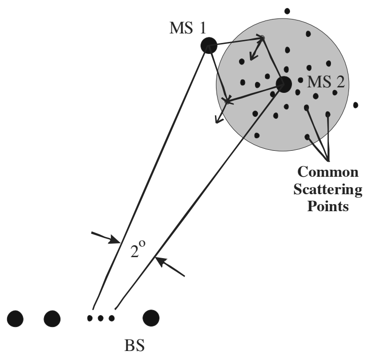

Fig. 7. Common multipath scenario

Definitely no DOA estimation algorithm can cope with this scenario, but the different channel response matrices allowed signal separation by our algorithm. Fig. 8 shows the BER performance of this scenario as the broken line for a λ/2 array with 6 elements. The solid line shows the situation in which we added independent far scattering areas for both users, but local contributions propagated still via the common scattering points. The figure shows that these independent multipath components increased the diverseness of the channel response matrices, and the BER curve approaches the corresponding simulation with the normal channel model (Fig. 4).

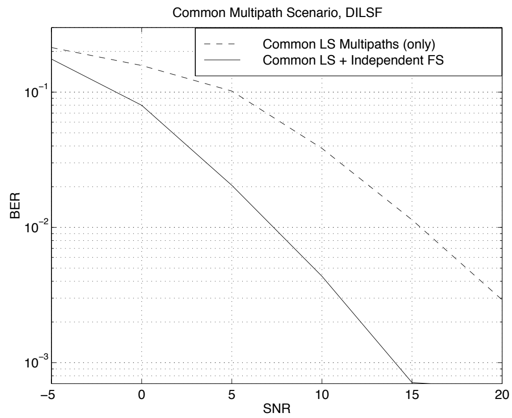

Fig. 8. BER with common multipath scenario, λ/2 spacing, M=6

Effect of Array Imperfections:

One benefit of blind techniques is the robustness against array imperfections, because these methods are not relying on a known array manifold. We demonstrated this by multiplying the received data samples with an error model containing hardware imperfections. Our error model included all the important deviations from ideal hardware like receiver gain and phase imbalances, mutual coupling, oscillator drifts, I/Q imbalances, quantisation noise and cable length differences. We performed comparative simulations [20] between spatial reference techniques and the algorithm described in this paper. Our results show that SR techniques suffer from imperfections disturbing array manifold structure whereas the semi-blind technique was much more robust. Our algorithm was insensitive against all errors affecting the phase of the received signal, because the phase errors can be hidden in the estimated channel impulse responses. The only imperfection considered that have an effect on the semi-blind receiver was amplitude imbalances between different receiver trains. In this simulation the received samples at antenna outputs were multiplied by different magnitude factors taken from a Gaussian distribution and new values were randomly selected every tenth simulation round. However, this impairment effect was only observed with strong imbalance values exceeding a mean value of 5dB (Fig. 9). The reason for this behaviour is that the singular vectors spanning the required subspace are disturbed by multiplicative magnitude factors.

VI. SUMMARY AND CONCLUSIONS

In this paper we presented a semi-blind estimation principle. First we perform joint space-time equalisation for all incoming signals and estimate a basis for the row span of the data matrix. After that we separate the signals by using the finite alphabet constellation (FA). We also described a method for the required projections, called DILSF (Decoupled Iterative Least Squares with Subspace Fitting) algorithm. This technique is based on the decoupled iterative least-squares projections between obtained subspace and finite alphabet constellation and it requires only matrix-vector multiplications during the iteration rounds. For the initialisation of these projections we use the system specific user identification fields included in the slot structure.

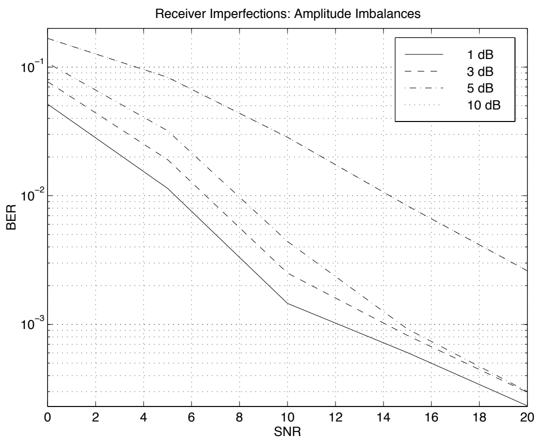

Fig. 9. BER with magnitude imbalances between receiver trains, λ/2 spacing, M=6

We evaluated the performance of the proposed algorithm by simulations with a realistic directional channel model using different array structures. Our results show that increasing the distance between antenna elements improves performance because of the diminished correlation of the signals in the independent sensors. This enables signal separation and detection also with small numbers of antennas, leading to reduced dimensions of the data matrices and less stringent hardware requirements**.** We also replaced the singular value decomposition of the subspace estimation part with computationally more efficient subspace tracking methods and considered the trade-off between complexity and performance. Utilisation of tracking methods leads to an error floor, but it still gives reasonable bit error rate performance. Tracking techniques decrease the number of required operations by up to a factor of 70. We obtained promising bit error rate performance also in a scenario where signals were not separable in angle. By comparative simulations with spatial reference techniques we demonstrated that our semi-blind detector was much more robust against different hardware imperfections.

ACKNOWLEDGEMENTS

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REFERENCES

[1] R.O. Schmidt, Multiple Emitter Location and Signal Parameters Estimation, IEEE Trans. on Antennas and Prop., vol.34, pp. 276-280, March 1986

[2] R. Roy, A. Paulraj, T. Kailath, ESPRIT - A Subspace Rotation Approach to Estimation of Parameters of Cisoids in Noise, IEEE Trans. on Acoustics, Speech and Signal Proc., vol.32, pp. 1340-1342, May 1986

[3] M. Haardt, J.A. Nossek, Unitary ESPRIT: How to Obtain an Increased Estimation Accuracy with a Reduced Computational Burden. IEEE Trans. on Signal Proc., vol.43, pp.1232-1242, May 1995

[4] L. Tong, G. Xu, T. Kailath, Blind Identification and Equalization Based on Second-order Statistics: A Time Domain Approach, IEEE Trans. on Information Theory, vol. 40, pp. 340-349, March 1994

[5] A.J. van der Veen, A. Paulraj, An Analytical Constant Modulus Algorithm. IEEE Trans. on Signal Proc., vol.44, pp. 1136-1155, May 1996

[6] B.G. Agee. *et.al.*, Spectral self-coherence restoral: A new approach to blind adaptive signal extraction using antenna arrays., Proc. IEEE, vol. 78, pp. 753-767, April 1990

[7] J.F. Cardoso, Super-symmetric decomposition of the fourth order cumulant tensor. Blind identification of more sources than sensors, Proc. IEEE ICASSP, Toronto, Canada, 1991, vol.5, pp. 3109-3112

[8] J. Laurila, E. Bonek, SDMA Using Blind Adaptation, ACTS Mobile Communication Summit, Aalborg, Denmark, October 7-10, 1997, pp. 314-319

[9] J. Laurila, E. Bonek, Pseudo-Blind Algorithm for SDMA Application. 8th Virginia Tech. Symposium on Wireless Personal Communications, Blacksburg, VA, USA, June 10-12, 1998, p. 23-34

[10] A-J. van der Veen, S. Talwar, A. Paulraj. A Subspace Approach to Blind Space-Time Signal Processing for Wireless Communications Systems. IEEE Trans. Signal Proc., vol.45, pp. 173-190, Jan. 1997

[11] G. Golub, C.F. Van Loan, Matrix Computations, 2nd Ed, 1989, 642 p.

[12] B. Yang, Projection Approximation Subspace Tracking, IEEE Trans. Signal Proc., vol.43, pp. 95-107, Jan. 1995

[13] S. Talwar, M. Viberg, A. Paulraj, Blind Separation of Synchronous Co-Channel Digital Signals Using an Antenna Array - Part I: Algorithms. IEEE Trans. on Signal Proc., vol.44, pp. 1184-1197, May 1996

[14] A-J. van der Veen, A. Paulraj. Singular Value Analysis of Time-Space Equalization in the GSM Mobile System. Proc. IEEE ICASSP '96. May 1996, Atlanta, GA. p. 1073-1076

[15] P. Pelin, Spatial Diversity Receivers for Base Station Antenna Arrays, Licentiate Thesis, Chalmers University of Technology, Göteborg, Sweden, May 1997, 72 p.

[16] J. Fuhl, A.F. Molisch, E. Bonek, A Unified Channel Model for Mobile Radio Systems with Smart Antennas, IEE Proc. - Radar, Sonar and Navigation: Special Issue on Antenna Array Processing Techniques, vol.145, Feb. 1998, pp. 32-41

[17] A.F. Molisch, J. Laurila, A. Kuchar, R. Schmalenberger, Test scenarios for mobile radio systems with adaptive antennas, First COST 252/259 Joint Workshop, Bradford, UK, April 21-22, 1998, p. 162-170

[18] M. Mouly, P. Pautet, The GSM System for Mobile Communications, 1992, 701 p.

[19] P.A. Laurent, Exact and Approximate Construction of Digital Phase Modulations by Superposition of Amplitude Modulated Pulses (AMP). IEEE Trans. on Comm., vol.34, pp. 150-160, Feb. 1986

[20] R. Schürhuber, Receiver Imperfections and Calibration of Adaptive Antennas, M.Sc. Thesis, INTHFT, Technische Universität Wien. Vienna, Austria. 1998, 98 p.