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UNBIASED SCATTERING FUNCTION ESTIMATION DURING DATA TRANSMISSION*

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ABSTRACT

We propose an unbiased, data-driven estimator of the scattering function of random time-varying channels satisfying the usual WSSUS assumption. The estimator allows continuous on-line operation during data transmission, with segments of the data signal serving as sounding signals. A matched filterbank preprocessing stage is used to compensate for the nonideal properties of these sounding signals. The proposed estimator is well suited to fast time-varying channels, and it exploits the underspread property of typical mobile radio channels for a substantial reduction of computational complexity. The performance of the new estimator is assessed by a bias/variance analysis and simulation results.

1 INTRODUCTION

The scattering function characterizes the second-order statistics of random, time-varying, linear channels obeying the usual wide-sense stationary uncorrelated scattering (WSSUS) assumption [1, 2]. An estimate of the scattering function is required or helpful for (e.g.) optimum receiver design [2]–[4], channel estimation [5], and performance analysis [6].

Extending results in [7], we propose an unbiased, data-driven scattering function estimator for on-line operation during data transmission without a separate sounding procedure. This allows to continuously track the time-varying statistics of practical *quasi-WSSUS* channels [1]. In contrast to existing estimators [8]–[10], our estimator exploits the *underspread property* of typical mobile radio channels [2, 11] to obtain an efficient implementation.

The paper is organized as follows. Section 2 summarizes some fundamentals. Section 3 reviews the scattering function estimator of [7]. The new data-driven estimator is introduced and studied in Section 4. Finally, simulation results are presented in Section 5.

2 FUNDAMENTALS

We shall first discuss a discrete-time framework of linear time-varying (LTV) channels and a channel sounding technique related to our estimator.

2.1 Discrete-Time LTV Channels

LTV channels are commonly characterized by their time-varying impulse response $h(t,\tau)$ (equivalent baseband channels will be considered throughout). For applicability of digital block processing algorithms such as the FFT, we shall use a discrete-time, finite-length framework that is based on the following assumptions:

1. The input signal $x(t)$ is bandlimited (bandwidth B).
2. The impulse response $h(t,\tau)$ is bandlimited with respect to t (bandwidth B_D, i.e., maximum Doppler shift $\pm B_D$) and with respect to τ (bandwidth B), and (effectively) delay-limited with maximum delay $\tau_{\max}$.
3. We consider the output signal $y(t)$ on $[0, t_{\max}]$.

We then obtain the discrete-time input-output relation [12]

$$y[n] = (\mathbf{H}x)[n] = \sum_{m=0}^{N_m-1} h[n,m]\, x[n-m], \quad n \in [0, N-1],$$

where $\mathbf{H}$ is the LTV channel filter. Furthermore, $y[n] = y(n/f_s)$, $x[n] = x(n/f_s)$, and $h[n,m] = (1/f_s)\, h(n/f_s, m/f_s)$ with the sampling frequency $f_s = 2(B + B_D)$; additionally $N_m = \lfloor \tau_{\max} f_s \rfloor + 1$ and $N = \lfloor t_{\max} f_s \rfloor + 1$. In this discrete-time setting, the WSSUS property [1] can be defined as

$$\mathbb{E}\{S[m,l]\, S^*[m',l']\} = C[m,l]\, \delta[m-m']\, \delta[l-l'], \quad (1)$$

where $\mathbb{E}$ denotes expectation, $S[m,l] = \sum_{n=0}^{N-1} h[n,m]\, e^{-j2\pi ln/N}$ is the *(delay-Doppler) spreading function*, and $C[m,l]$ is the *scattering function* [1]. Equivalently,

$$\mathbb{E}\{H[n,k]\, H^*[n',k']\} = R_H[n-n', k-k'], \quad (2)$$

where $H[n,k] = \sum_{m=0}^{N_m-1} h[n,m]\, e^{-j2\pi km/N}$ is the *time-varying transfer function* and $R_H[n,k]$ is the *time-frequency correlation function* [1]. We note that $S[m,l] \leftrightarrow H[n,k]$ as well as $C[m,l] \leftrightarrow R_H[n,k]$ are related by a 2-D DFT of length N. The relations (1) and (2) show that $H[n,k]$ is a stationary 2-D random process with autocorrelation function $R_H[n,k]$ and power spectral density $C[m,l] \geq 0$. Hence, *estimation of the scattering function is a 2-D spectral estimation problem.* Note, however, that realizations of the process $H[n,k]$ (whose spectrum is to be estimated) must themselves be estimated by means of some kind of channel sounding. As we shall see in Section 4, this channel sounding need not be accurate however.

The discrete-time channel is termed *underspread* [2, 11] if the scattering function $C[m,l]$ (and thus, with probability one, also the spreading function $S[m,l]$) is zero outside a region[1] $[0, N_m-1] \times [0, N_l-1]$ with

$$N_m N_l \leq N. \quad (3)$$

2.2 Channel Sounding

We next review a channel sounding technique that motivates the data-driven sounding performed by the new estimator. An elementary channel sounding technique [13]

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[1] For convenience, we assume one-sided m and l intervals. Other locations of these intervals can easily be accommodated by a simple shift.

uses as sounding signal an idealized impulse train, i.e., $x[n] = K\delta_M[n] = K\sum_{r=0}^{N/M-1}\delta[n-rM]$, where the block length N is assumed to be an integer multiple of the impulse spacing M. *Throughout this paper we shall assume $N_m \le M \le N/N_l$, which presupposes the underspread property (3)* [7]. Including zero-mean, white measurement noise $w[n]$ with variance N_0, the resulting output signal is obtained as

$$y[n] = K\sum_{r=0}^{N/M-1} h[n, n-rM] + w[n] = K\,h\Big[n, n-\Big\lfloor\frac{n}{M}\Big\rfloor M\Big] + w[n],$$

where the last expression holds since (due to $M \ge N_m$) the responses to the individual impulses do not overlap. Thus, $y[n]$ is a noisy, undersampled version of the impulse response. However, due to the underspread property the undersampling does not cause an information loss.

Unfortunately, an idealized impulse-train sounding signal is impractical due to its high crest factor. Instead, most practical channel sounders use a periodic sounding signal $x[n] = \sum_{r=0}^{N/M-1}p[n-rM]$ with low crest factor and pulse compression by means of a suitable receive filter $f[n]$ [14–16]. The output of the receive filter is

$$y_f[n] \triangleq (y*f)[n] = \big((\mathbf{H}x)*f\big)[n] + w_f[n],$$

where $w_f[n] = (w*f)[n]$ is the noise filtered by the receive filter. It has been shown in [7,17] that if the product of maximum Doppler shift N_l and length of receive filter N_f satisfies $N_lN_f \ll N$, then the LTV channel filter $\mathbf{H}$ and the receive filter f approximately commute. That is, $\big((\mathbf{H}x)*f\big)[n] \approx \big(\mathbf{H}(x*f)\big)[n] = (\mathbf{H}x_v)[n]$ for all $x[n]$, with the *virtual* sounding signal

$$x_v[n] \triangleq (x*f)[n] \tag{4}$$

(note that conceptually, we have moved the receive filter to the transmitter side). Hence,

$$y_f[n] \approx y_v[n] \triangleq (\mathbf{H}x_v)[n] + w_f[n]. \tag{5}$$

For $x[n] = \sum_{r=0}^{N/M-1}p[n-rM]$, the virtual sounding signal is

$$x_v[n] = \sum_{r=0}^{N/M-1}(p*f)[n-rM].$$

If the impulse responses $p[n]$ and $f[n]$ are chosen such that $(p*f)[n]$ is impulse-like, $x_v[n]$ will approximate an ideal impulse train. In particular, this will be the case if $p[n]$ is a maximum-length pseudonoise sequence and if we use a *matched* receive filter $f[n] = p^*[-n]$, so that $(p*f)[n]$ becomes the autocorrelation of $p[n]$.

3 AN UNBIASED SCATTERING FUNCTION ESTIMATOR

Application of the pulse-compression sounding described above to *fast* time-varying channels requires a small receive filter length N_f in order to satisfy the "commutation condition" $N_lN_f \ll N$. However, a small N_f results in poor pulse compression and, hence, systematic sounding errors. Even larger sounding errors may be encountered with data-driven estimation (see Section 4). A scattering function estimator that compensates for non-perfect virtual sounding signals has recently been proposed [7]. We shall briefly review this estimator since it will provide the basis for the data-driven estimator to be introduced in Section 4.

The estimator is motivated by the following relation (cf. [8]) that follows from $y_v[n] = (\mathbf{H}x_v)[n] + w_f[n]$ in (5),

$$\bar{A}_{y_v}[m,l] = R_H[m,l]\,\bar{A}_{x_v}[m,l] + \bar{A}_{w_f}[m,l]. \tag{6}$$

Here, $A_s[m,l] \triangleq \sum_{n=0}^{N-1}s[n]\,s^*[n-m]\,e^{-j2\pi nl/N}$ denotes the (cyclic) discrete ambiguity function of a length-N signal $s[n]$, $\bar{A}_s[m,l] \triangleq \mathbb{E}\{A_s[m,l]\}$, and $\bar{A}_{w_f}[m,l] = NN_0\,r_f[m]\delta[l]$ with $r_f[m] = \sum_{n=0}^{N_f-1}f[n]\,f^*[n-m]$. Assuming a deterministic sounding signal, i.e., $\bar{A}_{x_v}[m,l] = A_{x_v}[m,l]$, and approximate commutation of channel and receive filter so that $y_v[n] \approx y_f[n]$ according to (5), relation (6) suggests the following estimator of $R_H[n,k]$,

$$\hat{R}_H[n,k] \triangleq \frac{\hat{A}_{y_f}[n,k] - NN_0\,r_f[n]\delta[k]}{A_{x_v}[n,k]}. \tag{7}$$

Here,

$$\hat{A}_{y_f}[n,k] = \frac{1}{L}\sum_{i=1}^{L}A_{y_f^{(i)}}[n,k]$$

is an estimate of $\bar{A}_{y_f}[n,k]$ that uses L filtered channel output signals $y_f^{(i)}[n]$ obtained by repeated channel sounding; furthermore, it is assumed that N_0 is known or has been estimated. From $\hat{R}_H[n,k]$, a scattering function estimator can be derived by a 2-D DFT.

Unfortunately, it is difficult to find virtual sounding signals $x_v[n]$ for which $\big|A_{x_v}[n,k]\big|$ is sufficiently bounded away from zero for all n and k. However, this problem can be circumvented since the channel is underspread, i.e., $C[m,l] = 0$ outside $[0, M-1] \times [0, \frac{N}{M}-1]$. This entails a bandlimited $R_H[n,k]$ that can be reconstructed without error from the subsampled version $R_H[rM, s\frac{N}{M}]$. We are thus led to replacing (7) by

$$\hat{R}_H\Big[rM, s\frac{N}{M}\Big] = \frac{\hat{A}_{y_f}\big[rM, s\frac{N}{M}\big] - NN_0\,r_f[rM]\delta[s]}{A_{x_v}\big[rM, s\frac{N}{M}\big]}, \tag{8}$$

with $r = 0, 1, \cdots, \frac{N}{M}-1$, $s = 0, 1, \cdots, M-1$. Here, $\big|A_{x_v}[n,k]\big|$ need only be sufficiently bounded away from zero for $n = rM$ and $k = s\frac{N}{M}$. It can be shown that $\hat{A}_{y_f}\big[rM, s\frac{N}{M}\big]$ can be efficiently computed as

$$\hat{A}_{y_f}\Big[rM, s\frac{N}{M}\Big] = \frac{M}{N}\sum_{m=0}^{M-1}\sum_{l=0}^{N/M-1} z[m,l]\,e^{j2\pi(\frac{lr}{N/M} - \frac{ms}{M})}, \tag{9}$$

with $z[m,l] \triangleq \frac{1}{L}\sum_{i=1}^{L}\big|Z_{y_f^{(i)}}[m,l]\big|^2$, where $Z_{y_f^{(i)}}[m,l] = \sum_{r=0}^{N/M-1}y_f^{(i)}[m+rM]\,e^{-j2\pi\frac{rl}{N/M}}$ is the discrete Zak transform [18] of the ith channel output signal $y_f^{(i)}[n]$. From $\hat{R}_H\big[rM, s\frac{N}{M}\big]$ in (8), a scattering function estimator is finally obtained via a 2-D DFT,

$$\hat{C}[m,l] = N\sum_{r=0}^{N/M-1}\sum_{s=0}^{M-1}\hat{R}_H\Big[rM, s\frac{N}{M}\Big]\,e^{-j2\pi(\frac{lr}{N/M} - \frac{ms}{M})}. \tag{10}$$

Assuming commutation of channel and receive filter, it can be shown that the estimator $\hat{C}[m,l]$ is unbiased [7]. A bound on the variance of $\hat{C}[m,l]$ has been given in [7]. If $N_f \le M$, then $r_f[rM] = E_f\,\delta[r]$ with $E_f = r_f[0]$. Here,

(10) becomes

$$\hat{C}[m,l] = N \sum_{r=0}^{N/M-1} \sum_{s=0}^{M-1} \frac{\hat{A}_{y_f}\left[rM, s\frac{N}{M}\right]}{A_{x_v}\left[rM, s\frac{N}{M}\right]} e^{-j2\pi\left(\frac{lr}{N/M} - \frac{ms}{M}\right)} - \gamma, \tag{11}$$

where $\gamma = N^2 N_0 E_f / E_{x_v}$ with $E_{x_v} = \sum_{n=0}^{N-1} |x_v[n]|^2 = A_{x_v}[0,0]$. This estimator is computationally efficient—its complexity is $\mathcal{O}\big(N\big[(L+2)\log(N/M) + 2\log M\big]\big)$, with substantial additional savings existing for $N_m N_l \ll N$.

4 DATA-DRIVEN SCATTERING FUNCTION ESTIMATION

We now adapt the scattering function estimator reviewed above to data-driven operation where a local segment of the *data signal* $x[n]$ serves as sounding signal. This segment is assumed known to the receiver (during start-up, the data symbols are known; during normal operation, it is assumed that they have been detected). We shall model $x[n]$ as a stationary random process with autocorrelation $R_x[m] = \mathbb{E}\{x[n]\, x^*[n-m]\}$ and correlation width N_x (i.e., $R_x[m]$ is effectively zero for $|m| > N_x$).

4.1 Data-Driven Channel Sounding

Let the receive filter $f[n]$ be matched to a segment of the data signal $x[n]$ that starts at a current time point $n = n_0$ and has length N_f, i.e.,

$$f[n] = \mathrm{rect}_{N_f}[-n]\, x^*[-(n-n_0)]\,,$$

where $\mathrm{rect}_{N_f}[n]$ is 1 on the interval $[0, N_f - 1]$ and zero elsewhere (note that the length of $f[n]$ is N_f). Assuming commutation of the channel filter $\mathbf{H}$ with the receive filter f, we obtain the virtual sounding signal (cf. (4))

$$\begin{aligned} x_v[n] &= (x * f)[n] = \sum_{n'=n}^{n+N_f-1} x[n']\, x^*[n' - (n-n_0)] \\ &= N_f\, \hat{R}_x[n - n_0]\,, \end{aligned}$$

where $\hat{R}_x[m] \triangleq \frac{1}{N_f} \sum_{n=n_0+m}^{n_0+m+N_f-1} x[n]\, x^*[n-m]$ is an unbiased estimate of the autocorrelation $R_x[m]$. If N_x is small so that $R_x[m]$ is narrow and if N_f is sufficiently large so that $\hat{R}_x[m] \approx R_x[m]$, then $x_v[n]$ will be a reasonable approximation to an impulse at $n = n_0$.

However, for channel sounding we need not one single impulse but a periodic impulse train with N/M impulses. An approximate periodization can be achieved by replacing the receive filter $f[n]$ with a bank of N/M receive filters $f_r[n]$ $(r = 0, 1, \cdots, N/M-1)$ whose outputs are added to yield the overall output signal $y_f[n]$ (see Fig. 1 with $g[n] \equiv 1$). The rth filter $f_r[n]$ is matched to the length-N_f block of $x[n]$ starting at $n = rM$, i.e.,

$$f_r[n] = \mathrm{rect}_{N_f}[-n]\, x^*[-(n-rM)], \quad r = 0,1,\cdots,\frac{N}{M}-1. \tag{12}$$

Assuming approximate commutation of $\mathbf{H}$ with f_r, the output signal of the rth receive filter is given by (cf. (5))

$$\begin{aligned} y_{f_r}[n] &= (y * f_r)[n] \\ &\approx \big(\mathbf{H}(x * f_r)\big)[n] + w_{f_r}[n] \\ &= N_f\, \mathbf{H}\big\{\hat{R}_x^{(r)}[n-rM]\big\} + w_{f_r}[n] \\ &= N_f \sum_{m=0}^{N_m-1} h[n,m]\, \hat{R}_x^{(r)}[n-m-rM] + w_{f_r}[n], \end{aligned} \tag{13}$$

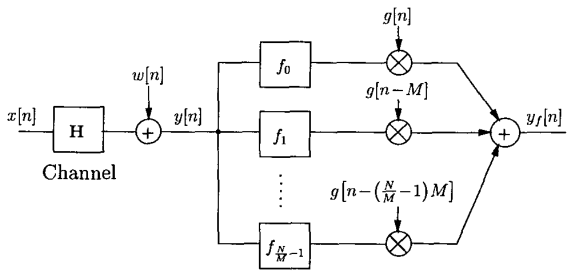

Figure 1. *Matched filterbank for data-driven sounding.*

where

$$\hat{R}_x^{(r)}[m] \triangleq \frac{1}{N_f} \sum_{n=m+rM}^{m+rM+N_f-1} x[n]\, x^*[n-m]$$

is an unbiased estimate of the autocorrelation $R_x[m]$ and $w_{f_r}[n] = (w * f_r)[n]$. Adding all $y_{f_r}[n]$, we obtain the overall output signal

$$y_f[n] = \sum_{r=0}^{N/M-1} y_{f_r}[n] \approx \big(\mathbf{H}x_v\big)[n] + w_f[n]\,, \tag{14}$$

with the virtual sounding signal

$$x_v[n] = N_f \sum_{r=0}^{N/M-1} \hat{R}_x^{(r)}[n - rM] \tag{15}$$

and $w_f[n] = \sum_{r=0}^{N/M-1} (w * f_r)[n] = \big(w * \sum_{r=0}^{N/M-1} f_r\big)[n]$. If N_x is small so that $R_x[m]$ is narrow and if N_f is large enough so that $\hat{R}_x^{(r)}[m] \approx R_x[m]$, then $x_v[n]$ approximates an idealized impulse train.

We now explain the windows $g[n-rM]$ in Fig. 1. Recall that the output of the rth matched filter, $y_{f_r}[n]$, is given by (13). The channel is causal and has maximum delay $N_m - 1$. Let us assume that $\hat{R}_x^{(r)}[m] \approx R_x[m]$ which is effectively zero for $|m| > N_x$. It follows that the term $N_f \sum_{m=0}^{N_m-1} h[n,m]\, \hat{R}_x^{(r)}[n-m-rM]$ in (13) is approximately zero outside the interval $[-N_x + rM, N_x + N_m - 1 + rM]$. Hence, outside this interval $y_{f_r}[n]$ is essentially noise which should be windowed out to reduce sounding errors. We therefore replace $y_{f_r}[n]$ by

$$y_r[n] \triangleq y_{f_r}[n]\, g[n - rM]\,,$$

where $g[n]$ is a window that is effectively supported on $[-N_x, N_x + N_m - 1]$. While a general window $g[n]$ will be considered in [19], here we shall assume $g[n] = \mathrm{rect}_M[n + N_x]$, i.e., $g[n]$ is 1 on $[-N_x, -N_x + M - 1]$ and 0 elsewhere (we assume that $2N_x + N_m \le M$ so that the interval $[-N_x, N_x + N_m - 1]$ is contained in the interval $[-N_x, -N_x + M - 1]$). With this choice of $g[n]$, which will simplify subsequent calculations, we obtain the overall output signal as

$$\begin{aligned} y_f[n] &= \sum_{r=0}^{N/M-1} y_r[n] = \sum_{r=0}^{N/M-1} (y * f_r)[n]\, g[n-rM] \\ &\approx \sum_{r=0}^{N/M-1} \big(\mathbf{H}(x * f_r)\big)[n]\, g[n-rM] + \tilde{w}_f[n], \end{aligned} \tag{16}$$

with the windowed noise

$$\tilde{w}_f[n] = \sum_{r=0}^{N/M-1} (w * f_r)[n]\, g[n - rM].$$

The windows $g[n - rM]$ truncate the noise components $w_{f_r}[n] = (w * f_r)[n]$ but approximately pass the 'signal' components $\big(\mathbf{H}(x * f_r)\big)[n] = N_f \sum_{m=0}^{N_m-1} h[n,m]\, \hat{R}_x^{(r)}[n - m - rM]$. Hence, we obtain (cf. (14))

$$y_f[n] \approx \big(\mathbf{H}x_v\big)[n] + \tilde{w}_f[n],$$

with the virtual sounding signal $x_v[n]$ as in (15). It can be shown that, due to the windowing, the variance of $\tilde{w}_f[n]$ is smaller by a factor of N/M than that of $w_f[n]$.

The choice of the receive filter length N_f is governed by two conflicting requirements: N_f should be small to keep commutation errors small but large so that $\hat{R}_x^{(r)}[m] \approx R_x[m]$. However, for data-driven scattering function estimation (see next), sounding errors resulting from a small N_f are compensated by averaging over several successive data-driven soundings.

4.2 The Data-Driven Scattering Function Estimator

The new data-driven scattering function estimator is motivated by a relation that is analogous to (6) and that will now be derived. As before, we assume commutation of channel $\mathbf{H}$ and receive filters f_r. Furthermore, we assume that $x[n]$, $h[n,m]$, and $w[n]$ are Gaussian and statistically independent. Using (16), one can show

$$\bar{A}_{y_f}\Big[rM, s\frac{N}{M}\Big] = \sum_{n=0}^{N-1} \sum_{r'=0}^{N/M-1} \Big[D[n,r,r'] + E[r] + \delta[r] N_f N_0 R_x[0]\Big] \big|g[n - r'M]\big|^2 e^{-j2\pi ns/M}, \quad (17)$$

with

$$D[n,r,r'] \triangleq \frac{N_f^2}{N} \sum_{m=0}^{N_m-1} \Bigg[\big|R_x[n - m - r'M]\big|^2 \cdot \sum_{k=0}^{N-1} R_H[rM,k]\, e^{j2\pi km/N}\Bigg],$$

$$E[r] \triangleq R_H[rM,0] \sum_{m=-N_f+1}^{N_f-1} |R_x[m + rM]|^2 (N_f - |m|).$$

If N_f is chosen such that $N_x + N_f \le M$, then $E[r] = \frac{1}{N}\delta[r]\gamma_c$ with

$$\gamma_c \triangleq 2NR_H[0,0] \sum_{m=0}^{N_f-1} \big|R_x[m]\big|^2 (N_f - m).$$

Furthermore (17) becomes

$$\bar{A}_{y_f}\Big[rM, s\frac{N}{M}\Big] = R_H\Big[rM, s\frac{N}{M}\Big] B[s] + \gamma\, \delta[r]\delta[s]. \quad (18)$$

Here,

$$B[s] \triangleq \frac{N_f^2 N}{M} \sum_{r=0}^{N/M-1} A_{R_x}\Big[rM, s\frac{N}{M}\Big]$$

and

$$\gamma = \gamma_c + N_0 N_f N R_x[0].$$

Eq. (18) motivates the following estimator of the sub-sampled time-frequency correlation function $R_H[n,k]$,

$$\hat{R}_{H,\mathrm{d}}\Big[rM, s\frac{N}{M}\Big] \triangleq \frac{\hat{A}_{y_f}\big[rM, s\frac{N}{M}\big] - \gamma\,\delta[r]\delta[s]}{B[s]}.$$

Note the similarity to the estimator in (8). Again,

$$\hat{A}_{y_f}\Big[rM, s\frac{N}{M}\Big] = \frac{1}{L}\sum_{i=1}^{L} A_{y_f^{(i)}}\Big[rM, s\frac{N}{M}\Big]$$

is an estimate of $\bar{A}_{y_f}\big[rM, s\frac{N}{M}\big]$ that uses L filtered channel output signals $y_f^{(i)}[n]$. As a difference from the estimator in (8), the $y_f^{(i)}[n]$ are obtained by repeated data-driven channel sounding according to Subsection 4.1 (i.e., using the matched filterbank). Furthermore, it is assumed that N_0, $R_x[n]$, and the path loss $R_H[0,0]$ are either known or have been estimated[2]. Again, the Zak transform can be used for efficient computation of $\hat{A}_{y_f}\big[rM, s\frac{N}{M}\big]$ (cf. (9)).

Finally, from $\hat{R}_{H,\mathrm{d}}\big[rM, s\frac{N}{M}\big]$ a scattering function estimator is obtained via a 2-D DFT, which yields (cf. (11))

$$\hat{C}_\mathrm{d}[m,l] = N \sum_{r=0}^{N/M-1} \sum_{s=0}^{M-1} \frac{\hat{A}_{y_f}\big[rM, s\frac{N}{M}\big]}{B[s]} e^{-j2\pi(\frac{lr}{N/M} - \frac{ms}{M})} - \frac{\gamma}{B[0]}. \quad (19)$$

4.3 Bias/Variance Analysis

We now calculate the bias and variance of the proposed data-driven estimator $\hat{C}_\mathrm{d}[m,l]$ under the assumption of perfect commutation of channel and receive filters. With $\mathbb{E}\{\hat{A}_{y_f}[m,l]\} = \bar{A}_{y_f}[m,l]$ and (18), it easily follows that *the estimator is unbiased*, i.e.,

$$\mathbb{E}\{\hat{C}_\mathrm{d}[m,l]\} \equiv C[m,l].$$

For variance calculation, we assume $x[n]$ given and $h[n,m]$ and $w[n]$ to be statistically independent and circularly symmetric complex Gaussian. Furthermore we assume all channel soundings to be separated by at least the channel's coherence time [2] so that the $y_f^{(i)}[n]$ are statistically independent. Finally, we assume for simplicity that $\hat{R}_x^{(r)}[m]$ does not depend on r, or equivalently $\hat{R}_x^{(r)}[m] = \hat{R}_x^{(0)}[m]$ (this will be true approximately if N_f is not too small). The mean variance of $C_\mathrm{d}[m,l]$,

$$V \triangleq \frac{1}{N} \sum_{m=0}^{M-1} \sum_{l=0}^{N/M-1} \mathrm{var}\{\hat{C}_\mathrm{d}[m,l]\},$$

can then be shown to be bounded as

$$V \le \beta \frac{P_x^2}{L} \sum_{s=0}^{M-1} \frac{1}{\big|A_{R_x}[0, s\frac{N}{M}]\big|}$$

with

$$\beta = \frac{1}{M}(12N_x + 3)(2N_x + 1)^2 P_x^2 \|C\|^2 + \frac{1}{N_f} N^2 M 2 N_0 (2N_x + 1)^2 P_x R_H[0,0] + \frac{N_0^2 M}{N_f N^2},$$

[2] Experiments indicate that, for typical receive filter lengths N_f, the data-driven estimator is quite insensitive to errors in estimating $R_H[0,0]$.

where $P_x = \hat{R}_x^{(0)}[0] = \frac{1}{N_f}\sum_{n=0}^{N_f-1}|x[n]|^2$ and $\|C\|^2 \triangleq \sum_{m=0}^{N-1}\sum_{l=0}^{N-1}|C[m,l]|^2$. From this bound it can be seen that the mean variance decreases for increasing matched filter length N_f and for increasing number L of soundings.

5 SIMULATION RESULTS

In order to assess the performance of the proposed scattering function estimators $\hat{C}[m,l]$ in (11) and $\hat{C}_d[m,l]$ in (19), we simulated a random time-varying channel that corresponds to a system operating at carrier frequency 1.8 GHz with maximum Doppler shift $B_D = 100$ Hz (i.e., maximum velocity 60 km/h) and maximum delay $\tau_{\max} = 7\mu$s. We assumed a sampling frequency of $f_s = 1$ MHz and a sounding duration of about 150 ms, resulting in $N = 149358$, $N_m = 7$, and $N_l = 15$. There were about 6 soundings per second.

For the estimator $\hat{C}[m,l]$, the sounding pulse $p[n]$ was a maximum-length pseudonoise sequence of length $M = 15$ (corresponding to a duration of 15μs), and the receive filter was matched to $p[n]$ and thus had length $N_f = 15$. For the data-driven estimator $\hat{C}_d[m,l]$ we conducted two experiments; for the first the sounding signal (= data signal) $x[n]$ was chosen white and for the second $x[n]$ was chosen nonwhite with correlation width $N_x = 2$. The receive filters were chosen according to (12) with $N_f = 14$ for white $x[n]$ and $N_f = 12$ for nonwhite $x[n]$ (such that $N_x + N_f \le M$ was satisfied for both cases).

In the following, $\varepsilon_{\hat{C}}$ and $\varepsilon_{\hat{C}_d}$ will denote the mean-square error[3] (MSE) of $\hat{C}[m,l]$ and $\hat{C}_d[m,l]$, respectively, summed over m and l and normalized by $\|C\|^2$. These MSEs have been estimated using a sufficient number of Monte Carlo runs. Fig. 2 compares $\varepsilon_{\hat{C}}$ and $\varepsilon_{\hat{C}_d}$ as a function of L (the number of soundings), both for the case of no noise and for an $\mathrm{SNR} \triangleq \frac{1}{N}\sum_{n=0}^{N-1}\mathbb{E}\{|y_f[n]|^2\}/N_0$ of 0 dB. It is seen that initially, $\varepsilon_{\hat{C}}$ and $\varepsilon_{\hat{C}_d}$ decay according to $1/L$. However, for larger values of L they level off, which is mainly due to a noticeable commutation error that increases the MSE of both estimators. (This commutation error could be decreased by choosing a smaller receive filter length N_f, cf. [7].) It is seen that $\varepsilon_{\hat{C}_d}$ is larger than $\varepsilon_{\hat{C}}$, and furthermore $\varepsilon_{\hat{C}_d}$ is larger for a nonwhite data signal than for a white data signal.

[3] The theoretical MSE is the sum of the squared bias and the variance. Since (for perfect commutation of channel and receive filter) $\hat{C}[m,l]$ and $\hat{C}_d[m,l]$ are unbiased, the MSEs theoretically equal the respective variances.

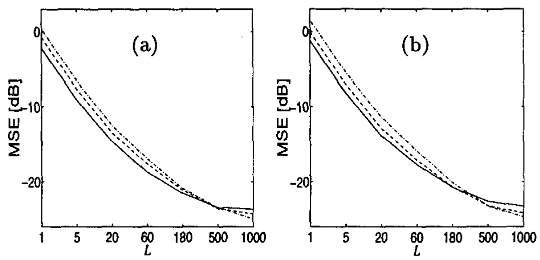

Figure 2. *Comparison of MSEs $\varepsilon_{\hat{C}}$ (solid line) and $\varepsilon_{\hat{C}_d}$ (dashed line: white data signal $x[n]$; dash-dotted line: nonwhite data signal $x[n]$ with correlation width $N_x = 2$) as a function of L: (a) no noise, (b) $\mathrm{SNR} = 0dB$.*

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