# The Wigner Distribution

## THEORY AND APPLICATIONS IN SIGNAL PROCESSING

W. Mecklenbräuker & F. Hlawatsch, editors

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Edited by

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# PREFACE

Signals usually describe the temporal (and/or spatial) evolution of some physical quantity. In most situations, it is the time domain in which signals naturally appear. However, the frequency domain is a complementary domain which has tremendous importance in signal analysis and signal processing.

While the frequency-domain description (the Fourier transform or spectrum) nicely shows the strength of various spectral components contained in a signal, the *temporal localization* of these components is hidden in the phase of the spectrum. The desire to characterize temporal and spectral signal components on a joint basis has led to the definition of *time-frequency representations* that display signals over a joint time-frequency plane. Besides many important advantages that will be highlighted in the contributions of this book, such representations have inherent limitations of resolution and interpretation. Specifically, a pointwise energetic interpretation of *any* time-frequency representation is prohibited by the uncertainty principle. This fact may also be held responsible for the large variety of time-frequency representations that have been proposed over the years.

In the last two decades, the field of time-frequency analysis has evolved into a widely recognized and applied discipline of signal processing. Textbooks and edited books, symposia, special conference sessions, and special journal issues are being devoted to this research field. Besides linear time-frequency representations like the short-time Fourier transform, the Gabor transform, and the wavelet transform, an important contribution to this development has undoubtedly been the *Wigner distribution* (WD) which holds an exeptional position within the field of bilinear/quadratic time-frequency representations.

The WD was first defined in quantum mechanics as early as 1932 by the later Nobel laureate E. Wigner (in cooperation with L. Szilard) [1]. In 1948, J. Ville introduced this concept in signal analysis [2]. Based on investigations of its mathematical structure and properties by N. G. de Bruijn in 1967 [3] and 1973 [4], the WD was brought to the attention of a larger signal processing community in 1980 [5].

The WD was soon recognized to be important for two reasons: on the one hand, it provides a powerful theoretical basis for quadratic time-frequency analysis, and on the other hand, its discrete-time form (supplemented by suitable windowing and smoothing) is an eminently practical signal analysis tool.

Hence, there appears to be ample motivation and justification to devote an edited book primarily to the WD. The seven chapters of this book cover a wide range of different aspects of the WD and other bilinear time-frequency distributions: properties like positivity, spread, and interference term geometry; signal synthesis methods and their application to signal design, time-frequency filtering, and signal separation; WD based analysis of nonstationary random processes; singular value decompositions and their application to WD based detection and classification; and optical applications of the WD. The size of the chapters has been chosen such that an in-depth treatment of the various topics is achieved.

The first two chapters discuss important properties of the WD and other bilinear time-frequency distributions. The chapter by A. J. E. M. JANSSEN, **"Positivity and Spread of Bilinear Time-Frequency Distributions,"** presents several fundamental results concerning the local negativity and the concentration properties of the WD and other bilinear time-frequency distributions. Given the fact that bilinear time-frequency distributions may not have perfect (infinite) concentration and may not be always nonnegative, it is both theoretically and practically important to identify the distribution(s) having maximum concentration and minimum negativity. It is shown that, under certain assumptions and within a specific class of time-frequency distributions, the WD is optimum with respect to both criteria. Besides other important theoretical results, the chapter also presents theorems on the existence of signals for which time-frequency distributions are nonnegative.

The local negativity of the WD and other time-frequency distributions can be associated with the occurrence of cross or "interference" terms. An in-depth discussion of interference terms is provided in the chapter **"The Interference Structure of the Wigner Distribution and Related Time-Frequency Signal Representations"** by F. HLAWATSCH and P. FLANDRIN. This chapter focuses on the geometry of interference terms and their attenuation by means of smoothing, two issues that are of great importance for practical applications of the WD. Besides discussing the interference structure of the WD and the family of generalized WDs, the chapter also considers the shift-invariant class (known as Cohen's class), the shift-scale-invariant class, and the class of smoothed WDs. Furthermore, interference terms are shown to be related to energetic quantities (marginals) and to aliasing terms occurring in the discrete-time WD.

The chapter by F. HLAWATSCH and W. KRATTENTHALER, entitled **"Signal Synthesis Algorithms for Bilinear Time-Frequency Signal Representations,"** discusses the time-frequency signal synthesis problem and various signal synthesis methods within a discrete-time framework. Time-frequency signal synthesis allows a time-frequency design and time-frequency processing of signals. The chapter presents a general solution of the signal synthesis problem (with optional subspace constraint) for "subspace-unitary" time-frequency representations and specializes this general solution to the discrete-time WD. Iterative signal synthesis algorithms are developed for the practically important case of smoothed WDs and the spectrogram, and an "on-line" signal synthesis algorithm for the pseudo WD is presented. Several phase matching algorithms are proposed for resolving phase ambiguities of the synthesized signal. Finally, the application of signal synthesis to



time-frequency filtering is considered. (More on time-frequency signal synthesis can be found in the later chapter by G. F. BOUDREAUX-BARTELS.)

While so far only deterministic signals have been considered, P. FLANDRIN and W. MARTIN extend WD based time-frequency analysis to nonstationary random processes in their chapter **"The Wigner-Ville Spectrum of Nonstationary Random Signals."** This chapter first reviews possible approaches to the problem of defining a time-dependent spectrum, i.e., extending the power spectral density of stationary processes to the nonstationary case. Subsequently, a particular time-dependent spectrum related to the WD, known as the Wigner-Ville spectrum, is discussed in detail. Special emphasis is placed on its theoretical properties, its relations to other time-dependent spectra, and the results obtained for important special processes. The estimation of the Wigner-Ville spectrum by means of Cohen's class time-frequency representations is discussed, and a smoothed pseudo Wigner estimator is proposed and analyzed. Finally, the use of the Wigner-Ville spectrum in optimum detection procedures is considered.

The chapter by G. F. BOUDREAUX-BARTELS, **"Time-Varying Signal Processing Using Wigner Distribution Synthesis Techniques,"** considers WD based time-frequency signal synthesis and its applications. Since this chapter devotes considerable space to signal processing applications of time-frequency signal synthesis, it is somewhat complementary to the earlier chapter (by F. HLAWATSCH and W. KRATTENTHALER) on time-frequency signal synthesis. After a general review of time-frequency analysis and synthesis, an optimal signal synthesis algorithm for the discrete-time WD is presented. The subsequent discussion of signal processing applications of time-frequency signal synthesis addresses time-varying filtering, signal separation, and the use of time-frequency signal synthesis in well logging, filter design, and window design. Finally, a signal synthesis algorithm based on the cross WD and incorporating an optional time-frequency mask is presented and shown to be advantageous when the time-frequency model function is only partly known.

Applications are also emphasized by N. M. MARINOVICH in his chapter entitled **"The Singular Value Decomposition of the Wigner Distribution and its Applications."** The singular value decomposition of the WD, followed by a truncation of the singular values, is proposed and shown to achieve a data reduction and an enhancement of the WD of noisy or multicomponent signals. Several applications of this and similar techniques are discussed. Application of the singular value decomposition to the WD of a "boundary sequence" (describing the shape of an object) is shown to result in shape descriptors that possess properties desirable in a classification context. Another application where the singular value decomposition of the WD can improve performance is biomedical ultrasonic tissue characterization. Finally, a generalized singular value decomposition, applied to the Wigner-Ville spectrum (expected WD), is shown to result in a signal/noise subspace decomposition that can be used to improve detection performance.

Last but not least, M. J. BASTIAANS considers the WD from an optical perspective in his chapter entitled **"Application of the Wigner Distribution Function in Optics."**

Here the WD is applied to a spatial signal and is hence a function of space and spatial frequency. The WD of deterministic optical signals (completely coherent light) provides a link between Fourier optics and geometrical optics, while the expected WD of stochastic optical signals (partially coherent light) provides a link between partial coherence and radiometry. After a review of elementary properties of the WD and the results obtained for some fundamental optical signals, the propagation of the WD through linear systems is discussed and results for various special optical systems are provided. Transport equations for the WD are derived from differential equations satisfied by the signal. A section is devoted to the expected WD of partially coherent light (also known as "generalized radiance"). Finally, various applications of the WD to problems arising in optics are considered.

The book also includes a bibliography of the Wigner distribution and other time-frequency representations. This bibliography, which continues an earlier reference list provided in [6], covers the years from 1985 to 1992. No references after 1992 were included due to the dramatically growing number of papers on this subject and the availability of textbooks and review papers with extensive bibliographies [7]–[11].

This brief summary of the contents of this book shows that both theoretical and practical aspects of the WD are addressed by the authors. Hence, we are confident that this book will serve a twofold purpose: first, contribute to a better understanding of the theoretical properties of the WD, and second, encourage use of the WD in practical applications. Our sincere thanks are due to the authors for their outstanding contributions to this book and for their patience during the long final phase of editorial work. We are furthermore grateful to W. Kozek and C. Mecklenbräuker for compiling the bibliography.

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