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Oversampled Wilson-Type Cosine Modulated Filter Banks with Linear Phase*

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Abstract

We introduce Wilson filter banks *(WFBs) as a new type of cosine modulated filter banks (CMFBs) corresponding to the discrete-time Wilson expansion. WFBs allow linear phase filters in all channels. We formulate perfect reconstruction (PR) conditions for oversampled and critically sampled WFBs and show that PR WFBs correspond to PR DFT filter banks with twice the oversampling factor.*

Generalizing WFBs, we then propose the new family of "even-stacked" CMFBs allowing both PR and linear phase filters in all channels. This CMFB family contains WFBs as well as CMFBs recently introduced by Lin and Vaidyanathan. Finally, after extending conventional ("odd-stacked") CMFBs to the oversampled case, we formulate unified PR conditions for both even- and odd-stacked, oversampled and critically sampled CMFBs. We show that PR CMFBs are always related to PR DFT filter banks of the same stacking type and with twice the oversampling factor.

1. Introduction

Recent interest in oversampled filter banks (FBs) [1]-[5] is mainly due to their increased design freedom and noise immunity [1, 3, 4]. Oversampled DFT FBs [6]-[8], [3] and oversampled cosine modulated FBs (CMFBs) [9]-[15], [8] are especially attractive as they allow an efficient implementation. An advantage of CMFBs over DFT FBs is the fact that their subband signals are real for real input signals and real analysis prototype.

This paper introduces and studies a new (possibly oversampled) CMFB which we call *Wilson FB* (WFB) since it corresponds to the discrete-time Wilson expansion [16]. Wilson expansions are based on cosine and sine modulation of a prototype function and can be constructed to have good time and frequency localization besides being orthonormal [17]-[19], [16]. An important advantage (especially in image coding applications) of WFBs over conventional CMFBs [9]-[15] is that they have linear phase filters in all channels if the prototypes have linear phase.

Organization of paper. After the introduction of (possibly oversampled) WFBs in Section 2, Section 3 shows a close relation between WFBs and DFT FBs with twice the oversampling factor. Section 4 provides perfect reconstruction (PR) conditions for (possibly oversampled) WFBs and shows that PR WFBs can always be derived from PR DFT FBs. Section 5 presents a new generalized framework for linear phase CMFBs that contains WFBs and a CMFB type recently introduced by Lin and Vaidyanathan [20].

2. Wilson Filter Banks

We consider a WFB with $2N$ channels and decimation factor $2M$. The WFB is critically sampled if $N = M$ and oversampled if $N > M$. The analysis FB consists of two partial FBs with impulse responses $\{h_k[n]\}_{k=0,..,N}$ and $\{\bar{h}_k[n]\}_{k=1,..,N-1}$, respectively, that are derived from an analysis prototype $h[n]$ as

$$h_k[n] = \begin{cases} h[n], & k=0 \\ \sqrt{2}\, h[n] \cos\left(\frac{k\pi}{N}\left(n-\frac{N}{2}\right)\right), & k=1,...,N-1 \\ h[n-sM]\,(-1)^{n-sM}, & k=N \end{cases}$$

and

$$\bar{h}_k[n] = \sqrt{2}\, h[n-M] \sin\left(\frac{k\pi}{N}\left(n-M-\frac{N}{2}\right)\right)$$

for $k = 1, ..., N-1$. Here $s = 0$ for N even and $s = 1$ for N odd. Similarly, the synthesis FB consists of the two partial FBs $\{f_k[n]\}_{k=0,..,N}$ and $\{\bar{f}_k[n]\}_{k=1,..,N-1}$ derived from a synthesis prototype $f[n]$ as

$$f_k[n] = \begin{cases} f[n], & k=0 \\ \sqrt{2}\, f[n] \cos\left(\frac{k\pi}{N}\left(n+\frac{N}{2}\right)\right), & k=1,...,N-1 \\ f[n+sM]\,(-1)^{n+sM}, & k=N \end{cases}$$

and

$$\bar{f}_k[n] = -\sqrt{2}\, f[n+M] \sin\left(\frac{k\pi}{N}\left(n+M+\frac{N}{2}\right)\right)$$

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for $k = 1, ..., N-1$. Note that there are $2N$ channels but only $N+1$ different channel frequencies $\theta_k = \frac{k}{2N}$ $(k = 0, ..., N)$ as depicted schematically in Fig. 1(a). In particular, the $k = 0$ channel is centered at frequency $\theta_0 = 0$, which is a difference from conventional CMFBs (see Section 5). If the analysis prototype has linear phase, i.e., $h[2lN - n] = h[n]$ with some $l \in \mathbb{Z}$, we have

$$h_k[2lN-n] = (-1)^k h_k[n], \quad k = 0,..,N-1,$$
$$h_N[2(lN+sM)-n] = h_N[n],$$
$$\bar{h}_k[2(lN+M)-n] = (-1)^{k+1}\bar{h}_k[n], \quad k = 1,..,N-1.$$

Thus, all analysis filters have linear phase as well. Similarly, for a linear phase synthesis prototype $f[n]$ all synthesis filters $f_k[n]$ and $\bar{f}_k[n]$ have linear phase. This is an important advantage of WFBs over conventional CMFBs [9]-[15].

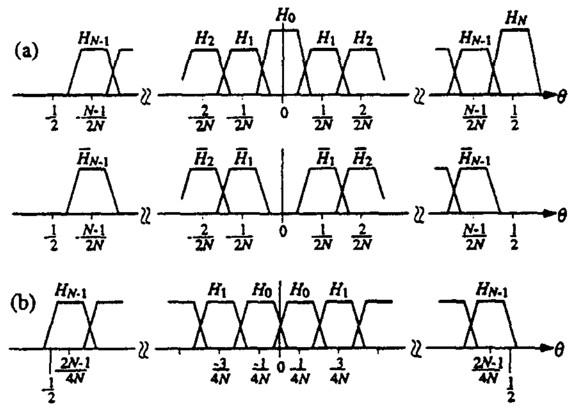

Figure 1. Transfer functions of the channel filters in (a) a WFB or, more generally, an even-stacked CMFB with $2N$ channels, (b) an N-channel odd-stacked CMFB.

3 Relation to DFT Filter Banks

We shall next show that there is a close relationship between WFBs and DFT FBs with twice the oversampling factor. The basic idea—combining positive and negative frequencies in a DFT FB to obtain a CMFB—has been introduced by Daubechies et al. [17] in a signal expansion context, and has also been used in filter bank theory for many years [14] (in filter bank theory, however, emphasis has been placed on the near-PR case).

The input-output relation in a WFB with $2N$ channels and decimation factor $2M$ is

$$\hat{x}[n] = \sum_{k=0}^{N}\sum_{i=-\infty}^{\infty} \langle x, h_{k,2i}\rangle f_{k,2i}[n] + \sum_{k=1}^{N-1}\sum_{i=-\infty}^{\infty} \langle x, h_{k,2i-1}\rangle f_{k,2i-1}[n], \qquad (1)$$

where

$$f_{k,2i}[n] = f_k[n-2iM], \quad k = 0,..,N$$
$$f_{k,2i-1}[n] = \bar{f}_k[n-2iM], \quad k = 1,..,N-1,$$

and

$$h_{k,2i}[n] = h_k^*[2iM-n], \quad k = 0,..,N$$
$$h_{k,2i-1}[n] = \bar{h}_k^*[2iM-n], \quad k = 1,..,N-1.$$

Note that (1) describes an expansion of the reconstructed signal $\hat{x}[n]$ into the synthesis functions $f_{k,m}[n]$; this expansion can alternatively be written as

$$\hat{x}[n] = \sum_{k=1}^{N-1}\sum_{m=-\infty}^{\infty} \langle x, h_{k,m}\rangle f_{k,m}[n] + \sum_{i=-\infty}^{\infty}\Big[\langle x, h_{0,2i}\rangle f_{0,2i}[n] + \langle x, h_{N,2i}\rangle f_{N,2i}[n]\Big]. \qquad (2)$$

Similarly, the input-output relation in an even-stacked DFT FB [6] with $2N$ channels and decimation factor M, using analysis prototype $h[n]$ and synthesis prototype $f[n]$, is

$$\hat{x}_{\mathrm{DFT}}[n] = \sum_{k=0}^{2N-1}\sum_{m=-\infty}^{\infty} \langle x, h_{k,m}^{\mathrm{DFT}}\rangle f_{k,m}^{\mathrm{DFT}}[n]. \qquad (3)$$

Here $f_{k,m}^{\mathrm{DFT}}[n] = f_k^{\mathrm{DFT}}[n-mM]$ with $f_k^{\mathrm{DFT}}[n] = f[n]\,W_{2N}^{-kn}$ and $h_{k,m}^{\mathrm{DFT}}[n] = h_k^{\mathrm{DFT}*}[mM-n]$ with $h_k^{\mathrm{DFT}}[n] = h[n]\,W_{2N}^{-kn}$, where $W_{2N} = e^{-j\frac{2\pi}{2N}}$.

It can now be shown that the WFB's synthesis functions can be derived from the DFT FB's synthesis functions as

$$f_{k,m}[n] = \frac{c_m}{\sqrt{2}}\Big[f_{k,m}^{\mathrm{DFT}}[n]\,(-1)^m e^{j\frac{k\pi}{2}} + f_{2N-k,m}^{\mathrm{DFT}}[n]\,e^{-j\frac{k\pi}{2}}\Big], \quad k = 1, ..., N-1, \qquad (4)$$

$$f_{0,2i}[n] = f_{0,2i}^{\mathrm{DFT}}[n], \qquad (5)$$

$$f_{N,2i}[n] = f_{N,2i-s}^{\mathrm{DFT}}[n], \qquad (6)$$

where $c_{2i} = 1$ and $c_{2i-1} = -j$, and s has been defined in Section 2. Indeed, for $m = 2i$ the right-hand side of (4) is

$$\frac{1}{\sqrt{2}}\Big[f_{k,2i}^{\mathrm{DFT}}[n]\,e^{j\frac{k\pi}{2}} + f_{2N-k,2i}^{\mathrm{DFT}}[n]\,e^{-j\frac{k\pi}{2}}\Big]$$
$$= \frac{1}{\sqrt{2}}\Big[f[n-2iM]\,W_{2N}^{-k(n+\frac{N}{2}-2iM)} + f[n-2iM]\,W_{2N}^{k(n+\frac{N}{2}-2iM)}\Big]$$
$$= \sqrt{2}\,f[n-2iM]\cos\Big(\frac{k\pi}{N}\Big(n+\frac{N}{2}-2iM\Big)\Big)$$
$$= f_k[n-2iM] = f_{k,2i}[n].$$

For $m = 2i-1$, Eq. (4) can be verified in a similar way, and verification of (5) and (6) is straightforward.

Similarly, the WFB's analysis functions can be derived from the DFT FB's analysis functions as

$$h_{k,m}[n] = \frac{c_m}{\sqrt{2}}\left[h_{k,m}^{\mathrm{DFT}}[n]\,(-1)^m e^{j\frac{k\pi}{2}} + h_{2N-k,m}^{\mathrm{DFT}}[n]\,e^{-j\frac{k\pi}{2}}\right], \quad k = 1,...,N-1, \tag{7}$$

$$h_{0,2i}[n] = h_{0,2i}^{\mathrm{DFT}}[n]\,, \tag{8}$$

$$h_{N,2i}[n] = h_{N,2i-s}^{\mathrm{DFT}}[n]\,. \tag{9}$$

Inserting the relations (4)-(6) and (7)-(9) in (2) and arranging terms leads to the following important result.

Theorem [21]. For a WFB, the reconstructed signal can be decomposed as

$$\hat{x}[n] = \frac{1}{2}\left[(\mathbf{S}x)[n] + (\mathbf{T}x)[n]\right], \tag{10}$$

where the operators $\mathbf{S}$ and $\mathbf{T}$ are defined as

$$(\mathbf{S}x)[n] = \sum_{k=0}^{2N-1}\sum_{m=-\infty}^{\infty} \langle x, h_{k,m}^{\mathrm{DFT}}\rangle\, f_{k,m}^{\mathrm{DFT}}[n],$$

$$(\mathbf{T}x)[n] = \sum_{k=0}^{2N-1}\sum_{m=-\infty}^{\infty} (-1)^{k+m}\langle x, h_{k,m}^{\mathrm{DFT}}\rangle\, f_{2N-k,m}^{\mathrm{DFT}}[n]\,.$$

Note that the DFT FB input-output relation in (3) can be written as $\hat{x}_{\mathrm{DFT}}[n] = (\mathbf{S}x)[n]$, and thus the operator $\mathbf{S}$ corresponds to an even-stacked DFT FB with $2N$ channels and decimation factor M, i.e., with twice the oversampling factor of the CMFB.

4 Perfect Reconstruction Conditions

With (10), it can be shown [21] that a necessary and sufficient condition for PR with zero delay, $\hat{x}[n] = x[n]$, is

$$\mathbf{S} = 2\mathbf{I} \quad \text{and} \quad \mathbf{T} = \mathbf{0}\,, \tag{11}$$

where $\mathbf{I}$ and $\mathbf{0}$ denote the identity and zero operator, respectively. For $\mathbf{T} = \mathbf{0}$ the WFB's input-output relation (10) reduces to $\hat{x}[n] = \frac{1}{2}(\mathbf{S}x)[n]$, i.e., the input-output relation of an even-stacked DFT FB with $2N$ channels and decimation factor M. Thus, *PR WFBs correspond to even-stacked PR DFT FBs with twice the oversampling factor.* In particular, it can be shown [21] that paraunitary [14] WFBs correspond to paraunitary DFT FBs (here, $f[n] = h^*[-n]$).

The operators $\mathbf{S}$ and $\mathbf{T}$ can be expressed in the time domain as

$$(\mathbf{S}x)[n] = 2N\sum_{l=-\infty}^{\infty} x[n-2lN]\sum_{m=-\infty}^{\infty} f[n-mM] \cdot h[-n+mM+2lN]\,, \tag{12}$$

$$(\mathbf{T}x)[n] = 2N\sum_{l=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} (-1)^m x[-n+2mM - (2l+1)N]\, f[n-mM]\, h[n-mM+(2l+1)N]\,. \tag{13}$$

With (12), the first PR condition in (11), $\mathbf{S} = 2\mathbf{I}$, is satisfied if and only if

$$\sum_{m=-\infty}^{\infty} f[n-mM]\,h[-n+mM+2lN] = \frac{1}{N}\,\delta[l]\,,$$

where $\delta[l]$ is the unit sample. This is equivalent to the PR condition for a DFT FB with $2N$ channels and decimation factor M, i.e., with twice the CMFB's oversampling factor.

In general, (13) does not lead to a similarly simple condition for the second PR condition $\mathbf{T} = \mathbf{0}$. In the case of *integer oversampling*, however, i.e., $N = KM$ with $K \in \mathbb{N}$, it can be shown [21] that $\mathbf{T} = \mathbf{0}$ is satisfied if and only if

$$\sum_{i=-\infty}^{\infty} (-1)^{iK} f[n-(l+iK)M] \cdot h[n-lM+(2m+i+1)KM] = 0$$

for $l = 0,...,K-1$ and $-\infty < m < \infty$. Note that critical sampling, $N = M$, is a special case with $K = 1$.

The operators $\mathbf{S}$ and $\mathbf{T}$ can also be expressed in the frequency and in the polyphase domains; this leads to corresponding formulations of the PR conditions in (11) [21].

5 A Unified Framework for CMFBs

In a WFB all channel filters have linear phase if the prototypes have linear phase. We now introduce a generalized framework for "linear-phase" CMFBs. In analogy to even-stacked DFT FBs, we call this new CMFB class *even-stacked*. We show that the class of even-stacked CMFBs contains WFBs and the linear phase CMFBs recently introduced by Lin and Vaidyanathan [20]. Subsequently, we extend the conventional "odd-stacked" CMFBs to the oversampled case, and finally we present unified PR conditions that are valid for both even- and odd-stacked CMFBs.

Even-stacked CMFBs. The analysis FB in an even-stacked CMFB with $2N$ channels and decimation factor $2M$ consists of two partial FBs $\{h_k^e[n]\}_{k=0,..,N}$ and $\{\bar{h}_k^e[n]\}_{k=1,..,N-1}$ derived from an analysis prototype $h[n]$,

$$h_k^e[n] = \begin{cases} h[n-rM], & k=0 \\ \sqrt{2}\,h[n]\,\cos\!\left(\frac{k\pi}{N}n + \phi_k^e\right), & k=1,..,N-1 \\ h[n-sM]\,(-1)^{n-sM}, & k=N \end{cases}$$

and

$$\bar{h}_k^e[n] = \sqrt{2}\,h[n-M]\,\sin\!\left(\frac{k\pi}{N}(n-M) + \phi_k^e\right)$$

for $k = 1,..,N-1$. Similarly, the synthesis FB consists of partial FBs $\{f_k^e[n]\}_{k=0,..,N}$ and $\{\bar{f}_k^e[n]\}_{k=1,..,N-1}$ defined in terms of a synthesis prototype $f[n]$ as

$$f_k^e[n] = \begin{cases} f[n+rM], & k=0 \\ \sqrt{2}\,f[n]\,\cos\!\left(\frac{k\pi}{N}n - \phi_k^e\right), & k=1,..,N-1 \\ f[n+sM]\,(-1)^{n+sM}, & k=N \end{cases}$$

and

$$\bar{f}_k^e[n] = -\sqrt{2}\, f[n+M]\, \sin\Big(\frac{k\pi}{N}(n+M) - \phi_k^e\Big)$$

for $k = 1, .., N-1$. Here, we define the phases as

$$\phi_k^e = -\alpha\frac{\pi}{2N}k + r\frac{\pi}{2} \quad \text{with } \alpha \in \mathbb{Z};$$

furthermore $r, s \in \{0,1\}$ with $s = r$ for α even and $s = 1-r$ for α odd. Thus, even-stacked CMFBs are parameterized in terms of the two parameters $\alpha \in \mathbb{Z}$ and $r \in \{0,1\}$. For *any* choice of $\alpha \in \mathbb{Z}$ and $r \in \{0,1\}$, *all analysis filters have linear phase* if $h[\alpha + (2l-1)N - n] = h[n]$ with some $l \in \mathbb{Z}$. Similarly, all synthesis filters have linear phase if $f[-\alpha - (2l-1)N - n] = f[n]$. Note that even-stacked CMFBs have $2N$ channels but only $N+1$ different channel frequencies $\theta_k = \frac{k}{2N}$ $(k = 0, ..., N)$, with the $k = 0$ channel centered at frequency $\theta_0 = 0$ (see Fig. 1(a)).

A special even-stacked CMFB is the WFB introduced in Section 2; it is obtained by choosing the parameters as $\alpha = N$ and $r = 0$. Another important special case is the CMFB recently introduced (for critical sampling) by Lin and Vaidyanathan [20]; its generalization to arbitrary oversampling is obtained for $\alpha = 0$ and $r = 0$.

Odd-stacked CMFBs. *Odd-stacked* CMFBs are the traditional CMFB type previously defined for critical sampling [9]-[15]. In the general case of an odd-stacked CMFB with N channels and decimation factor M (note that the CMFB is oversampled for $N > M$), we define the analysis and synthesis filters respectively as

$$h_k^o[n] = \sqrt{2}\, h[n]\, \cos\Big(\frac{(k+1/2)\pi}{N}n + \phi_k^o\Big),$$
$$f_k^o[n] = \sqrt{2}\, f[n]\, \cos\Big(\frac{(k+1/2)\pi}{N}n - \phi_k^o\Big)$$

for $k = 0, ..., N-1$. Here, $h[n]$ and $f[n]$ denote the analysis and synthesis prototype, respectively, and the phases are defined as

$$\phi_k^o = -\alpha\frac{\pi}{2N}\Big(k + \frac{1}{2}\Big) + r\frac{\pi}{2} \quad \text{with } \alpha \in \mathbb{Z},\ r \in \{0,1\};$$

this extends the phase definition given in [11] for the special case of critical sampling. Note that the channel frequencies are $\theta_k = \frac{k+1/2}{2N}$; in particular, the $k = 0$ channel is centered at frequency $\theta_0 = \frac{1}{4N}$ (see Fig. 1(b)). A disadvantage of odd-stacked CMFBs is that the channel filters do not have linear phase even if the prototypes have linear phase [9]-[15].

Relation to DFT FBs. For both even- and odd-stacked CMFBs, the following decomposition of the reconstructed signal (generalizing (10)) can be shown [21],

$$\hat{x}[n] = \frac{1}{2}\left[(\mathbf{S}x)[n] + (\mathbf{T}x)[n]\right],$$

where the operators $\mathbf{S}$ and $\mathbf{T}$ are defined as

$$(\mathbf{S}x)[n] = \sum_{k=0}^{2N-1}\sum_{m=-\infty}^{\infty} \langle x, h_{k,m}^{\mathrm{DFT}}\rangle\, f_{k,m}^{\mathrm{DFT}}[n],$$
$$(\mathbf{T}x)[n] = \sum_{k=0}^{2N-1}\sum_{m=-\infty}^{\infty} e^{j2\phi_k} b_m\, \langle x, h_{k,m}^{\mathrm{DFT}}\rangle\, f_{2N-k,m}^{\mathrm{DFT}}[n].$$

(The operators $\mathbf{S}, \mathbf{T}$ in Sections 3 and 4 are a special case.) In the even-stacked case we have $f_{k,m}^{\mathrm{DFT}}[n] = f[n - mM]\, W_{2N}^{-k(n-mM)}$, $h_{k,m}^{\mathrm{DFT}}[n] = h^*[mM-n]\, W_{2N}^{k(mM-n)}$, $\phi_k = \phi_k^e$ and $b_m = (-1)^m$. In the odd-stacked case, $f_{k,m}^{\mathrm{DFT}}[n]$ and $h_{k,m}^{\mathrm{DFT}}[n]$ are obtained by formally replacing k with $k+1/2$ in the above expressions; furthermore $\phi_k = \phi_k^o$ and $b_m = 1$. For an even-stacked (odd-stacked) CMFB the operator $\mathbf{S}$ corresponds to an even-stacked (odd-stacked) DFT FB [6] with $2N$ channels and decimation factor M; this DFT FB has thus the same stacking type and twice the oversampling factor of the CMFB.

PR conditions. We next provide unified PR conditions for oversampled and critically sampled, even- and odd-stacked CMFBs. These PR conditions generalize the PR conditions for WFBs given in Section 4. For PR with zero delay, $\hat{x}[n] = x[n]$, it is necessary and sufficient that [21]

$$\mathbf{S} = 2\mathbf{I} \qquad \text{and} \qquad \mathbf{T} = \mathbf{0}. \tag{14}$$

(Note that (11) is a special case.) For $\mathbf{T} = \mathbf{0}$ the CMFB's input-output relation reduces to $\hat{x}[n] = \frac{1}{2}(\mathbf{S}x)[n]$, which is the input-output relation of the corresponding DFT FB. Hence, *PR CMFBs correspond to PR DFT FBs of the same stacking type and with twice the oversampling factor.* In particular, paraunitary CMFBs can be shown to correspond to paraunitary DFT FBs.

The operators $\mathbf{S}$ and $\mathbf{T}$ can be expressed in the time domain as

$$(\mathbf{S}x)[n] = 2N\sum_{l=-\infty}^{\infty} d_l\, x[n-2lN] \sum_{m=-\infty}^{\infty} f[n-mM] \cdot h[-n+mM+2lN], \tag{15}$$

$$(\mathbf{T}x)[n] = (-1)^r\, 2N \sum_{l=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} d_l a_m\, x[-n+2mM - 2lN - \alpha]\, f[n-mM]\, h[n-mM+\alpha+2lN], \tag{16}$$

with $d_l = 1$ and $a_m = (-1)^m$ in the even-stacked case and $d_l = (-1)^l$ and $a_m = 1$ in the odd-stacked case. With (15), the first PR condition in (14), $\mathbf{S} = 2\mathbf{I}$, is satisfied if and only if

$$\sum_{m=-\infty}^{\infty} f[n-mM]\, h[-n+mM+2lN] = \frac{1}{N}\delta[l].$$

Note that this condition is independent of the stacking type; it is the PR condition for a DFT FB (even- or odd-stacked) with $2N$ channels and decimation factor M, i.e., twice the CMFB's oversampling factor.

Furthermore, for *integer oversampling*, $N = KM$, it follows from (16) that $\mathbf{T} = \mathbf{0}$ is satisfied if and only if

$$\sum_{i=-\infty}^{\infty} p_i\, f[n-(l+iK)M]\, h[n-(l-2mK-iK)M+\alpha] = 0$$

for $l = 0, ..., K-1$ and $-\infty < m < \infty$, where $p_i = (-1)^{iK}$ in the even-stacked case and $p_i = (-1)^i$ in the odd-stacked case. Note that critical sampling, $N = M$, is a special case with $K = 1$.

6 Conclusion

We introduced and studied *Wilson filter banks* (WFBs), a new type of cosine modulated filter banks (CMFBs) that corresponds to the discrete-time Wilson expansion and allows both perfect reconstruction (PR) and linear phase filters in all channels. We formulated PR conditions for oversampled and critically sampled WFBs, and we showed that PR WFBs are associated to PR DFT FBs.

Generalizing WFBs, we then defined the new class of *even-stacked* CMFBs that all have the desirable property of allowing both PR and linear phase filters in all channels. The CMFBs recently introduced for critical sampling by Lin and Vaidyanathan were extended to the oversampled case and shown to be a further special case (besides WFBs) of even-stacked CMFBs.

The conventional ("odd-stacked") CMFBs were also extended to the oversampled case. Finally, we presented a unified set of PR conditions that applies to both even- and odd-stacked, oversampled and critically sampled CMFBs. These PR conditions showed that PR CMFBs are always related to PR DFT filter banks of the same stacking type and with twice the oversampling factor. We note that frame-theoretic properties of oversampled CMFBs are discussed in [21].

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