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OVERSAMPLED FIR AND IIR DFT FILTER BANKS AND WEYL-HEISENBERG FRAMES*

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Abstract—We apply the theory of Weyl-Heisenberg frames (WHFs) to oversampled FIR and IIR DFT filter banks (FBs). We show that the polyphase matrices provide a matrix representation of the frame operator, and we find conditions on a DFT FB to provide a WHF expansion. We also show that paraunitary and biorthogonal DFT FBs correspond to tight and exact WHFs, respectively, and that the frame bounds can be obtained by an eigenanalysis of the polyphase matrices. Simulation results demonstrate the importance of the frame bounds for the design of DFT FBs.

1 INTRODUCTION AND OUTLINE

This paper applies the theory of *Weyl-Heisenberg frames* (WHFs) [1] to FIR and IIR, oversampled or critically sampled, *DFT filter banks* (FBs) [2]-[10]. DFT FBs (also known as modulated FBs) are an important class of *uniform FBs* [5]-[7]. Although the connection between DFT FBs and short-time Fourier transforms (or Gabor expansions [11]) is well established [5]-[7],[12], a frame-theoretical approach to the study of DFT FBs has been proposed only recently [13]-[15]. (While preparing this paper, we became aware of [16] where a frame-theoretical study of continuous-time FBs is presented.)

While only DFT FBs will be considered in this paper, most of our results can readily be extended to general uniform FBs [17]. The relation between discrete-time signal expansions and maximally decimated uniform FBs has been studied in [18, 6]. Recently the theory of frames [1] has been applied to the study of uniform FBs [13]-[15], [19, 20].

Whereas most previous discussions have been restricted to the FIR case [13, 14, 19, 20], we present an alternative approach that applies also to the IIR case. Our theory is based on the fact (shown in the paper) that the FB's polyphase matrices provide a matrix representation of the frame operator. This fundamental result allows a very efficient frame-theoretical analysis of uniform FBs that is consistent with many results previously derived in the FB literature and also provides a number of new results.

This paper is organized as follows. Section 2 briefly reviews the relation between DFT FBs and WHFs. Section 3 introduces the matrix representation of the frame operator on which our theory is based. Section 4 formulates conditions for perfect reconstruction and for the frame property, and shows how the frame bounds of a FB can be obtained from the FB's polyphase matrices. Section 5 discusses the equivalence of paraunitary DFT FBs and tight WHFs, and proposes a new procedure for the design of paraunitary DFT FBs. Sections 6 and 7 consider the cases of integer and rational oversampling, respectively. Simulation results demonstrating the importance of the frame bounds for the design of DFT FBs are presented in Section 8.

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2 OVERSAMPLED DFT FILTER BANKS AND WEYL-HEISENBERG FRAMES

We consider N-channel DFT (or modulated) FBs [2]-[10] with subsampling by the integer factor M in each channel, perfect reconstruction (PR) and zero delay, so that $\hat{x}[n] = x[n]$ where $x[n]$ and $\hat{x}[n]$ denote the input and reconstructed signal, respectively. (We note that our theory can easily be extended to nonzero delay.) The transfer functions of the analysis and synthesis filters are respectively

$$H_k(z) = H(zW_N^k)\,, \quad F_k(z) = F(zW_N^k)\,, \quad k = 0, 1, ..., N-1$$

with $W_N = e^{-j2\pi/N}$, and the corresponding impulse responses are $h_k[n] = h[n]\,W_N^{-kn}$ and $f_k[n] = f[n]\,W_N^{-kn}$. The polyphase decomposition [2], [5]-[7] of the analysis prototype reads $H(z) = \sum_{l=0}^{M-1} z^l\, E_l(z^M)$ where

$$E_l(z) = \sum_{n=-\infty}^{\infty} h[nM - l]\, z^{-n}, \qquad l = 0, 1, ..., M-1\,.$$

The $N \times M$ analysis polyphase matrix $\mathbf{E}(z)$ is defined as $[\mathbf{E}(z)]_{kl} = E_{k,l}(z)$, where $E_{k,l}(z) = W_N^{kl}\, E_l(z\, W_N^{Mk})$ is the l-th polyphase component of the k-th analysis filter. Similarly, the polyphase decomposition of the synthesis prototype reads $F(z) = \sum_{l=0}^{M-1} z^{-l} R_l(z^M)$ with

$$R_l(z) = \sum_{n=-\infty}^{\infty} f[nM + l]\, z^{-n}, \qquad l = 0, 1, ..., M-1\,,$$

and the $M \times N$ synthesis polyphase matrix $\mathbf{R}(z)$ is defined as $[\mathbf{R}(z)]_{kl} = R_{l,k}(z)$ where $R_{k,l}(z) = W_N^{-kl}\, R_l(z\, W_N^{Mk})$ is the l-th polyphase component of the k-th synthesis filter. We restrict our attention to the oversampled case ($N > M$) and the critically sampled case ($N = M$) since for undersampling ($N < M$) PR cannot be achieved [17].

The reconstructed signal is given by

$$\hat{x}[n] = \sum_{k=0}^{N-1} \sum_{m=-\infty}^{\infty} v_k[m]\, f_k[n - mM]$$

with the subband signals

$$v_k[m] = \sum_{n=-\infty}^{\infty} x[n]\, h_k[mM - n] = \langle x, h_{k,m}^{-*} \rangle,$$

where $h_{k,m}^{-}[n] = h_k[mM-n]$ ($k = 0, 1, ..., N-1$) and * stands for complex conjugation. Setting $f_{k,m}[n] = f_k[n - mM]$ and using the PR property, we obtain

$$x[n] = \hat{x}[n] = \sum_{k=0}^{N-1} \sum_{m=-\infty}^{\infty} \left\langle x, h_{k,m}^{-*} \right\rangle f_{k,m}[n].$$

Hence, the PR DFT FB provides an expansion of the input signal $x[n]$ into the *Weyl-Heisenberg set* [1] $f_{k,m}[n] = f[n - mM]\, W_N^{-k(n-mM)}$ $(k = 0, 1, ..., N-1,\ -\infty < m < \infty)$ generated by $f[n]$. In general the $f_{k,m}[n]$ are not orthogonal, so that the expansion coefficients (= subband signals), $v_k[m] = \left\langle x, h_{k,m}^{-*} \right\rangle$, are obtained by projecting $x[n]$ onto a "dual" Weyl-Heisenberg set, $h_{k,m}^{-*}[n] = h^{-*}[n - mM]\, W_N^{-k(n-mM)}$, generated by $h^{-*}[n] = h^*[-n]$. Critically sampled DFT FBs correspond to orthogonal or biorthogonal Weyl-Heisenberg sets, whereas oversampled DFT FBs correspond to redundant (overcomplete) Weyl-Heisenberg sets.

The theory of *Weyl-Heisenberg frames* (WHFs) [1] provides a powerful vehicle for the study of oversampled and critically sampled DFT FBs. The Weyl-Heisenberg set $\{f_{k,m}[n]\}$ is called a WHF for $l^2(\mathbb{Z})$ if

$$A\|x\|^2 \le \sum_{k=0}^{N-1} \sum_{m=-\infty}^{\infty} \left|\left\langle x, f_{k,m} \right\rangle\right|^2 \le B\|x\|^2 \qquad \forall\ x[n] \in l^2(\mathbb{Z})$$

with the *frame bounds* $A > 0$ and $B < \infty$. The frame bound ratio B/A characterizes the numerical properties of the WHF and the corresponding DFT FB. For synthesis prototype $f[n]$ such that $\{f_{k,m}[n]\}$ is a WHF for $l^2(\mathbb{Z})$, the analysis prototype with minimum energy (norm) is given by

$$h[n] = (\boldsymbol{S}^{-1} f)^*[-n]. \tag{1}$$

Here, $\boldsymbol{S}^{-1}$ is the inverse of the *frame operator* $\boldsymbol{S}$ defined as

$$(\boldsymbol{S}x)[n] = \sum_{k=0}^{N-1} \sum_{m=-\infty}^{\infty} \left\langle x, f_{k,m} \right\rangle f_{k,m}[n].$$

The inverse frame operator is a linear, positive definite operator mapping $l^2(\mathbb{Z})$ onto $l^2(\mathbb{Z})$. If $\{f_{k,m}[n]\}$ is a WHF, then $\{h_{k,m}^{-*}[n]\}$ is a WHF as well (the "dual" frame), with frame bounds $A' = 1/B$ and $B' = 1/A$. The subband signals $v_k[m] = \left\langle x, h_{k,m}^{-*} \right\rangle$ of a FB corresponding to a WHF satisfy

$$A'\|x\|^2 \le \sum_{k=0}^{N-1} \sum_{m=-\infty}^{\infty} |v_k[m]|^2 \le B'\|x\|^2,$$

which generalizes the energy conservation equation in orthonormal filter banks [18]. A frame is called *snug* if $\frac{B}{A} = \frac{B'}{A'} \approx 1$ and *tight* if $\frac{B}{A} = \frac{B'}{A'} = 1$. For a tight WHF $h[n] = A' f^*[-n]$. We note that complete orthogonal and biorthogonal sets are special cases of frames.

3 MATRIX REPRESENTATION

The analysis polyphase matrix $\mathbf{E}(z)$ can be shown to provide a matrix representation of the inverse frame operator [17]:

Theorem 1 [17]. Let $\mathbf{S}^{-1}(e^{j2\pi\theta})$ denote the (positive definite) matrix representing the inverse frame operator $\boldsymbol{S}^{-1}$ with respect to the orthonormal basis $\{e_{l,\theta}[n]\}$ of $l^2(\mathbb{Z})$ given by[1] $e_{l,\theta}[n] = \sum_{r=-\infty}^{\infty} \delta[n - l - rM]\, e^{j2\pi\frac{\theta}{M}(n-l)}$ $(l = 0, 1, ..., M-1,\ 0 \le \theta < 1)$. Then[2]

$$\mathbf{S}^{-1}(e^{j2\pi\theta}) = \mathbf{E}^H(e^{j2\pi\theta})\, \mathbf{E}(e^{j2\pi\theta}). \tag{2}$$

Similarly, the frame operator $\boldsymbol{S}$ is represented by the matrix $\mathbf{S}(e^{j2\pi\theta}) = \mathbf{R}(e^{j2\pi\theta})\, \mathbf{R}^H(e^{j2\pi\theta})$. It follows that the eigenvalues of the inverse frame operator $\boldsymbol{S}^{-1}$ and the eigenvalues of the matrix $\mathbf{S}^{-1}(e^{j2\pi\theta})$ are equal. As we will show in the next sections, the matrix representation (2) is a powerful tool for the analysis and the design of PR DFT FBs that correspond to WHFs. A similar approach has been used for the study of continuous-time WHFs in [22].

4 PERFECT RECONSTRUCTION AND FRAME PROPERTIES

A DFT FB satisfies the PR property with zero delay, $\hat{x}[n] = x[n]$, if and only if

$$\mathbf{R}(z)\, \mathbf{E}(z) = \mathbf{I}_M,$$

where $\mathbf{I}_M$ is the $M \times M$ identity matrix. This implies that $\mathbf{E}(z)$ has full rank almost everywhere. In the critically sampled case $(N = M)$, the synthesis polyphase matrix is given by $\mathbf{R}(z) = \mathbf{E}^{-1}(z)$. In the oversampled case $(N > M)$, $\mathbf{E}(z)$ is rectangular and therefore $\mathbf{R}(z)$, and hence the synthesis FB, is not uniquely determined—in fact, $\mathbf{R}(z)$ can be any left-inverse of $\mathbf{E}(z)$. In what follows, we shall restrict our discussion to the *minimum norm* synthesis prototype $f[n]$ provided by frame theory (see (1)), which is given by[3]

$$\mathbf{R}(z) = \mathbf{S}(z)\, \tilde{\mathbf{E}}(z), \tag{3}$$

where $\mathbf{S}(z) = \left[\tilde{\mathbf{E}}(z)\, \mathbf{E}(z)\right]^{-1} = \mathbf{R}(z)\, \tilde{\mathbf{R}}(z)$. The minimum norm synthesis FB corresponds to the WHF $\{f_{k,m}[n]\}$ dual to the WHF $\{h_{k,m}^{-*}[n]\}$. In [14] this result has been stated independently for FIR oversampled FBs.

Besides PR, the *frame property* is desirable since it guarantees a certain degree of numerical stability (as characterized by the frame bound ratio B'/A'). We now formulate conditions on a DFT FB to correspond to a WHF for $l^2(\mathbb{Z})$, such that $\{h_{k,m}^{-*}[n]\}$ and $\{f_{k,m}[n]\}$ are (dual) WHFs.

Theorem 2 [17]. A DFT FB with BIBO stable analysis prototype $h[n]$ provides a WHF expansion in $l^2(\mathbb{Z})$ if and only if $\mathrm{rank}\{\mathbf{E}(e^{j2\pi\theta})\} = M$, i.e., the polyphase matrix on the unit circle, $\mathbf{E}(e^{j2\pi\theta})$, has full rank for all θ.

We emphasize that this result holds for both FIR and IIR $h[n]$. A similar result has been found in [14] for the FIR case, where stability is always guaranteed.

Alternatively, it can be shown that a DFT FB corresponds to a WHF for $l^2(\mathbb{Z})$ if $\mathbf{E}(e^{j2\pi\theta})$ has full rank and the $E_{k,l}(e^{j2\pi\theta})$ are continuous and bounded functions of θ. Alternatively, a DFT FB corresponds to a WHF for $l^2(\mathbb{Z})$ if and only if

$$0 < A' \le \lambda_l(\theta) \le B' < \infty$$

for $l = 0, 1, ..., M-1$ and $\theta \in [0, 1)$, where the $\lambda_l(\theta)$ are the eigenvalues of the matrix $\mathbf{S}^{-1}(e^{j2\pi\theta}) = \mathbf{E}^H(e^{j2\pi\theta})\, \mathbf{E}(e^{j2\pi\theta})$.

In the following, a PR DFT FB corresponding to a WHF will be called a *WHF FB*. The next theorem states that critically sampled WHF FBs correspond to *exact* WHFs, i.e., the WHFs $\{h_{k,m}^{-*}[n]\}$ and $\{f_{k,m}[n]\}$ are both linearly independent sets.

[1] This basis induces the polyphase representation, $\left\langle f, e_{l,\theta} \right\rangle = R_l(e^{j2\pi\theta})$. This is essentially the *Zak transform* of $f[n]$ [21].

[2] The superscript ^{H} stands for conjugate transposition.

[3] $\tilde{\mathbf{E}}(z) = \mathbf{E}^H(1/z^*)$.

Theorem 3 [17]. A WHF FB corresponds to an *exact* WHF for $l^2(\mathbb{Z})$ if and only if it is *critically sampled*, i.e., $N = M$.

We note that the linear independence of the set $\{h_{k,m}^{-*}[n]\}$ for critically sampled uniform FBs has been previously shown in [18]. Exact WHFs satisfy the *biorthogonality relation*

$$\langle h_{k,m}^{-*}, f_{k',m'} \rangle = \delta[k-k']\,\delta[m-m'],$$

which is in accordance with the fact that for a critically sampled uniform FB biorthogonality is equivalent with PR [18].

We now show how the frame bounds A', B' characterizing the numerical properties of a WHF FB can be derived from the polyphase matrix $\mathbf{E}(z)$.

Theorem 4 [17]. The frame bounds A', B' of a WHF FB are

$$A' = \inf_{l=0,1,\ldots,M-1,\,\theta\in[0,1)} \lambda_l(\theta)$$
$$B' = \sup_{l=0,1,\ldots,M-1,\,\theta\in[0,1)} \lambda_l(\theta),$$

where the $\lambda_l(\theta)$ are the eigenvalues of $\mathbf{S}^{-1}(e^{j2\pi\theta}) = \mathbf{E}^H(e^{j2\pi\theta})\,\mathbf{E}(e^{j2\pi\theta})$.

Note that $A{=}1/B'$ and $B{=}1/A'$ can similarly be obtained from the eigenvalues of $\mathbf{S}(e^{j2\pi\theta}) = \mathbf{R}(e^{j2\pi\theta})\,\mathbf{R}^H(e^{j2\pi\theta})$.

5 PARAUNITARY DFT FILTER BANKS

We now address the case of *tight* frames ($A' = B'$) which yields best numerical properties.

Theorem 5 [17]. A WHF FB corresponds to a *tight* WHF with frame bound A' if and only if it is *paraunitary*, i.e.,

$$\mathbf{S}^{-1}(z) \equiv A'\,\mathbf{I}_M. \qquad (4)$$

Note that (4) implies $\mathbf{R}(z) = \frac{1}{A'}\tilde{\mathbf{E}}(z), f[n] = \frac{1}{A'}h^*[-n]$ and $\lambda_l(\theta) \equiv A'$. The equivalence of tight WHFs and paraunitary DFT FBs has also been discussed in [15] and independently in [13, 14]. This equivalence holds both in the FIR and the IIR case. Frame theory [1] suggests a procedure for constructing paraunitary WHF FBs from given nonparaunitary WHF FBs:

Theorem 6 [17]. Consider a WHF FB with polyphase matrices $\mathbf{E}(z)$ and $\mathbf{R}(z)$, and let $\mathbf{P}(z)$ be an invertible, parahermitian,[4] $M\times M$ matrix such that $\mathbf{R}(z)\,\tilde{\mathbf{R}}(z) = \mathbf{P}^2(z)$. Then, the WHF FB with analysis polyphase matrix $\mathbf{E}_p(z) \triangleq \mathbf{E}(z)\,\mathbf{P}(z)$ is paraunitary with frame bound $A' = 1$, i.e., $\mathbf{S}_p^{-1}(z) = \tilde{\mathbf{E}}_p(z)\,\mathbf{E}_p(z) = \mathbf{I}_M$.

It can also be shown that the procedure described in this theorem transforms a biorthogonal WHF FB into an orthogonal WHF FB. Note that orthogonality and paraunitarity are equivalent only in the critical case.

6 INTEGER OVERSAMPLING

For integer oversampled DFT FBs ($N = KM$) [22, 21] the matrix $\mathbf{S}^{-1}(z) = \tilde{\mathbf{E}}(z)\,\mathbf{E}(z)$ is diagonal with diagonal elements $[\mathbf{S}^{-1}(z)]_{ll} = \Lambda_l(z)$, where

$$\Lambda_l(z) \triangleq M\sum_{r=0}^{K-1} E_l(zW_K^r)\,\tilde{E}_l(zW_K^r), \qquad l = 0,1,..,M-1$$

with $\tilde{E}_l(z) = E_l^*(\frac{1}{z^*})$. An integer oversampled DFT FB corresponds to a WHF with frame bounds A', B' if and only if $0 < A' \le \lambda_l(\theta) \le B' < \infty$ for $l = 0,1,\ldots,M-1$ and $\theta \in [0,1)$, where the eigenvalues of $\mathbf{S}^{-1}(e^{j2\pi\theta})$ are given by

$$\lambda_l(\theta) = \Lambda_l(e^{j2\pi\theta}) = M\sum_{r=0}^{K-1}\left|E_l\left(e^{j2\pi(\theta-\frac{r}{K})}\right)\right|^2.$$

We recall from Theorem 4 that the frame bounds A', B' are given by the infimum and supremum, respectively, of the eigenvalues $\lambda_l(\theta)$. From (3) it follows that

$$R_l(z) = \frac{\tilde{E}_l(z)}{\Lambda_l(z)}.$$

An integer oversampled WHF FB is paraunitary with frame bound A' if and only if

$$\Lambda_l(z) \equiv A' \qquad \text{for } l = 0,1,\ldots,M-1. \qquad (5)$$

In the critical case ($K = 1$), $\Lambda_l(z) = M\,E_l(z)\,\tilde{E}_l(z)$ and the above relations simplify accordingly. In particular, (5) reduces to $E_l(z)\,\tilde{E}_l(z) = A'/M$ for $l = 0,1,\ldots,M-1$, which implies $|E_l(e^{j2\pi\theta})|^2 \equiv A'/M$. Hence, the polyphase filters $E_l(z)$ are allpass filters, and thus the design of critically sampled paraunitary WHF FBs reduces to finding an arbitrary set of M allpass filters.

For K=2, a paraunitary WHF FB with frame bound A' can be constructed by choosing the polyphase filters such that the following power symmetry condition [5] holds:

$$E_l(z)\tilde{E}_l(z) + E_l(-z)\tilde{E}_l(-z) = \frac{A'}{M} \quad \text{for} \quad l = 0,1,\ldots,M-1.$$

7 RATIONAL OVERSAMPLING

In the more general case of rational oversampling ($N = \frac{p}{q}M$), the matrix $\mathbf{S}^{-1}(z)$ is not diagonal in general. However, by imposing restrictions on the temporal support (length) of $h[n]$, explicit expressions for the frame bounds and for the analysis prototype can be found.[5] It can be shown [21] that an FIR DFT FB satisfying length$\{h[n]\} \le N$ corresponds to a WHF with frame bounds A', B' if and only if $0 < A' \le \lambda_n \le B' < \infty$, where the eigenvalues of $\mathbf{S}^{-1}(e^{j2\pi\theta})$ are given by the M-periodic function $\lambda_n = N\sum_{r=-\infty}^{\infty}|h[-n-rM]|^2$. Note that this condition can be satisfied only if length$\{h[n]\} \ge M$, since otherwise temporal gaps would cause $A' = 0$. The frame bounds are given by $A' = \min\lambda_n$ and $B' = \max\lambda_n$. Furthermore

$$f[n] = \frac{h^*[-n]}{\lambda_n},$$

which shows that length$\{f[n]\}$ = length$\{h[n]\}$. The FB is paraunitary with frame bound A' if and only if $\lambda_n \equiv A'$.

8 SIMULATION RESULTS

We now present simulation results demonstrating the importance of "good" frame bounds ($A' \approx B'$). Fig. 1 shows[6] the 64-tap analysis and synthesis prototypes of a WHF FB with $N = 64, M = 8$ (i.e., integer oversampling by a factor of 8). The frame bounds of this FB satisfy $\frac{B'}{A'} = 1.0509$. Thus, the corresponding WHF is snug, and equivalently the FB is "nearly paraunitary" in the sense that the analysis and synthesis prototypes are nearly identical (recall that for a paraunitary WHF FB there is $\frac{B'}{A'} = 1$ and $f[n] = \frac{1}{A'}h^*[-n]$).

[4] A matrix $\mathbf{P}(z)$ is said to be parahermitian if $\tilde{\mathbf{P}}(z) = \mathbf{P}(z)$.

[5] Similar expressions are obtained by imposing restrictions on the prototype's spectral support (i.e., bandwidth) [21].

[6] All plots are normalized.

Fig. 2 considers a WHF FB, again with 64-tap prototypes and $N = 64, M = 8$, where now $\frac{B'}{A'} = 4.0787$, i.e., the frame bounds are quite poor. Here, the analysis and synthesis prototypes are very different; in particular, the frequency selectivity of the synthesis prototype is extremely poor.

Finally, Fig. 3 depicts two paraunitary prototypes which were obtained by applying the procedure described in Theorem 6 to the FBs in Fig. 1 and Fig. 2, respectively. (Note that paraunitarity implies that the analysis and synthesis prototypes are identical except for conjugation, time reversal and a constant factor.) We see from Fig. 3(d) that the paraunitary prototype derived from the FB with poor frame bounds has very poor frequency selectivity.

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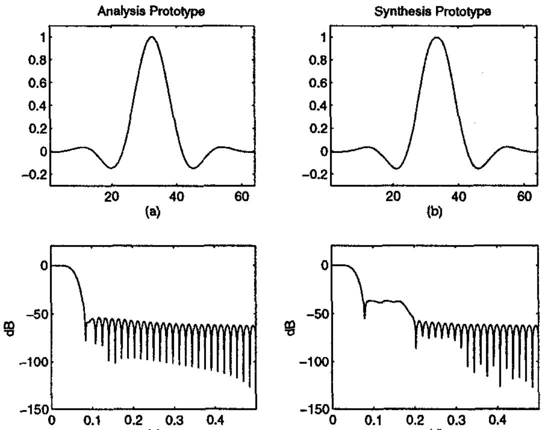

Fig. 1. DFT filter bank with $B'/A' = 1.0509$: (a) $h[n]$, (b) $f[n]$, (c) $H(e^{j2\pi\theta})$, (d) $F(e^{j2\pi\theta})$.

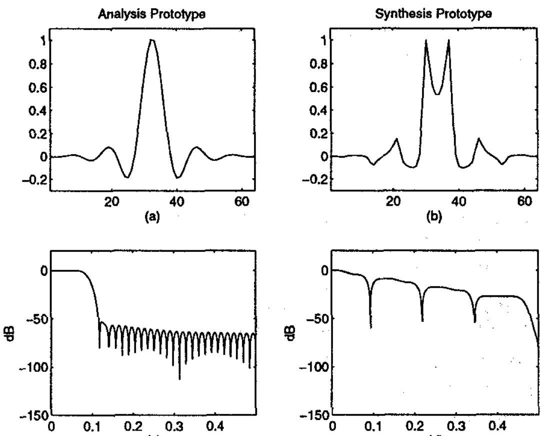

Fig. 2. DFT filter bank with $B'/A' = 4.0787$: (a) $h[n]$, (b) $f[n]$, (c) $H(e^{j2\pi\theta})$, (d) $F(e^{j2\pi\theta})$.

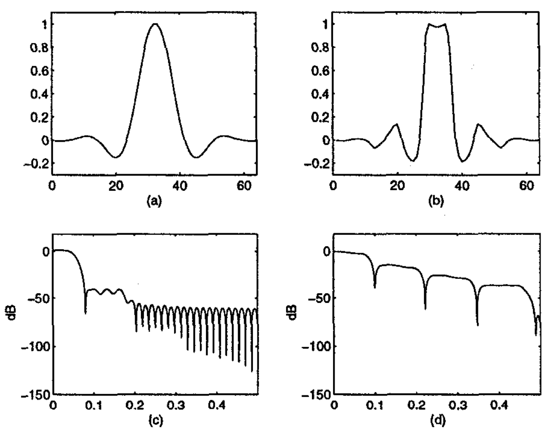

Fig. 3. (a) Paraunitary prototype derived from $h[n]$ in Fig. 1, (b) paraunitary prototype derived from $h[n]$ in Fig. 2, (c) Fourier transform of (a), (d) Fourier transform of (b).