AEÜ Int. J. Electron. Commun.
53 (1999) No. 6, 1–6

Time-Frequency Subspace Detectors and Application to Knock Detection

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Dedicated with great respect to Prof. J. F. Böhme on the occasion of his sixtieth birthday.

Abstract **We consider a composite hypotheses detection problem involving a subspace and an energy level, and we show that for this problem the matched subspace detector is uniformly most powerful and the generalized likelihood ratio detector. We present a generalization of the matched subspace detector and time-frequency methods for the formulation and design of subspace detectors. Finally, we apply the various detectors to the problem of detecting knock in car engines, and we demonstrate the potential advantages of time-frequency detectors.**

Keywords **Signal detection, time-frequency analysis, nonstationary processes, knock detection.**

1. Introduction

Signal detection [1–3] is important in many applications. This paper is motivated by contributions of J. F. Böhme and his co-workers to the problem of detecting knock in car engines [4–7]. We study the discrimination between two signals $s_0(t)$ (non-knocking combustion) and $s_1(t)$ (knocking combustion) embedded in stationary white Gaussian noise $n(t)$ with power spectral density η. The hypotheses thus are

$$H_0: \quad r(t) = s_0(t) + n(t)\,,$$
$$H_1: \quad r(t) = s_1(t) + n(t)\,.$$

If $s_0(t)$ and $s_1(t)$ are modeled as Gaussian nonstationary random processes with known correlation operators[1] $\mathbf{R}_{s_0}$ and $\mathbf{R}_{s_1}$, the likelihood ratio detection statistic [1–3] is the quadratic form[2]

$$\Lambda(r) = \langle \mathbf{H} r, r\rangle = \int_t \int_{t'} H(t,t')\, r(t')\, r^*(t)\, dt\, dt' \qquad (1)$$

induced by the linear operator $\mathbf{H} = \mathbf{H}_{\rm LR} \triangleq \mathbf{R}_0^{-1}(\mathbf{R}_1 - \mathbf{R}_0)\mathbf{R}_1^{-1} = \mathbf{R}_0^{-1} - \mathbf{R}_1^{-1}$ with $\mathbf{R}_i = \mathbf{R}_{s_i} + \eta\,\mathbf{I}$. A decision is obtained by comparing $\Lambda(r)$ to a threshold.

However, as the signals $s_0(t)$ and $s_1(t)$ may not be Gaussian and/or $\mathbf{R}_{s_0}$ and $\mathbf{R}_{s_1}$ may not be known with sufficient accuracy, we here use a less detailed signal model. We merely assume that $s_0(t)$ and $s_1(t)$ lie in a known linear signal subspace $\mathcal{S} \subseteq L_2(\mathbb{R})$ of dimension p, and that their energies are (respectively) below and above some fixed level γ^2 [8], i.e.,

$$s_i(t) \in \mathcal{S}\,, \quad \|s_0\|^2 \le \gamma^2\,, \quad \|s_1\|^2 > \gamma^2\,. \qquad (2)$$

Thus, $s_0(t)$ and $s_1(t)$ lie (respectively) inside and outside a p-dimensional hypersphere of radius γ in $\mathcal{S}$.

The above model can be motivated by the problem of knock detection. It is known that the pressure and vibration signals induced by knocking as well as non-knocking combustions consist of several resonances with decaying frequencies [4–7, 9, 10]. Hence, the signal subspace $\mathcal{S}$ is spanned by these time-varying resonance modes. The use of the energy threshold γ^2 is motivated by the fact that the signal energy for knocking combustions tends to be larger than for non-knocking combustions [4–7]. We note that several detectors directly based on resonance energies were recently considered in [6].

The rest of this paper is organized as follows. Section 2 discusses subspace detectors for the detection problem formulated above. Section 3 considers a time-frequency formulation and design of subspace detectors. The estimation of prior knowledge from training data is considered in Section 4. Finally, in Section 5 the various detectors are applied to the problem of knock detection.

2. Subspace Detectors

An alternative formulation of (2) is

$$s_i(t) = \sum_{l=1}^{p} \theta_l^{(i)} u_l(t)\,, \quad \|\underline{\theta}^{(0)}\| \le \gamma\,, \quad \|\underline{\theta}^{(1)}\| > \gamma\,,$$

where $\{u_l(t)\}_{l=1,..,p}$ is some fixed orthonormal basis of $\mathcal{S}$ and $\underline{\theta}^{(i)} = \big(\theta_1^{(i)}, \ldots, \theta_p^{(i)}\big)^T$. Since the expansion coefficients $\underline{\theta}^{(i)}$ are unknown, our hypotheses are composite. Two possible detection strategies are the following:

- One may search for a maximally invariant detector that is *uniformly most powerful* (UMP), i.e., that maximizes the detection probability for all possible signal parameters $\underline{\theta}^{(i)}$ while keeping the false alarm probability below a certain threshold [2].
- An alternative is the *generalized likelihood ratio* (GLR) detector, where the parameters $\underline{\theta}^{(i)}$ occurring in the likelihood ratio are replaced by their (conditional) maximum-likelihood (ML) estimates [1, 3].

[1] The correlation operator $\mathbf{R}_x$ of a (generally nonstationary) random process $x(t)$ is the linear operator whose kernel equals the correlation function $R_x(t,t') = \mathrm{E}\{x(t)\, x^*(t')\}$. In a discrete-time setting, the correlation operator corresponds to a matrix.

[2] The inner product is defined as $\langle x, y\rangle = \int_t x(t)\, y^*(t)\, dt$. All integrals go from $-\infty$ to ∞. $\mathbf{H}$ is a linear, time-varying system (linear operator) with kernel $H(t,t')$.

Received month 00, 1997.

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AEÜ Int. J. Electron. Commun.
53 (1999) No. 6, 1–6

Matched Subspace Detector. The *matched subspace detector* [2, 11] is given by the detection statistic[3]

$$\Lambda_{\mathcal{S}}(r) = \langle \mathbf{P}_{\mathcal{S}} r, r \rangle = \|\mathbf{P}_{\mathcal{S}} r\|^2 \,, \qquad (3)$$

where $\mathbf{P}_{\mathcal{S}}$ denotes the orthogonal projection operator on $\mathcal{S}$. Note that $\Lambda_{\mathcal{S}}(r)$ is a quadratic detection statistic as in (1), however with $\mathbf{H} = \mathbf{P}_{\mathcal{S}}$. For the special case of our detection problem where $s_0(t) \equiv 0$ (or equivalently $\underline{\theta}^{(0)} = \underline{0}$) and $\gamma = 0$, the matched subspace detector was shown in [2, 11] to be (i) maximally invariant to observation components in the orthogonal complement space $\mathcal{S}^{\perp}$ and to rotations within $\mathcal{S}$, (ii) UMP, and (iii) the GLR detector.

This result can be shown to be true also in the general case where $s_0(t) \neq 0$ and $\gamma \neq 0$. Indeed, the maximal invariance proof in [2] straightforwardly carries over to our generalized problem. A similar fact holds for the UMP property: Since $\Lambda_{\mathcal{S}}/\eta$ under H_i is χ^2 distributed with p degrees of freedom and noncentrality parameter $\|s_i\|^2/\eta$, it has a monotone likelihood ratio and thus is UMP [2]. Finally, the equivalence to the GLR detector is stated in the next theorem that complements the discussion in [11] and is proved in the Appendix.

Theorem 1. *$\Lambda_{\mathcal{S}}(r)$ is the GLR detection statistic for testing $\|s\|^2 \leq \gamma^2$ vs. $\|s\|^2 > \gamma^2$ with $s(t) \in \mathcal{S}$.*

We note that the detection statistic $\Lambda_{\mathcal{S}}(r)$ does not depend on γ^2. It can be efficiently computed as

$$\Lambda_{\mathcal{S}}(r) = \sum_{l=1}^{p} |\langle r, u_l \rangle|^2, \qquad (4)$$

which requires $p\,(N+1)$ multiplications and additions (with N the length of the sampled observation $r(t)$). For $p < \log_2 N$, this is even less expensive than an FFT of the observation.

The false alarm and detection probabilities achieved by the matched subspace detector using threshold ζ_0 are

$$P_f = P\{\Lambda_{\mathcal{S}} > \zeta_0 | H_0\} = 1 - F^{(0)}_{\Lambda_{\mathcal{S}}}(\zeta_0) \,,$$
$$P_d = P\{\Lambda_{\mathcal{S}} > \zeta_0 | H_1\} = 1 - F^{(1)}_{\Lambda_{\mathcal{S}}}(\zeta_0) \,,$$

where $F^{(i)}_{\Lambda_{\mathcal{S}}}(\zeta)$ denotes the cumulative distribution function of the detection statistic $\Lambda_{\mathcal{S}}$ under hypothesis H_i. We recall that $\Lambda_{\mathcal{S}}/\eta$ is χ^2 distributed with p degrees of freedom and noncentrality parameter $\|s_i\|^2/\eta$.

Extended Matched Subspace Detector. As an intermediate model between the subspace model (known subspace affiliation but no statistical prior knowledge) and the likelihood ratio detector (complete statistical prior knowledge), we next assume that $s_0(t)$, $s_1(t)$ are nonstationary random processes whose energies in K disjoint signal subspaces $\mathcal{S}_k$, $\langle \mathbf{P}_{\mathcal{S}_k} s_i, s_i \rangle = \|\mathbf{P}_{\mathcal{S}_k} s_i\|^2$, have known statistical properties. Motivated by the approach taken in [6], we then use the subspace energies of $r(t)$,

$$\xi_k \triangleq \langle \mathbf{P}_{\mathcal{S}_k} r, r \rangle = \|\mathbf{P}_{\mathcal{S}_k} r\|^2, \qquad k = 1, \ldots, K, \qquad (5)$$

as intermediate statistics and devise a likelihood ratio detector based on the ξ_k [8]. Specifically, if the $s_i(t)$ are Gaussian and the $\mathcal{S}_k$ have small dimensions so that $\mathbf{P}_{\mathcal{S}_k} \mathbf{R}_i \mathbf{P}_{\mathcal{S}_k}$ has only one dominant eigenvalue $\lambda_k^{(i)}$, then under H_i the ξ_k are approximately independent and exponentially distributed with parameters $\mu_k^{(i)} = 1/\lambda_k^{(i)}$ [2, 12]. Here, the likelihood ratio detection statistic is [8]

$$\Lambda_{\mathrm{ES}}(r) = \sum_{k=1}^{K} \left[\mu_k^{(0)} - \mu_k^{(1)}\right] \xi_k \,. \qquad (6)$$

Note that $\Lambda_{\mathrm{ES}}(r)$ can be written as a quadratic form as in (1) with operator $\mathbf{H} = \sum_{k=1}^{K} \left[\mu_k^{(0)} - \mu_k^{(1)}\right] \mathbf{P}_{\mathcal{S}_k}$.

3. Time-Frequency Formulation

Any quadratic detection statistic of the type (1) can be rewritten as an inner product in the time-frequency (TF) domain [13–15],

$$\Lambda(r) = \langle \mathbf{H} r, r \rangle = \langle L_{\mathbf{H}}, W_r \rangle \qquad (7)$$
$$= \int_t \int_f L_{\mathbf{H}}(t,f)\, W_r(t,f)\, dt\, df.$$

Here,

$$L_{\mathbf{H}}(t,f) \triangleq \int_\tau H\Big(t + \frac{\tau}{2},\, t - \frac{\tau}{2}\Big)\, e^{-j2\pi f \tau}\, d\tau$$

is the *Weyl symbol* of the operator $\mathbf{H}$ [16–19] and

$$W_r(t,f) \triangleq \int_\tau r\Big(t + \frac{\tau}{2}\Big)\, r^*\Big(t - \frac{\tau}{2}\Big)\, e^{-j2\pi f \tau}\, d\tau$$

is the *Wigner distribution* of $r(t)$ [20–22]. Thus, any quadratic detection statistic can be interpreted as a weighted integral of $W_r(t,f)$. For the matched subspace detector $\Lambda_{\mathcal{S}}$ in (3), the TF weight function is given by

$$L_{\mathbf{P}_{\mathcal{S}}}(t,f) = W_{\mathcal{S}}(t,f) \,,$$

where $W_{\mathcal{S}}(t,f)$ is the Wigner distribution of the signal space $\mathcal{S}$ as defined in [23, 24] (this can be interpreted as the Wigner distribution of a signal averaged over $\mathcal{S}$).

TF Detector. We shall now develop an approximation to the matched subspace detector $\Lambda_{\mathcal{S}}$ that is simpler and more intuitive since the subspace $\mathcal{S}$ is replaced by a TF region. Indeed, a *non-sophisticated* signal subspace $\mathcal{S}$ (i.e., a space where $\mathbf{P}_{\mathcal{S}}$ causes only small TF displacements [23]) can be associated with a TF region $\mathcal{R}$ such that $W_{\mathcal{S}}(t,f)$ is approximately equal to the indicator function of $\mathcal{R}$,

$$W_{\mathcal{S}}(t,f) \approx I_{\mathcal{R}}(t,f) \triangleq \begin{cases} 1 & \text{for } (t,f) \in \mathcal{R}\,, \\ 0 & \text{for } (t,f) \notin \mathcal{R}\,. \end{cases} \qquad (8)$$

The area of $\mathcal{R}$ approximately equals the dimension p of the space $\mathcal{S}$ [23, 24]. Replacing $W_{\mathcal{S}}(t,f)$ with $I_{\mathcal{R}}(t,f)$ yields the particularly simple *TF detection statistic*

$$\tilde{\Lambda}_{\mathcal{S}}(r) \triangleq \langle I_{\mathcal{R}}, W_r \rangle = \iint_{\mathcal{R}} W_r(t,f)\, dt\, df \,,$$

[3] The second identity in (3) follows from the fact that $\mathbf{P}_{\mathcal{S}}$ is idempotent ($\mathbf{P}_{\mathcal{S}}^2 = \mathbf{P}_{\mathcal{S}}$) and self-adjoint ($\mathbf{P}_{\mathcal{S}}^+ = \mathbf{P}_{\mathcal{S}}$).

which due to (8) approximates the matched subspace detector $\Lambda_{\mathcal{S}}$. $I_{\mathcal{R}}(t,f)$ can be written as the Weyl symbol of an operator $\widetilde{\mathbf{P}}$ whose kernel is given by

$$\tilde{P}(t,t') = \int_f I_{\mathcal{R}}\left(\frac{t+t'}{2}, f\right) e^{j2\pi(t-t')f}\, df\,.$$

With $I_{\mathcal{R}}(t,f) = L_{\widetilde{\mathbf{P}}}(t,f)$ and (7), $\tilde{\Lambda}_{\mathcal{S}}$ can also be written as a quadratic form, $\tilde{\Lambda}_{\mathcal{S}}(r) = \langle \widetilde{\mathbf{P}} r, r\rangle$. Using the eigenvalues and eigenfunctions of $\widetilde{\mathbf{P}}$, and approximating small eigenvalues by zero, $\tilde{\Lambda}_{\mathcal{S}}(r)$ can also be written similarly as in (4). This yields a particularly efficient implementation.

Extended TF Detector. A TF version of the extended matched subspace detector Λ_{ES} in (6) is obtained by approximating the ξ_k in (5) by

$$\tilde{\xi}_k \triangleq \iint_{\mathcal{R}_k} W_r(t,f)\, dt\, df\,, \qquad k = 1,\ldots,K\,,$$

where the TF regions $\mathcal{R}_k$ are associated to the subspaces $\mathcal{S}_k$ in the sense that $I_{\mathcal{R}_k}(t,f) \approx W_{\mathcal{S}_k}(t,f)$. The parameters $\mu_k^{(i)}$ of the ξ_k are approximated by $\tilde{\mu}_k^{(i)} = 1/\tilde{\lambda}_k^{(i)}$ with

$$\tilde{\lambda}_k^{(i)} \triangleq \iint_{\mathcal{R}_k} \overline{W}_r^{(i)}(t,f)\, dt\, df\,, \qquad k = 1,\ldots,K\,, \quad (9)$$

where $\overline{W}_r^{(i)}(t,f)$ is the *Wigner-Ville spectrum* (WVS) of the observed process $r(t)$ under hypothesis H_i. The WVS is defined as the Weyl symbol of the correlation operator $\mathbf{R}_r$ of $r(t)$, i.e., $\overline{W}_r(t,f) \triangleq L_{\mathbf{R}_r}(t,f)$; it is also the expectation of $W_r(t,f)$ [21, pp. 212–267], [22, 25, 26].

4. Estimation of Prior Knowledge

We now discuss the estimation of the prior knowledge that is necessary for designing the various detection statistics. We assume that, for each hypothesis H_i, N_i observations (training signals) $r_j^{(i)}(t)$ ($j = 1,\ldots,N_i$) are available.

Likelihood Ratio Detector. The correlations $\mathbf{R}_i$ required for designing the likelihood ratio detector (see Section 1) can be estimated by the sample correlation operators $\hat{\mathbf{R}}_i$ whose kernels are given by

$$\hat{R}_i(t,t') = \frac{1}{N_i}\sum_{j=1}^{N_i} r_j^{(i)}(t)\, r_j^{(i)*}(t')\,.$$

Matched Subspace Detector. The p-dimensional subspace $\mathcal{S}$ required for the matched subspace detector $\Lambda_{\mathcal{S}}$ can be estimated by means of *ML subspace identification* [2]. The estimated subspace $\hat{\mathcal{S}}$ is spanned by the p dominant eigenfunctions of the sample correlation operator $\hat{\mathbf{R}}$ of all $N = N_0 + N_1$ training signals $r_j(t)$. The kernel of $\hat{\mathbf{R}}$ is given by

$$\hat{R}(t,t') = \frac{1}{N}\sum_{j=1}^{N} r_j(t)\, r_j^*(t')\,.$$

Note that this estimator does not require a labeling of the training signals.

TF Detector. For estimation of the TF region $\mathcal{R}$ required for the TF detector $\tilde{\Lambda}_{\mathcal{S}}$, we propose a TF counterpart of ML subspace identification. First, the WVS $\overline{W}_r(t,f)$ is estimated from all training signals $r_j(t)$ by the Weyl symbol of the sample correlation operator $\hat{\mathbf{R}}$,

$$\widehat{W}(t,f) = L_{\hat{\mathbf{R}}}(t,f) = \frac{1}{N}\sum_{j=1}^{N} W_{r_j}(t,f)\,,$$

with $W_{r_j}(t,f)$ the Wigner distribution of $r_j(t)$. (In the case of underspread processes—i.e., processes where components located in different TF regions are almost uncorrelated [26, 27]—or low SNR, it is advantageous to use an additional TF smoothing [22, 25, 28].) An estimate of $\mathcal{R}$ is then obtained by thresholding $\widehat{W}(t,f)$,

$$\hat{\mathcal{R}} = \left\{(t,f) : \widehat{W}(t,f) \geq \varepsilon\right\},$$

with ε chosen such that $\iint I_{\hat{\mathcal{R}}}(t,f)\, dt\, df = p$ (recall that the area of $\mathcal{R}$ approximates the dimension p of $\mathcal{S}$).

This method is a TF version of ML subspace identification since (for underspread processes [19, 26, 27]) the values of $\widehat{W}(t,f)$ approximate the eigenvalues of $\hat{\mathbf{R}}$ and $\hat{\mathcal{R}}$ approximates the union of the effective TF support regions of the p dominant eigenfunctions of $\hat{\mathbf{R}}$ [19]. However, the TF method is more intuitive, physically meaningful, and numerically stable than ML subspace identification since it uses a TF function instead of an operator and it does not contain a (numerically sensitive) eigendecomposition. Like ML subspace identification, the TF estimator does not require labeled training signals.

Extended Matched Subspace Detector and Extended TF Detector. The design of the extended matched subspace detector from training data may be difficult when it is not clear how to meaningfully split the estimated space $\hat{\mathcal{S}}$ into disjoint subspaces $\hat{\mathcal{S}}_k$. In contrast, the design of the extended TF detector is fairly straightforward since (in a knock detection application) the estimated TF region $\hat{\mathcal{R}}$ typically breaks up into disjoint resonance subregions $\hat{\mathcal{R}}_k$. The parameters $\tilde{\lambda}_k^{(i)}$ in (9) can be estimated from the labeled training signals $r_j^{(i)}(t)$ as

$$\hat{\tilde{\lambda}}_k^{(i)} = \iint_{\hat{\mathcal{R}}_k} \widehat{W}_i(t,f)\, dt\, df,$$

where $\widehat{W}_i(t,f) = L_{\hat{\mathbf{R}}_i}(t,f)$ is an estimate of $\overline{W}_r^{(i)}(t,f)$.

5. Application to Knock Detection

The detection of knocking combustions in car engines is important since a car engine is most efficient near knocking conditions but frequent knock can damage the engine. Knocking combustions give rise to strong transient resonances with decreasing resonance frequencies [4–7, 9, 10]

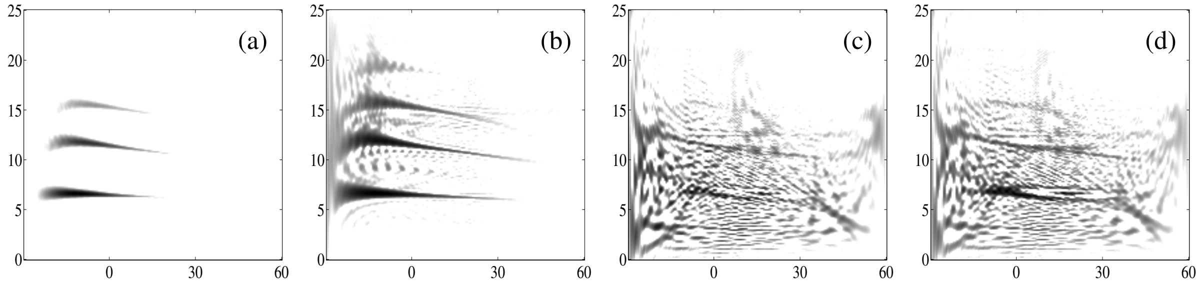

Fig. 1. *Estimated WVS of pressure and vibration signals corresponding to non-knocking (H_0) and knocking (H_1) combustions: (a) Non-knocking pressure signals, (b) knocking pressure signals, (c) non-knocking vibration signals, (d) knocking vibration signals. Horizontal axis: crank angle (in degrees, proportional to time), vertical axis: frequency (in kHz). The engine speed was 4000 rpm.*

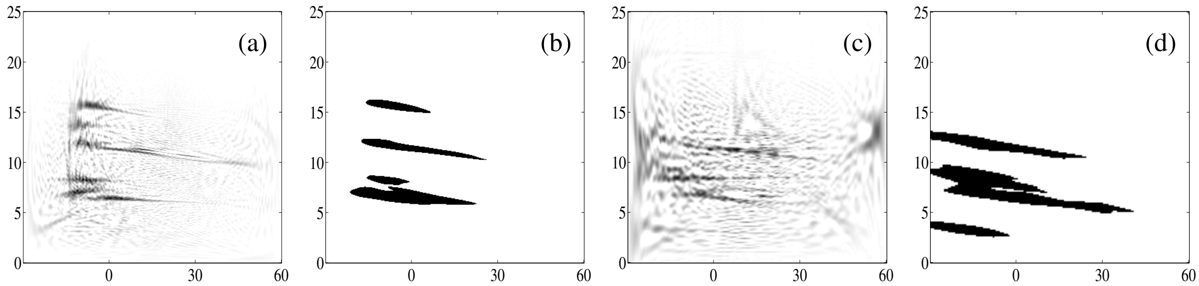

Fig. 2. *Estimated TF weight functions of matched subspace detector and TF detector at two different engine speeds: (a) Matched subspace detector at 2000 rpm, (b) TF detector at 2000 rpm, (c) matched subspace detector at 4000 rpm, (d) TF detector at 4000 rpm. Horizontal axis: crank angle (in degrees, proportional to time), vertical axis: frequency (in kHz).*

(see Fig. 1). It is desirable that knock detectors match this time-varying behavior.

Our data set consisted of vibration signals obtained from acceleration sensors mounted on the engine (such signals are used in practice for knock detection) and corresponding pressure signals obtained from pressure sensors mounted inside the cylinder. Each signal corresponded to one combustion cycle. Pressure and vibration signals were recorded at several different engine speeds. All signals were discrete-time and finite-length, and hence all operators reduced to finite-size matrices.

In our experiments, for each engine speed the following steps have been carried out:

- As a preparatory step, the vibration signals were labeled (i.e., split into knocking and non-knocking combustions) using the pressure signals as reference. To this end, a TF detector was designed from all available pressure signals as described in Sections 3 and 4. This TF detector was applied to the pressure signals with the detection threshold chosen such that 4% of the combustions were classified as knocking (see [6]). The labeling of the vibration signals was based on the pressure signals since these were much cleaner than the vibration signals (cf. Fig. 1).
- The labeled vibration signals were then split into a training set consisting of one fifth of the knocking and one fifth of the non-knocking signals and a validation set consisting of the remaining signals. The training set was used to estimate the prior knowledge on which the design of the various detectors was subsequently based. For the design of the likelihood ratio detector, pseudo-inverses of the correlations had to be used since the estimated correlations were strongly ill-conditioned. Furthermore, for the subspace and TF detectors we used dimension $p = 8$ and for the extended TF detector we used $p = 8$ and $K = 3$ (since we observed that $\hat{\mathcal{R}}$ consisted of three dominant TF subregions $\hat{\mathcal{R}}_k$ corresponding to the three dominant resonances, which depend on the engine speed).
- Finally, the performance of the detectors was assessed by applying them to the vibration signals from the validation set and computing empirical receiver operating characteristics (ROCs).

Fig. 2 shows the TF weight functions $W_{\hat{\mathcal{S}}}(t, f)$ and $I_{\hat{\mathcal{R}}}(t, f)$ estimated from the vibration signals by ML subspace identification and its TF counterpart, respectively (see Section 4). It is seen that the TF estimation method (using smoothed WVS estimates [22, 25, 28]) yields a better characterization of the resonances. Note also that at different engine speeds different resonances are dominant.

Fig. 3 shows the ROCs for the likelihood ratio, matched subspace, TF, and extended TF detectors at three different engine speeds. The TF detector tends to perform slightly better than the matched subspace detector. Furthermore, both the matched subspace detector and the TF detector yield better results than the likelihood ratio detector even though their design did not use labeled training signals. This can be attributed to numerical errors caused by the ill-conditioning of the correlation estimates and, possibly, to non-Gaussianity of the measured data. The performance of the extended TF detector is seen to be intermediate be-

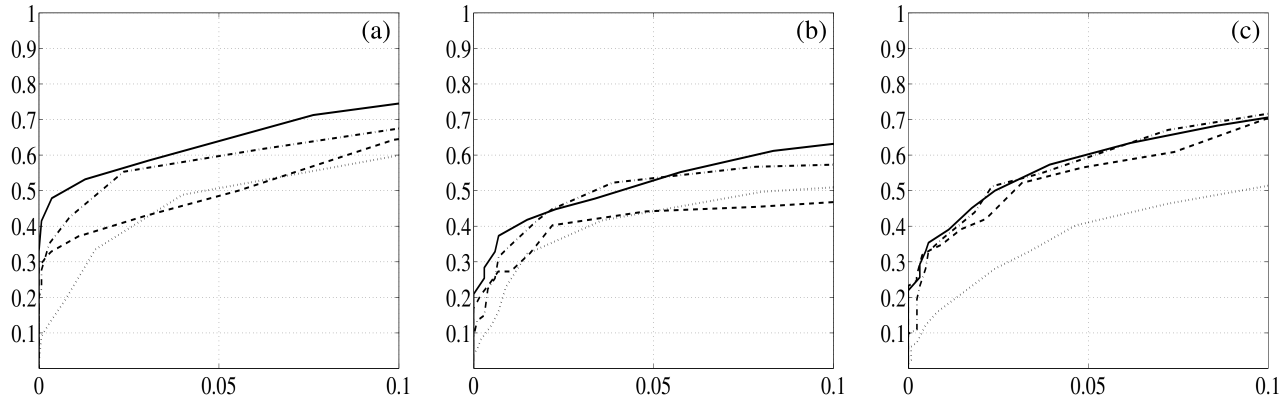

Fig. 3. *ROCs of various detectors at (a) 3000 rpm, (b) 4000 rpm, (c) 5000 rpm. Horizontal axis: false alarm probability P_f (probability of erroneously deciding a non-knocking combustion to be knocking) considered for practically relevant values between* 0 *and* 0.1*, vertical axis: detection probability P_d (probability of correctly detecting a knocking combustion). Dotted line: likelihood ratio detector, dash-dotted line: matched subspace detector, solid line: TF detector, dashed line: extended TF detector.*

tween the matched subspace and TF detectors on the one hand and the likelihood ratio detector on the other hand.

We can thus conclude that detectors requiring less prior knowledge may be advantageous since this reduced prior knowledge can be determined more reliably. This is even more true for TF detectors whose design tends to be numerically more stable. Finally, we note that detection performance tends to degrade with increasing engine speed (5000 rpm partly being an exception). This degradation can be attributed to an increase of engine noise.

6. Conclusion

Motivated by the application of knock detection, we have studied a composite hypotheses detection problem involving a subspace and an energy level. The matched subspace detector was shown to be theoretically appropriate for this problem and numerically efficient. Furthermore, an extension of the matched subspace detector was proposed. We also presented a time-frequency formulation and design of subspace detectors. Application of the detectors to knock detection showed the good performance of the time-frequency detector proposed.

Appendix: Proof of Theorem 1

In what follows, let $f(r;\underline{\theta}|H_i)$ denote the probability density function of $r(t)$ under hypothesis H_i, parameterized by $\underline{\theta}$. The GLR detection statistic is the likelihood ratio $f(r;\underline{\theta}|H_1)/f(r;\underline{\theta}|H_0)$ in which $\underline{\theta}$ is replaced by its (conditional) ML estimate $\hat{\underline{\theta}}_{\mathrm{ML}}^{(0)} = \arg\max_{\|\underline{\theta}\|\leq\gamma} f(r;\underline{\theta}|H_0)$ or $\hat{\underline{\theta}}_{\mathrm{ML}}^{(1)} = \arg\max_{\|\underline{\theta}\|>\gamma} f(r;\underline{\theta}|H_1)$ [1, 3, 11], i.e.,

$$\Lambda_{\mathrm{GLR}}(r) \triangleq \frac{f\big(r;\hat{\underline{\theta}}_{\mathrm{ML}}^{(1)}|H_1\big)}{f\big(r;\hat{\underline{\theta}}_{\mathrm{ML}}^{(0)}|H_0\big)}\,. \tag{10}$$

Under H_i, the observation obeys the linear model $r(t) = \sum_{l=1}^{p}\theta_l^{(i)}u_l(t) + n(t)$ and thus is Gaussian with mean $s_i(t) = \sum_{l=1}^{p}\theta_l^{(i)}u_l(t)$ and covariance $\eta\mathbf{I}$. For this linear-Gaussian model, it is known [2, 3] that the ML estimates $\hat{\underline{\theta}}_{\mathrm{ML}}^{(0)}$ and $\hat{\underline{\theta}}_{\mathrm{ML}}^{(1)}$ equal the least-squares estimates with side-constraints $\|\underline{\theta}\| \leq \gamma$ and $\|\underline{\theta}\| > \gamma$, respectively. It can be shown that the corresponding signal estimates $\hat{s}_i(t) = \sum_{l=1}^{p}\hat{\theta}_{\mathrm{ML},l}^{(i)}u_l(t)$ are given by

$$\hat{s}_0(t) = c_0\, r_{\mathcal{S}}(t)\,, \qquad \hat{s}_1(t) = c_1\, r_{\mathcal{S}}(t)\,,$$

where $r_{\mathcal{S}}(t) \triangleq (\mathbf{P}_{\mathcal{S}}r)(t)$ and $c_0 \triangleq \min\{1, \gamma/\|r_{\mathcal{S}}\|\}$, $c_1 \triangleq \max\{1, \gamma/\|r_{\mathcal{S}}\|\}$.

According to [11], Λ_{GLR} in (10) can be written as

$$\Lambda_{\mathrm{GLR}}(r) = \exp\left(\frac{\|\hat{n}_1\|^2 - \|\hat{n}_0\|^2}{2\eta}\right),$$

with the noise estimates

$$\begin{aligned}\hat{n}_i(t) &= r(t) - \hat{s}_i(t) = r(t) - c_i\, r_{\mathcal{S}}(t)\\ &= r(t) - r_{\mathcal{S}}(t) + (1-c_i)\, r_{\mathcal{S}}(t)\,.\end{aligned}$$

It follows that (note that $r(t) - r_{\mathcal{S}}(t)$ and $(1-c_i)\, r_{\mathcal{S}}(t)$ are deterministically orthogonal)

$$\begin{aligned}2\eta\,\log\Lambda_{\mathrm{GLR}}(r) &= \|\hat{n}_1\|^2 - \|\hat{n}_0\|^2\\ &= \|r - r_{\mathcal{S}}\|^2 + \|(1-c_1)\, r_{\mathcal{S}}\|^2\\ &\quad - \big(\|r - r_{\mathcal{S}}\|^2 + \|(1-c_0)\, r_{\mathcal{S}}\|^2\big)\\ &= \big(\|r_{\mathcal{S}}\| - \max\{\|r_{\mathcal{S}}\|, \gamma\}\big)^2\\ &\quad - \big(\|r_{\mathcal{S}}\| - \min\{\|r_{\mathcal{S}}\|, \gamma\}\big)^2\\ &= \mathrm{sgn}(\|r_{\mathcal{S}}\| - \gamma)\,\big(\|r_{\mathcal{S}}\| - \gamma\big)^2,\end{aligned}$$

which is easily seen to be a strictly monotone function of, and thus equivalent to, $\Lambda_{\mathcal{S}}(r) = \|r_{\mathcal{S}}\|^2$.

Acknowledgement

We are grateful to S. Carstens-Behrens, M. Wagner, and J. F. Böhme for stimulating and illuminating discussions and for providing us with the car engine data (courtesy of Aral-Forschung, Bochum).

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