Impulsive coding in optical free-space links: Optimum choice of the receive filter and impact of transmit booster amplifier

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ABSTRACT
A theory for the signal-to-noise ratio of optical direct detection receivers employing return-to-zero coding (and possibly optical preamplification) is developed. The results are valid for both signal-independent noise limited and signal-dependent noise limited receivers, as well as for arbitrary optical pulse shapes and receive filter characteristics, taking into account intersymbol interference. Even if the same receiver bandwidth is used, return-to-zero coding is seen to yield higher signal-to-noise ratio than nonreturn-to-zero coding. Asymptotic expressions for the signal-to-noise ratio for very high and very low receiver bandwidths show that the full sensitivity enhancement potential of return-to-zero coding is exhausted at fairly moderate duty cycles. A realistic example taking into account intersymbol interference shows that a receiver sensitivity gain (compared to nonreturn-to-zero coding) of e.g. 3.2dB can be obtained in a signal-independent noise limited receiver with a bandwidth of 80% of the data rate, using a duty cycle of 3. For the signal-independent noise limited case, the sensitivity enhancement potential depends on the receive filter characteristics; we provide a design rule for filters with high sensitivity enhancement potential. Further, we investigate the role of rare-earth doped booster amplifiers in impulsively coded communication links: It is shown that, due to the average power limitation of these devices, a less powerful booster amplifier as with nonreturn-to-zero coding can be employed if certain conditions regarding the data rate and the return-to-zero duty cycle are met.

Keywords: nonreturn-to-zero (NRZ), return-to-zero (RZ), on/off keying (OOK), receiver sensitivity, noise, optical amplifier, amplified spontaneous emission (ASE)

1. INTRODUCTION
It was found by Personick\textsuperscript{1} in 1973, and has recently been pointed out again\textsuperscript{2} that – for a given average optical power at the receiver – return-to-zero (RZ) coding, where the optical pulses occupy only a fraction of the bit duration, yields improved sensitivity over nonreturn-to-zero (NRZ) coding, where the optical pulse duration equals the bit duration. This gain is obtained even if the same receiver hardware is used. Experiments fostering this finding have also been reported recently\textsuperscript{2,3}. However, the theory presented in Ref. 1 constrains the receiver output to a raised-cosine shape, and both the theory and the qualitative explanation of Ref. 2 are based, again, on a specific receiver structure (which is basically a transimpedance amplifier with a first-order low-pass characteristic). Further, Ref. 2 only considers the case where signal-independent noise (e.g. thermal noise) dominates signal-dependent noise (e.g. shot noise).

In this paper, we give a general theory of the sensitivity improvement in terms of signal-to-noise ratio (SNR) obtained by impulsive coding, valid for arbitrary optical pulse shapes and arbitrary receive filter characteristics. We calculate the (different!) sensitivity improvements obtainable for receivers whose performance is limited by signal-independent noise and for receivers whose performance is limited by signal-dependent noise. Conventional optical direct detection receivers are – apart from some designs using avalanche photodiodes – usually dominated by signal-independent noise. The same is true for some optically preamplified direct detection systems, where the signal-independent amplified spontaneous emission self-beat noise (ASE-ASE beat noise) constitutes the dominant noise source\textsuperscript{a}. Usually, however, the signal-dependent signal-ASE beat noise (s-ASE beat noise) determines the signal-to-noise ratio in optically preamplified receivers. Furthermore, we address the question of how narrow the optical RZ pulses have to be to arrive at a significant sensitivity improvement and show that, unlike invoked by

\textsuperscript{a}This is the case if the bandwidth of the optical filter following the optical amplifier is much larger than the data rate.
Ref. 2, fairly moderate duty cycles of about 3 are sufficient to fully exhaust the potential of sensitivity enhancement for a given receiver bandwidth. A by-product of our calculations is the optimum receiver bandwidth for given optical pulse shape and given type of electrical filter.

Section 2 of this paper provides general expressions for the signal and noise in an optical direct detection receiver. In Section 3, we calculate the asymptotic behaviour of the SNR for very large and very small electrical bandwidths. In a simple example in Section 4 the sensitivity improvement of impulsive coding is evaluated analytically as a function of receiver bandwidth and RZ duty cycle. In Section 5, we include both inter-symbol-interference (ISI) and a realistic optical pulse shape and receive filter characteristic. Finally, in Section 6 we show that, due to the average power limitation of Erbium-doped optical fiber amplifiers (EDFAs), an EDFA boosted MOPA-type (master oscillator, power amplifier) transmitter for RZ pulses can be realized using the same or a less powerful booster amplifier as that required for an NRZ transmitter of the same average output power.

Throughout our work, bold print indicates stochastic processes (e.g. \( \mathbf{a}(t) \)), whose ensemble average is denoted \( \langle \mathbf{a}(t) \rangle \), and whose variance reads \( \sigma^2_a(t) \).

## 2. SIGNAL CURRENT AND NOISE FOR ARBITRARY RECEIVER BANDWIDTHS

We first have to clearly define the signal current and the noise terms in optical receivers. As we are interested in arbitrary ratios of the electrical receiver bandwidth to the bandwidth of the optical pulse, general expressions for signal-dependent noise — involving convolutions — have to be used: It can be shown\(^4,5\) that the ensemble average and the (signal-dependent) shot noise variance of the photocurrent \( \mathbf{i}(t) \) produced by a deterministic optical field with power \( p(t) \) generally read\(^b\)

\[
\langle \mathbf{i}(t) \rangle = S(p \ast h)(t) \tag{1}
\]

and

\[
\sigma^2_{i,\text{dep}}(t) = Se(p \ast h^2)(t) \tag{2}
\]

where \( S = \eta e / hf \) denotes the detector’s sensitivity (\( \eta \) is the detector’s quantum efficiency, \( e \) stands for the elementary charge and \( hf \) represents the energy of one photon), and \( h(t) \) is the impulse response of the electrical filter, normalized such that the integral over \( h(t) \) is unity\(^c\). Note from (2) that shot noise is non-stationary in general. For the case of a receiver employing an optical preamplifier with subsequent optical filtering, it can be shown\(^6\) that the signal-dependent s-ASE beat noise is of the same form as (2) and that the ASE-ASE beat noise is additive and stationary. The latter can thus be treated analogous to other additive noise sources (such as e.g. thermal noise), leading to a signal-independent variance of the form

\[
\sigma^2_{i,\text{indep}} = N_0 B_h \tag{3}
\]

where \( N_0 \) is the (single-sided) power spectral density of the noise, assumed independent of the receiver bandwidth\(^d\); \( B_h \) is the power equivalent bandwidth of \( h(t) \), defined\(^e\) as

\[
B_h = \int_0^\infty |H(f)|^2 df \tag{4}
\]

\(^b\)The symbol \(*\) denotes a convolution,

\[
\langle x \ast y(t) \rangle = \int_{-\infty}^{\infty} x(\tau)y(t - \tau) d\tau.
\]

\(^c\)Strictly speaking, \( h(t) \) stands for the normalized convolution of the detector’s impulse response with that of the electrical filter; however, since in most applications the detector bandwidth exceeds that of the electrical filter, \( h(t) \) is determined primarily by the electrical filter.

\(^d\)This assumption is correct if \( N_0 \) denotes the dominating ASE-ASE beat noise density in optically preamplified receivers, but has to be modified for receivers where the noise of the electronic circuitry dominates. For this case, some additional factors describing the dependence of \( N_0 \) on \( B_h \) would have to be introduced\(^1,8\), leading to worse results than predicted by our equations.

\(^e\)The usual normalization to the maximum of \( |H(f)| \) can be omitted in our case, as we assume that \( |H(f)| \) is peaked at \( f = 0 \), where it is unity due to the normalization of \( h(t) \).
The function $H(f)$ denotes the Fourier transform of $h(t)$,

$$H(f) = \int_{-\infty}^{\infty} h(t) \exp(-j2\pi ft) dt.$$ 

(5)

We define the SNR as

$$\text{SNR} = \frac{(\hat{h})^2 F^2}{\sigma_{\hat{h}}^2},$$

(6)

where $\hat{h}$ is the photocurrent for a mark at the optimum sampling instant (which is the one that maximizes the SNR). The parameter $F \in [0, 1]$ takes account of both finite extinction ratios and ISI. Usually, the bit-error probability (BEP) is calculated using a parameter $(Q)$, which can be expressed as

$$Q = \frac{(\hat{h}) \left(1 - \frac{\hat{i}_0}{\hat{h}}\right)}{\sigma_{\hat{h}} + \sigma_{\hat{i}_0}},$$

(7)

where $\hat{i}_0$ is the photocurrent for a space at the optimum sampling instant, and the term in parentheses represents the influence of a finite extinction ratio. In case of negligible intersymbol interference, the BEP is uniquely related to the $Q$-parameter via the complementary error function. In the presence of ISI, this relationship between the $Q$-parameter and the BEP does not hold. Nevertheless, we can approximately take into account ISI by modifying the $Q$-parameter (7) by an additional factor $F_{\text{ISI}}$ accounting for the eye closure due to ISI:

$$Q_{\text{ISI}} = Q \cdot F_{\text{ISI}}.$$ 

(8)

The product of the term accounting for a finite extinction ratio in (7) and of the factor $F_{\text{ISI}}$ then corresponds to our factor $F$ in (6). If signal-independent noise dominates $(\sigma_{\hat{h}} = \sigma_{\hat{i}_0})$, the $Q_{\text{ISI}}$-factor becomes $\sqrt{\text{SNR}}/2$, while it is $\sqrt{\text{SNR}}$ if the signal-dependent noise of the mark dominates $(\sigma_{\hat{h}} \gg \sigma_{\hat{i}_0})$.

3. ASYMPTOTIC EXPRESSIONS FOR THE SNR

For the calculation of the limiting behaviour of the SNR for very small and very large receiver bandwidths, consider the setup shown in Figure 1: A single optical pulse $p(t)$, which may also have passed through an optical preamplifier with subsequent (optical) bandpass filtering limiting the ASE power, impinges on a photodetector. Let the overall optical pulse energy be $E$, regardless of the pulse duration. (We thus compare systems with the same average optical power at the receiver.) The electronics following the photodetector are decomposed into a frequency independent amplifier and a filter with impulse response $h(t)$. The signal-independent power spectral density of the noise current behind the amplifier is $N_0$; it comprises all signal independent noise sources, such as amplifier noise or ASE-ASE beat noise. We are looking for the SNR at the output of the filter. For the calculation of the SNR’s asymptotic behaviour, we neglect the influence of finite extinction ratio and ISI and hence set $F = 1$.

3.1. Signal-independent Noise

If signal-independent noise dominates the other noise sources, the SNR at the optimum sampling instant follows from (6), inserting (1) and (3), as

$$\text{SNR} = \max_t \left\{ \frac{S^2 [(p * h)(t)]^2}{N_0 B_h} \right\},$$

(9)

where the maximum is taken over $t$. Employing the (inverse) Fourier transform, this expression can be written as

$$\text{SNR} = \max_t \left\{ \frac{\left[ \int_{-\infty}^{\infty} P(f) H(f) \exp(j2\pi ft) df \right]^2}{N_0 B_h} \right\},$$

(10)

where $P(f)$ stands for the Fourier transform of $p(t)$.

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*Mixed cases, where none of the noise sources clearly dominates, are not considered here. They will be subject of a future publication.*
Figure 1. Block diagram of the receiver: A single optical pulse \( p(t) \), which may also have passed through an optical preamplifier with subsequent (optical) bandpass filtering, impinges on a photodetector. The overall optical pulse energy is \( E \), regardless of the pulse duration. The electronics following the photodetector are decomposed into a frequency independent amplifier and a filter with impulse response \( h(t) \). The signal-independent power spectral density of the noise current behind the amplifier is \( N_0 \).

**Small Receiver Bandwidth**

If the electrical bandwidth \( B_h \) is much smaller than that of the optical pulse, we can approximate \( P(f) \) by \( P(0) \) in (10) and arrive at

\[
\text{SNR} = \frac{S^2 P^2(0)}{N_0 B_h} \max_i \left\{ \int_{-\infty}^{\infty} H(f) \exp(j2\pi ft) df \right\}^2.
\]

The integral is readily identified as \( h(t) \); \( P(0) \) always equals \( E \). We therefore arrive at

\[
\text{SNR} = \frac{S^2 E^2 R}{N_0} \alpha_h b_h,
\]

where the factor \( \alpha_h \) is a (bandwidth-independent) property of the electrical filter,

\[
\alpha_h = \max_i \{h(t)\}^2 / B_h^2,
\]

and \( b_h = B_h / R \) is the receiver bandwidth normalized to the bit rate \( R \). In the limit of small electrical bandwidths, the SNR is thus found to increase linearly with \( b_h \) at a rate determined solely by the electrical circuitry (and *not* by the optical pulse shape). To provide some examples, \( \alpha_h \) equals 4, 9, and 7.32 for rectangular, symmetrically triangular, and fifth-order Bessel-filter impulse responses \( h(t) \), respectively.

**Large Receiver Bandwidth**

For electrical bandwidths large compared to the bandwidth of the optical pulse, (10) reduces to

\[
\text{SNR} = \frac{S^2}{N_0 B_h} \max_i \left\{ \int_{-\infty}^{\infty} P(f) \exp(j2\pi ft) df \right\}^2,
\]

where we used \( H(0) = 1 \), brought by the normalization of \( h(t) \) to unit area. Identifying the integral as \( p(t) \) we arrive at

\[
\text{SNR} = \frac{S^2 E^2 R}{N_0} \frac{D^2}{b_h},
\]

where

\[
D = T_b / T_p
\]

is the RZ duty cycle; \( T_b = 1 / R \) is the bit duration and \( T_p \) is the optical pulse duration defined as

\[
T_p = \frac{\int_{-\infty}^{\infty} p(t) dt}{\max_i \{p(t)\}} = \frac{E}{\max_i \{p(t)\}}.
\]

As expected, in the limit of large electrical bandwidths, the SNR decreases inversely proportionally to \( b_h \).
3.2. Signal-dependent Noise

If signal-dependent noise dominates, the SNR reads

$$\text{SNR} = \max_i \left\{ \frac{S^2 \left[ (p \ast h)(t) \right]^2}{S e(p \ast h^2)(t)} \right\},$$

where use was made of (1), (2) and (6). In the Fourier domain, this equation reads

$$\text{SNR} = \max_i \left\{ \frac{S^2 \left[ \int_{-\infty}^{\infty} P(f) H(f) \exp(j2\pi ft) df \right]^2}{S e \int_{-\infty}^{\infty} P(f) \tilde{H}(f) \exp(j2\pi ft) df} \right\},$$

(19)

where $\tilde{H}(f)$ is the Fourier transform of $h^2(t)$ (and thus the convolution of $H(f)$ with itself); it can easily be shown that $\tilde{H}(0) = 2B_h$.

Small Receiver Bandwidth

For electrical bandwidths small compared to the bandwidth of the optical pulse, (19) can be simplified to

$$\text{SNR} = \frac{S}{e} \max_i \left\{ \frac{\left[ P(0)h(t) \right]^2}{P(0)h^2(t)} \right\},$$

(20)

and further to

$$\text{SNR} = \frac{SE}{e}.$$  

(21)

For dominating signal-dependent noise and small electrical bandwidths, the SNR is thus independent of $b_h$, regardless of the optical pulse shape and the electrical filter characteristics.

Large Receiver Bandwidth

If $B_h$ significantly exceeds the bandwidth of the optical pulse, (19) simplifies to

$$\text{SNR} = \frac{S}{2eB_h} \max_i \left\{ \left[ \int_{-\infty}^{\infty} P(f) \exp(j2\pi ft) df \right]^2 \right\},$$

(22)

as $H(0) = 1$ and $\tilde{H}(0) = 2B_h$, and further to

$$\text{SNR} = \frac{S}{2eB_h} \max_i \{p(t)\},$$

(23)

which closely resembles the well-known expression for the shot noise limited SNR.

4. ANALYTICAL SOLUTIONS FOR A SIMPLE EXAMPLE

We now demonstrate the principle of sensitivity enhancement by impulsive coding using a simple example, for which analytical solutions for the SNR as a function of $b_h$ can be found. This example does not take account of real-world pulse shapes and ISI; the analysis of a representative system including ISI is postponed to the next section.

Consider a single, symmetrically triangular optical pulse $p(t)$ of energy $E$ incident on a receiver with rectangular impulse response $h(t)$; the two functions are shown in Figure 2. Using (1), (2), and (3), the SNR (6) can readily be calculated. The results are

$$\text{SNR}_{\text{indep}} = \begin{cases} \frac{s^2 E^2 R^4 b_h}{N_0} & b_h < D/4, \\ \frac{s^2 E^2 R^4 D^2}{b_h} \left( 1 - \frac{D}{8b_h} \right)^2 & b_h > D/4 \end{cases},$$

(24)
for the signal-independent case, and

$$\text{SNR}_{\text{dep}} = \begin{cases} \frac{SE}{\frac{b_h}{4} + \frac{D}{2B_h}} & b_h < D/4, \\ \frac{SE}{\frac{b_h}{4} - \frac{1}{4\pi \sigma_b}} & b_h > D/4 \end{cases}$$

(25)

for the signal-dependent case. The results (24) and (25) are shown in Figure 3 as solid lines. The dashed curves represent the asymptotic expressions (12), (15), (21), and (23). Two sets of curves are presented, corresponding to $D = 1$ (which can be associated with NRZ) and $D = 2$ (which is RZ with duty cycle 2).

For the signal-independent case, Figure 3(a), we find that the choice of a receiver of normalized bandwidth $b_h = 0.4$, which is near the optimum in this case, leads to a SNR gain $G$ of 1.3$dB$ — corresponding to a receiver sensitivity gain of 0.65$dB$ — if we use RZ with duty cycle 2 instead of NRZ. Note that no additional gain can be obtained for higher duty cycles at this specific receiver bandwidth without changing the electrical filter characteristics! However, as pointed out in Section 3.1, modifying the electrical filter characteristics changes the slope of the low-bandwidth asymptote and thus permits higher limiting values for the SNR. In order to obtain a high sensitivity enhancement potential, we have to employ a receive filter with a large $\alpha$-factor (13). Hence, a design criterion of the receive filter is to maximize $\alpha$.

For the signal-dependent case, Figure 3(b), RZ with duty cycle 2 is seen to exhaust the full potential of sensitivity enhancement at a normalized receiver bandwidth of 0.4 by far, yielding a SNR (as well as receiver sensitivity) gain $G$ of 0.7$dB$. Note that, while in the signal-independent case arbitrary high sensitivity improvements can be obtained by increasing the duty cycle and increasing $b_h$, no further gain can be achieved in the signal-dependent case.

5. ANALYSIS OF A REPRESENTATIVE SYSTEM INCLUDING ISI

We now discuss the achievable sensitivity gain of a realistic optical direct detection receiver taking into account both a representative optical pulse form and electrical filter characteristic as well as ISI.

For the optical pulses $p(t)$ we choose the raised cosine dependence,

$$p(t) = \begin{cases} \frac{E T_p}{2} \sin^2 \left( \frac{t}{2 T_p} \right), & 0 < t < 2T_p, \\ 0 & \text{elsewhere}, \end{cases}$$

(26)

and assume a fifth-order Bessel-filter characteristic for the receiver, whose Laplace transform $H(s)$ is given by

$$H(s) = \frac{945}{s^5 + 15s^4 + 105s^3 + 420s^2 + 945s + 945}. \quad (27)$$

The two pulse shapes $p(t)$ and $h(t)$ are shown in Figure 4(a) and (b), respectively. We now include the influence
Figure 3. Normalized SNR for the pulse shapes depicted in Figure 2, and for either dominating signal-independent noise (a), or dominating signal-dependent noise (b). The solid lines represent (24) and (25), while the dashed lines show the asymptotic expressions (12), (15), (21), and (23). Two sets of curves corresponding to RZ duty cycles $D = 1$ and $D = 2$ are shown. Also indicated is the maximum SNR gain $G$ at $b_h = 0.4$. 
of ISI, as mentioned along with (6), by the factor \( F \in [0, 1] \). Obviously, \( F \) is bit pattern dependent. Worst ISI, i.e. minimum \( F \), is obtained at the highest eye-closure. We thus compare a \( ' \cdots 0001000 \cdots ' \) pattern (which is the ISI-free, single-pulse case discussed so far) with the worst case, a \( ' \cdots 1110111 \cdots ' \) pattern, and arrive at the general expression for \( F \):

\[
F = \frac{\max_t \left\{ (p \ast h)(t) - \sum_{k=-N}^{N} (p \ast h)(t - kT_h) \right\}}{(p \ast h)(t_{\max})},
\]

(28)

where \( t_{\max} \) is the optimum sampling instant and \( 2N \) is the number of pulses that significantly contribute to the eye-closure.

Numerically evaluating expression (6) for the SNR leads to the curves shown in Figure 5(a) for the signal-independent case and Figure 5(b) for the signal-dependent case. The plots show the asymptotes found in Section 3 (dashed) together with the numerically obtained curves for three different duty cycles \( D \) (solid). The curves neglecting ISI terminate at zero SNR when \( b_h \to 0 \), whereas the curves with worst ISI approach zero SNR at \( b_h \approx 0.2 \). The true (bit pattern dependent) curves lie all within the area between these two extremes; for the sake of clearness, this area is hatched for \( D = 1 \).

For the signal-independent case and \( D = 1 \), which represents NRZ coding, we find that the optimum receiver bandwidth is about 0.6 times the data rate; this agrees with general knowledge. Using RZ with \( D = 2 \) and the same receiver bandwidth, a sensitivity improvement of 1.7\( \text{dB} \) to 2.3\( \text{dB} \) (a SNR gain of 3.4\( \text{dB} \) to 4.6\( \text{dB} \)) can be achieved compared to NRZ. At \( D = 3 \), the sensitivity gain of 2.2\( \text{dB} \) to 2.8\( \text{dB} \) is already very close to the optimum gain of 2.8\( \text{dB} \) to 3.2\( \text{dB} \) achievable at this bandwidth. Thus, impulsive coding with moderate duty cycles yields high sensitivity enhancements\(^8\) and the limiting gain is reached at fairly low duty cycles. Higher gains can be achieved if one is willing to increase the receiver bandwidth, a well-known fact in communications engineering.

For the signal-dependent case (Figure 5(b)), the receiver sensitivity enhancement at \( b_h = 0.6 \) lies between 1.3\( \text{dB} \) and 2.4\( \text{dB} \) for RZ with \( D = 2 \), and between 1.5\( \text{dB} \) and 2.6\( \text{dB} \) for \( D = 3 \). The optimum sensitivity gain is 1.6\( \text{dB} \) to 2.7\( \text{dB} \) but, in contrast to the signal-independent case, with no further improvement potential.

6. RZ-CODING IN FREE-SPACE OPTICAL LINKS

Naturally, the RZ sensitivity gain described in the previous sections does not come for free: As the average transmit power and thus the energy per bit is held constant, the required peak power of the RZ-pulses linearly increases with the duty cycle \( D \). An optical transmitter for RZ-pulses has therefore to offer a peak output power which is significantly higher than that of an NRZ transmitter of the same average output power\(^3\). However, the requirement

\(^{1}\)It may happen for some configurations (where \( h(t) \) becomes significantly negative) that, at specific receiver bandwidths, other bit patterns yield worst ISI; for these cases, the factor \( F \) defined in (28) is rather an approximation for the worst ISI than its upper bound.

\(^{2}\)It has to be emphasized that the sensitivity gain values given here are based on the dependence of the bit error probability (BEP) on the Q-parameter and thus on the SNR. Recent simulations have shown that — in the presence of significant ISI, i.e. at very low \( b_h \) — the achievable sensitivity improvements in terms of BEP differ from the values given here.

\(^{3}\)In terrestrial, fiber based systems, the higher peak power of RZ pulses may also lead to undesired nonlinear effects within the fibers. In free-space communications links, however, due to the linearity of the transmission medium, no peak power limits apply.
Figure 5. Normalized SNR calculated using the waveforms given in Figure 4 for dominating signal-independent noise (a) and dominating signal-dependent noise (b). The solid lines represent (6), while the dashed lines show the asymptotic expressions (12), (15), (21), and (23). Three sets of curves corresponding to duty cycles $D = 1, 2, 3$ are shown.
of a higher peak transmit power is achievable with moderate effort using an optical transmitter based on the master oscillator, power amplifier (MOPA) architecture\textsuperscript{10} with a rare-earth doped fiber amplifier, e.g., an Erbium-doped fiber amplifier at a wavelength of 1.55\textmu m, as power amplifier. (A MOPA-type transmitter consists of a low-power primary laser source with internal or external modulation and of a subsequent power amplifier boosting the modulated laser output to the power level required to close the link.)

In contrast to the master oscillator laser diode, Erbium-doped fiber amplifiers are average power limited. In case of return-to-zero coding with the same energy per bit as the corresponding NRZ pulse this means that the booster amplifier gain is independent of the duty cycle. Naturally, the average power limitation will not hold if we increase the duty cycle arbitrarily: The saturated output pulse energy is limited by the pump energy stored in the excited states of Erbium-ions. This energy limit is connected to the spontaneous emission lifetime, which is about 10 ms in EDFAs. The average power limitation of EDFAs has also been verified experimentally\textsuperscript{11}: constant average output power is reported for 9 ps-wide pulses at repetition frequencies between 500 MHz and 2.1 GHz. Hence, at moderate duty cycles and under the condition of a limited number of successively transmitted zeros\textsuperscript{12}, no gain reduction has to be expected.

We will demonstrate this important advantage of EDFA-boosted systems in a short example (cf. Table 1): Assume that the average power at the EDFA booster output necessary to close the link is 33 dBm for NRZ coding. Using a simplified theoretical model of EDFA saturation\textsuperscript{12,13} together with data of a commercially available EDFA booster rated for $P_{sat} = 36$ dBm, we arrive at a minimum required average power at the EDFA input of $-7$ dBm, corresponding to a peak EDFA input power of $-4$ dBm. Assuming that 3 dB sensitivity improvement (in terms of average power) can be gained using RZ coding with a duty cycle of $D = 4$ (RZ4), the necessary average booster output power is reduced to 30 dBm, which is achieved using the same EDFA at an average input power of only $-13$ dBm. Because of $D = 4$, the corresponding peak input power of the EDFA is $-4$ dBm, exactly the value required for NRZ transmission. Hence we can employ the same primary laser source (peak power limited to $-4$ dBm) and also the same EDFA as in the NRZ case (but operated at a lower average output power level). On the other hand, if we take a slightly more powerful master oscillator laser with a peak output power of $-2$ dBm, we can replace the booster EDFA rated for 36 dBm with a less powerful one, rated at $P_{sat} = 33$ dBm.

### Table 1. RZ coding with a duty cycle of $D = 4$ requires (if at all) only slightly more powerful primary laser sources compared to NRZ, but allows to employ a less powerful EDFA booster

<table>
<thead>
<tr>
<th>Coding</th>
<th>Average Transmit Power Requirement</th>
<th>Peak Transmit Power Requirement</th>
<th>EDFA Rating ($P_{sat}$)</th>
<th>Average EDFA Input Power Required</th>
<th>Peak EDFA Input Power Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRZ</td>
<td>33 dBm</td>
<td>36 dBm</td>
<td>36 dBm</td>
<td>$-7$ dBm</td>
<td>$-4$ dBm</td>
</tr>
<tr>
<td>RZ4</td>
<td>30 dBm</td>
<td>39 dBm</td>
<td>36 dBm</td>
<td>$-13$ dBm</td>
<td>$-4$ dBm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>33 dBm</td>
<td>$-11$ dBm</td>
<td>$-2$ dBm</td>
</tr>
</tbody>
</table>

7. CONCLUSIONS

Using impulsive coding at the transmitter, the sensitivity of optical direct detection receivers (including those with optical preamplification) can be enhanced by several dB compared to NRZ coding for the same average power at the receiver. This is true even if the same receiver hardware is employed. We presented a theory of the achievable SNR gain, valid for the equally important classes of signal-independent noise limited and signal-dependent noise limited receivers, as well as for arbitrary optical pulse shapes and receive filter characteristics.

We arrived at asymptotic expressions for the SNR for very low and very high electrical receiver bandwidths and pointed out that the low-bandwidth asymptotes represent the ultimate sensitivity obtainable by impulsive coding. These asymptotes are reached at fairly moderate RZ duty cycles of about three for a technically representative system. The generation of extremely narrow optical pulses at the transmitter is thus not required.

In signal-dependent noise limited receivers, the maximum sensitivity gain is limited to about 2 dB, showing no further improvement possibilities by increasing the receiver bandwidth. The performance of signal-independent noise

\textsuperscript{10}The number of successive zeros and ones transmitted is usually limited to a few by channel coding algorithms, due to the requirements of clock recovery.
limited receivers, on the other hand, can be improved by about $3dB$ at a receiver bandwidth of about 0.6 times the data rate, with additional potential if the receiver bandwidth is increased (and if the signal-independent noise power spectral density is thereby not significantly increased, too.)

Our calculated sensitivity gain at a bandwidth of 0.8 times the data rate and an RZ duty cycle of 3 is about $3.2dB$, with a full (asymptotic) potential of about $3.8dB$. These values agree well with measurement results\(^{14}\) where $3.5dB$ were measured in a $10Gbit/s$ optically preamplified direct detection system with a duty cycle of slightly more than 3.

We further showed that, using a MOPA-type transmitter with a rare-earth doped booster amplifier, RZ coding relaxes the transmit power requirements: It requires (if at all) only slightly more powerful primary laser sources compared to NRZ, but significantly reduces the required average booster output power.

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**REFERENCES**