Efficient OFDM Transmission without Cyclic Prefix over Frequency-selective Channels

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Abstract

We introduce a computationally efficient OFDM transmission technique that does not require a cyclic prefix and thus provides high spectral efficiency. The ISI due to time dispersion of the channel is combatted by ISI cancellation. We show that the inversion of the channel matrix for ICI cancellation can be done by the "Operator Perturbation Technique" (OPT) much more efficiently than by conventional equation solvers. We supplement the OPT by blind techniques and hence reduce the error floor to zero for frequency selective channels. The expense for a 2MBit/s transmission is about 500MFlops. The technique can also deal with time-varying channels in a straightforward way.

1 Introduction

In the last years OFDM has become very attractive for high data rate transmission over wired (e.g. ADSL) and wireless (e.g. Hiperlan/2, Digital Audio Broadcasting DAB, Digital Video Broadcasting DVB, see [1] and references therein) channels.

OFDM splits the data stream into several substreams and modulates parallel subcarriers with them [2, 3]. The symbol duration on each channel is multiplied by the number of carriers, N, hence OFDM implies an enhanced robustness against time dispersion. By adding a cyclic prefix (CP) to every OFDM symbol, the residual ISI can be eliminated. Additionally the orthogonality of the subcarriers is guaranteed, so that no interchannel interference (ICI) occurs. For perfect ISI and ICI elimination the length of the CP has to be at least the length of the channel impulse response. In [4] the effect of the guard time length and the number of carriers on the system performance is investigated. The drawback of the CP (length G symbols) is that it reduces the spectral efficiency to $\frac{N}{N+G}$. This reduction can be eliminated only by omitting the CP, but as a consequence ICI and ISI appear, [5]. In this paper we propose a method for eliminating ICI and ISI without need of a CP, i.e. with full spectral efficiency. Under the assumption of correct decision of the last symbol, ISI can be subtracted by ISI cancellation [6], while elimination of the ICI requires an equalizer that performs an inversion of the channel transfer function matrix. Since standard matrix inversion techniques are much too computationally expensive, we propose the use of the Operator Perturbation Technique OPT [7] combined with techniques to accelerate the convergence [8].

In section 2 we explain the used system model, section 3 introduces the OFDM Channel Interference Matrix. In section 4 we treat the inversion of the channel matrix with the Operator Perturbation Technique, and section 5 shows simulation results.

2 System Model

Throughout the paper we assume perfect synchronization of carriers and blocks, and further, if not otherwise stated, absence of noise. The subcarrier modulation is done by means of an Inverse Discrete Fourier Transform (IDFT) of the data blocks (typical length $N = 64 \dots 1024$ carriers, but also 8000 carriers are possible for DVB). Before transmission, a guard period is inserted at the beginning of the transmission block (OFDM symbol) to avoid interference between consecutive symbols (ISI). The receiver detaches this part and performs the DFT with the rest of the received symbol. If the guard period is a copy of the end of the symbol, the linear convolution performed by the channel looks like a cyclic convolution to the data. This guard period is then called "Cyclic Prefix" (CP). Due to the cyclic convolution the connection between transmitter- and receiver data blocks is simply the pointwise multiplication by the channel frequency responses at the subcarrier frequencies. Thus the subcarriers stay independent, and no intercarrier interference (ICI) occurs. The equalizer in the receiver reduces to a pointwise multiplication by the inverse of the channel response. If the cyclic prefix is shorter than the impulse response or has zero length, ISI and ICI occur. ISI is the part of the interference that stems from the previous symbol while ICI is the self interference, where each tone is disturbed by each other tone of the own symbol [4], [5].

For the ith Block $X_{i,k}$ the transmission signal in the time domain $x_i[n]$ becomes

$$x_i[n] = \frac{1}{N}\sum_{k=0}^{N-1} X_{i,k} e^{j\frac{2\pi}{N}nk} \quad , \quad 0 \le n < N \tag{1}$$

where k is the subcarrier index. The channel performs the convolution with the impulse response $h[l]$,

$$y_i[n] = (h * x_i)[n] \ , \tag{2}$$

where $y_i[n]$ is the received sequence and $h[l]$ consists of L multipath components, $0 \le l < L$. We set the length of a possibly added CP to G samples. For treating the case of zero length CP, we simply set $G = 0$. The first $L - G - 1$ samples of $y_i[n]$ are distorted by ISI from the previous symbol x_{i-1}, thus y_i consists of two parts,

$$\begin{aligned} y_i[n] &= \sum_{l=0}^{L-1} h[l]x_i[n-l]_N \sigma[n-l+G] + \\ &+ \sum_{l=G+1}^{L-1} h[l]x_{i-1}[n-l+G]_N (1-\sigma[n-l+G]) = \\ &= y_i^{(i)}[n] + y_i^{(i-1)}[n] \ , \end{aligned} \tag{3}$$

where σ is the Heaviside function and $[n]_N$ means n modulo N. To reconstruct the sent symbols, the receiver does a DFT on $y_i[n]$, $Y_{i,k} = \mathcal{DFT}\{y_i[n]\}[k]$ where k again is the subcarrier index.

3 Channel Interference Matrix

3.1 ISI Cancellation

We now write the Discrete Fourier Transform of (3) in the receiver [6] as

$$\begin{aligned} Y_{i,k} &= \sum_{n=0}^{N-1} \left(y_i^{(i)}[n] + y_i^{(i-1)}[n]\right) e^{-j\frac{2\pi}{N}nk} = \\ &= \underbrace{\sum_{n=0}^{N-1}\sum_{l=0}^{L-1} h[l]x_i[n-l]_N \sigma[n-l+G] e^{-j\frac{2\pi}{N}nk}}_{Y_{i,k}^{(i)}} + \\ &+ \underbrace{\sum_{n=0}^{N-1}\sum_{l=G+1}^{L-1} h[l]x_{i-1}[n-l+G]_N \cdot (1-\sigma[n-l+G])\, e^{-j\frac{2\pi}{N}nk}}_{Y_{i,k}^{(i-1)}} . \end{aligned} \tag{4}$$

$Y_{i,k}^{(i)}$ and $Y_{i,k}^{(i-1)}$ are the parts of $Y_{i,k}$ that originate from the own symbol i and the previous symbol $i-1$, respectively. If we use (1), these parts can then be written as

$$\begin{aligned} Y_{i,k}^{(i)} &= \sum_{m=0}^{N-1}\sum_{l=0}^{L-1} X_{i,m} \cdot H_l^{(i)}(m-k) \cdot e^{-j\frac{2\pi}{N}lm} \ , \\ Y_{i,k}^{(i-1)} &= \sum_{m=0}^{N-1}\sum_{l=G+1}^{L-1} X_{i-1,m} \cdot \\ &\quad \cdot H_l^{(i-1)}(m-k) \cdot e^{-j\frac{2\pi}{N}m(l-G)} \ . \end{aligned} \tag{5}$$

$Y_{i,k}^{(i)}$ can be simplified to

$$Y_{i,k}^{(i)} = \sum_{m=0}^{N-1} a_{km} \cdot X_{i,m} \ , \tag{6}$$

where

$$a_{km} = \sum_{l=0}^{L-1} H_l^{(i)}(m-k) e^{-j\frac{2\pi}{N}lm}$$

$$H_l^{(i)}(m-k) = \frac{1}{N}\sum_{n=0}^{N-1} h[l]\sigma[n-l+G] e^{j\frac{2\pi}{N}n(m-k)}$$

$$H_l^{(i-1)}(m-k) = \frac{1}{N}\sum_{n=0}^{N-1} h[l]\left(1-\sigma[n-l+G]\right) \cdot e^{j\frac{2\pi}{N}n(m-k)} \quad . \tag{7}$$

a_{km} is the $(k,m)th$ coefficient of the ICI channel matrix $\mathbf{H}$, and (6) forms the vector-matrix-product of the data block $\vec{\mathbf{X}}_i$ and the channel $\mathbf{H}$,

$$\vec{\mathbf{Y}}_i^{(i)} = \mathbf{H}\cdot\vec{\mathbf{X}}_i \quad . \tag{8}$$

The second term $Y_{i,k}^{(i-1)}$ can also be simplified to

$$Y_{i,k}^{(i-1)} = \sum_{m=0}^{N-1} b_{km}\cdot X_{i-1,m} \quad , \tag{9}$$

where

$$b_{km} = \sum_{l=G+1}^{L-1} H_l^{(i-1)}(m-k) e^{-j\frac{2\pi}{N}(l-G)m} \quad .$$

b_{km} is the $(k,m)th$ coefficient of the ISI channel matrix $\mathbf{H}_{ISI}$, and (9) forms the vector-matrix-product of the data block $\vec{\mathbf{X}}_{i-1}$ and the channel $\mathbf{H}_{ISI}$.

$$\vec{\mathbf{Y}}_i^{(i-1)} = \mathbf{H}_{ISI}\cdot\vec{\mathbf{X}}_{i-1} \quad . \tag{10}$$

We clearly see that the received data vector $\vec{\mathbf{Y}}_i$ consists of two parts,

$$\vec{\mathbf{Y}}_i = \vec{\mathbf{Y}}_i^{(i)} + \vec{\mathbf{Y}}_i^{(i-1)} \quad , \tag{11}$$

where $\vec{\mathbf{Y}}_i^{(i-1)}$ is the ISI-term and $\vec{\mathbf{Y}}_i^{(i)}$ contains the desired data disturbed by ICI. Note that if a CP of sufficient length was used $\mathbf{H}_{ISI} = \mathbf{0}_{N\times N}$ and $\mathbf{H}$ is a diagonal matrix with $a_{kk} = \mathcal{DFT}\{h[l]\}_k$. Since we already decided on the previous data vector $\vec{\mathbf{X}}_{i-1}$, we can calculate $\vec{\mathbf{Y}}_i^{(i-1)}$ and subtract it from $\vec{\mathbf{Y}}_i$. This procedure is called "ISI cancellation" and can either be done in the frequency domain, as we showed here, or in the time domain, as in [6].

3.2 ICI Cancellation

A more difficult problem than ISI cancellation is the ICI cancellation. From (8) and (11) we get

$$\begin{aligned} \vec{\mathbf{Y}}_i^{(i)} &= \vec{\mathbf{Y}}_i - \vec{\mathbf{Y}}_i^{(i-1)} = \\ &= \mathbf{H}\cdot\vec{\mathbf{X}}_i \\ \Rightarrow \vec{\mathbf{X}}_i &= \mathbf{H}^{-1}\cdot\vec{\mathbf{Y}}_i^{(i)} \quad , \end{aligned} \tag{12}$$

where $\vec{\mathbf{Y}}_i$ is the received data vector, $\vec{\mathbf{Y}}_i^{(i)}$ is the ISI-cancelled data vector, and $\vec{\mathbf{X}}_i$ is the transmitted data vector that is to be estimated.

After ISI cancellation the data vector still contains interference from its own symbol, the intercarrier interference ICI. To equalize this interference we have to implement eq. (12). The advantage of our approach compared to [6] (RISIC-algorithm) is that it can be applied easily to time varying channels, too. To avoid stability problems with matrix inversion algorithms for badly conditioned channel matrices $\mathbf{H}$ we propose to use the Moore-Penrose pseudo inverse $\mathbf{H}^{\#} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$, where $\mathbf{H}^H$ means the hermitian transpose of $\mathbf{H}$.

Both a conventional and pseudo matrix inversion require $\mathcal{O}(n^3)$ operations, where n is the size of the square matrix. In a 128 carrier, 16QAM system a 100MFlops DSP needs about 20ms to invert $\mathbf{H}$. The resulting data rate would be $R_b = \frac{1}{20ms}\cdot 128\cdot ld(16) = 25600 bit/s$. This example shows the need of a more efficient inversion method. In the next section, we will introduce a technique (OPT) that needs $\mathcal{O}(n^2)$ operations per iteration. For $2\,Mbit/s$ data transmission, a 500MFlops DSP is required, if the OPT needs 8 iterations to converge.

4 Operator Perturbation – OPT

4.1 Standard OPT

The Operator Perturbation Technique is an iterative method to efficiently approximate and invert linear or nonlinear operators. It was originally introduced by J.C. Cannon in [7] and is well-known in the astrophysics community. For matrices it is also known as Jacobi-Iteration, see [10] ch. 10.1. We use it for inversion of our OFDM Channel Matrix $\mathbf{H}$, that was defined in (6) and (7). For notational convenience we write (8) as

$$Y = HX \tag{13}$$
$$\Rightarrow X = H^{-1}Y \tag{14}$$

and approximate (13) by

$$Y = \tilde{H}X - \epsilon \ .$$

$\tilde{H}$ is the approximate operator whose inverse is easy to compute, ϵ is the deviation from the exact solution. E.g. $\tilde{H}$ could consist of the diagonal of H and zero off-diagonal elements. The solution of (14) is then found by the following iteration:

1. $X_0 = \tilde{H}^{-1}Y$
2. $\epsilon_0 = \left(\tilde{H} - H\right) X_0$
3. $X_{i+1} = \tilde{H}^{-1}(\epsilon_i + Y)$
4. $\epsilon_{i+1} = \left(\tilde{H} - H\right) X_{i+1}$

After initialization (steps 1 and 2), steps 3 and 4 are repeated until a criterion is reached. Substituting step 4 in step 3 gives

$$X_{i+1} = X_i + \underbrace{\tilde{H}^{-1}(Y - HX_i)}_{e_x} \ , \tag{15}$$

where e_x is the error of the old solution X_i. If X_i converges to X_∞ then

$$\begin{aligned} X_\infty &= \tilde{H}^{-1}(\epsilon_\infty + Y) \\ \epsilon_\infty &= (\tilde{H} - H)X_\infty \\ X_\infty &= \tilde{H}^{-1}\left((\tilde{H} - H)X_\infty + Y\right) = \\ &= X_\infty + \tilde{H}^{-1}(Y - HX_\infty) \\ \Rightarrow 0 &= \tilde{H}^{-1}(Y - HX_\infty) \ . \end{aligned} \tag{16}$$

This means if $det\{\tilde{H}^{-1}\} \neq 0$ then we get the solution

$$Y - HX_\infty = 0 \ ,$$

and therefore $X_\infty = X$. Note that X and Y are vectors while H is a matrix.

Step 4 of the iteration procedure requires a vector-matrix-product that needs $\mathcal{O}(n^2)$ operations. This step determines the overall performance. The question, how many iteration loops are necessary for a sufficient accuracy, will be treated in section 5.

4.2 Acceleration of Convergence

Auer [8] and Ng [9] developed methods to improve the convergence speed of linear iterative schemes. These techniques have been invented in astrophysics and chemical physics but, astonishingly, do not seem to be widely known in mobile radio. Clearly linear iterative schemes are only linearly convergent. An improvement of convergent speed can be achieved, when we use more information than the last solution X_{i-1} to calculate the new X_i.

$$X_i = \alpha_0 X_{i-1} + \sum_{m=1}^{M} \alpha_m X_{i-1-m} \ . \tag{17}$$

$\alpha_0 = 1 - \sum \alpha_m$ is a normalization factor. The coefficients α_m are determined to fulfill the requirement of minimal distance between X and X_i , thus $r^2 = \|X - X_i\|^2 \rightarrow min$. For a M^{th} order acceleration we must supply $M + 2$ successive estimates of the solution. Ref. [8] gives a pseudocode for a 2^{nd} order acceleration. Unfortunately, there is no definitive statement to make about the optimal choice of M, but the utilization of higher orders than $M = 2$ usually do not significantly improve the acceleration.

4.3 OPT with blind techniques

As we know that the solution of $X = H^{-1}Y$ must lie in the set $\{-1, +1\}$ for BPSK (finite alphabet property) we can include a BPSK slicer after step 3 of the OPT algorithm. This slicer decides on the components of X_{i+1} and forces them to $+1$ or -1. We call this technique "OPT with decision".

4.4 Residual errors of OPT

Although the OPT in principle should be able to provide correct results, there is a residual error. This error is due to the fact that an initialization of the OPT algorithm by the diagonal of the channel matrix **H** can be insufficient in cases, where one or several of the diagonal elements are too close to zero. The longer the impulse response of the channel, the more diagonal elements are small, i.e. the more subcarriers can be in fading dips. This is the reason why the OPT gets worse when the impulse response length increases. Even if you increase the number of iterations in the OPT algorithm, the result does not converge to the correct solution but stays in a local minimum.

To mitigate this problem we modified the standard OPT to converge to the Pseudo-Inverse solution $X = \mathbf{H}^{\#}Y$ as follows.

$$\begin{aligned} X =& \ \mathbf{H}^{\#}Y = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H Y \\ (\mathbf{H}^H\mathbf{H})X =& \ \mathbf{H}^H Y \end{aligned}$$

So we get

$$\bar{Y} = \bar{\mathbf{H}}X \ , \tag{18}$$

with $\bar{\mathbf{H}} = \mathbf{H}^H\mathbf{H}$ and $\bar{Y} = \mathbf{H}^H Y$. Eq. (18) has exactly the same form as eq. (13), but we use $\bar{\mathbf{H}}$ and $\bar{Y}$ instead of $\mathbf{H}$ and Y for the OPT and call it OPT-ps.

5 Simulation Results

We made extensive Monte Carlo simulations to investigate the achievable BER and the number of iterations needed for the operator perturbation method (OPT). The considered system was as follows:

- OFDM-system with $N_c = 128$ carriers. Each channel was modulated by BPSK symbols.
- A frequency selective channel. Sample-spaced random 2-spike impulse responses with different maximum excess delays were used. The taps of the impulse responses were independently Rayleigh fading.
- The channel was constant during one OFDM symbol, but the impulse responses for different blocks were independent.
- The channel was perfectly known at the receiver.
- ISI cancellation was perfectly working (see sec. 3.1).

To get statistically reliable results, we averaged about 200000 simulation runs. Figure 1 shows simulations of the system without noise. To get comparable results we implemented a conventional OFDM

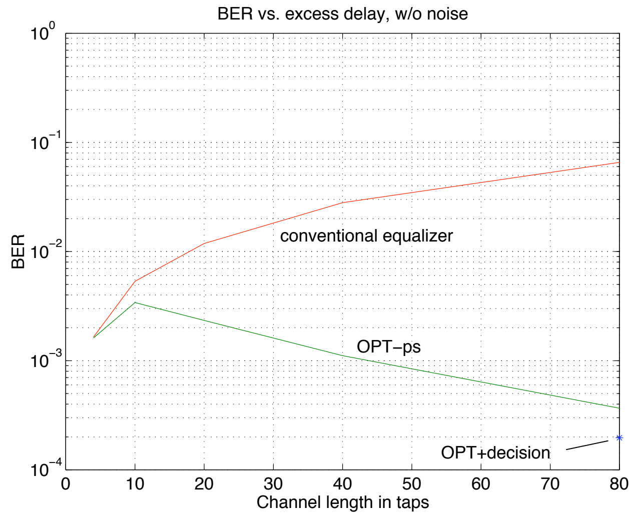

Figure 1: Bit Error Ratio of an OFDM system without cyclic prefix. Usage of Pseudo Inverse $\mathbf{H}^{\#}$ yields to $BER = 0$.

channel equalizer where we pointwise divided the Fourier Transform of the received signal by the Fourier Transform of the impulse response,

$$\hat{X}_{i,k} = \frac{Y_{i,k}^{(i)}}{\mathcal{DFT}\{h_i[l]\}_k} \ .$$

The upper line shows the BER of this simple equalizer.

The second curve (OPT-ps) shows the BER performance of the OPT-ps algorithm. We used second order Acceleration of Convergence (see sec. 4.2) with weighting [8]. One iteration cycle consists of 4 conventional iterations of the OPT (see steps 3 and 4 of section 4) to provide the required 4 successive estimates, followed by one acceleration step. Thorough trials showed that after 2 such iteration cycles the results were stable. This means that 8 iterations and 2 accelerations are enough to solve equation (14), where the number of operations for an acceleration step is negligible compared to the iterations of the OPT.

The mark at the channel length of 80 taps (OPT+decision) in fig. 1 indicates the BER of the OPT with blind techniques, see sec. 4.3. Acceleration of Convergence is no longer necessary. For impulse responses shorter than 80 taps our OPT+decision algorithm eliminates the ICI completely and we get $BER = 0$.

If we use the Moore-Penrose Pseudo Inverse $\mathbf{H}^{\#}$ to equalize the channel we can also eliminate ICI and get $BER = 0$.

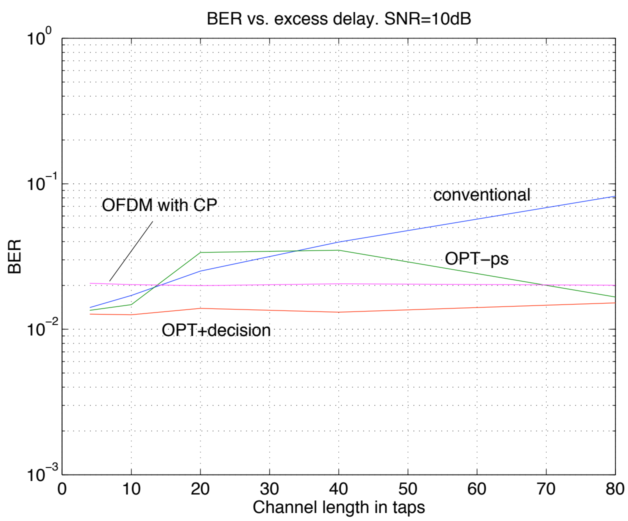

Figure 2: BER of OFDM without cyclic prefix in Frequency-selective AWGN-channels, SNR=10dB

Figure 2 shows simulation results of the same system as above, but also taking noise into account. The noise signal was added after the convolution with the impulse response

$$y_i[l] = (h * x_i)[l] + n[l] \quad ,$$

and the SNR is

$$SNR = \frac{E_l\{|(h * x_i)[l]|^2\}}{E_l\{|n[l]|^2\}} \quad ,$$

where E_l is the expectation over taps l. The BER does not anymore depend so strongly on the maximum excess delay. Again the same BER curves are shown. Additionally the BER of a conventional OFDM system with cyclic prefix is shown, where the length of the CP is 79. The SNR degradation due to the CP was also considered. Note that the OPT+decision outperforms OFDM+CP due to the SNR-loss of the CP.

6 Conclusions

A highly efficient OFDM transmission scheme has been introduced where we completely removed the cyclic prefix. This leads to maximum spectral efficiency. The arising ISI can be eliminated by ISI Cancellation in the time- or frequency domain, while the ICI must be removed by ICI Cancellation. We introduced the channel interference matrix $\mathbf{H}$ and showed that with the Operator Perturbation Technique (OPT) we are able to efficiently invert this matrix. We included a decision device into the OPT (OPT with decision) and were then able to completely eliminate bit errors in most cases. Furthermore, also time-variant channels can be equalized in a straitforward way, which is not possible with the CP or the RISIC algorithm. The OPT requires $\mathcal{O}(n^2)$ operations while a conventional equation solver based on matrix inversion requires $\mathcal{O}(n^3)$ operations. After 8 iteration cycles the results converged to stable values.

Our new method is very promising for wireless high speed data transmission where spectral efficiency is important.

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References

[1] R. van Nee, R. Prasad, *OFDM for Wireless Multimedia Communications*, Boston, London: Artech House Publishers, 2000.

[2] T. May, H. Rohling, “OFDM, Orthogonal Frequency Division Multiplexing”, in A.F. Molisch, Ed., *Wireless Wideband Digital Communications*, Prentice Hall, Summer 2000.

[3] J. M. Cioffi, *A Multicarrier Primer*, http://www-isl.stanford.edu/people/cioffi/papers.html.

[4] H. Steendam, M. Moeneclaey, “Optimization of OFDM on Frequency-Selective Time-Selective Fading Channels”, *Proc. URSI ’98*, p.398, 1998.

[5] M. Russel, G. L. Stüber, “Interchannel Interference Analysis of OFDM in a Mobile Environment”, *Proc. VTC 95*, p.820, 1995.

[6] D. Kim, G. L. Stüber, “Residual ISI Cancellation for OFDM with Application to HDTV Broadcasting”, *IEEE J. Select. Areas in Comm.*, vol.16, no.8, Oct. 1998.

[7] J.C. Cannon, “Non-local Perturbation Techniques”, in W. Kalkofen, ed., *Methods in Radiative Transfer*, Cambridge Univ. Press, Cambridge, U.K., 1984.

[8] L. Auer, “Acceleration Of Convergence”, in W. Kalkofen, ed., *Numerical Radiative Transfer*, Cambridge Univ. Press, Cambridge, U.K., 1987.

[9] K.C. Ng, 1974, *J. Chem. Phys.*, 61, 2680.

[10] G.H. Golub, C.F. van Loan, *Matrix Computations*, 2.ed., Baltimore: John Hopkins University Press, 1989.