

# Booster EDFAs in RZ coded links: Are they average power limited?

Martin Pauer, Peter J. Winzer, Walter R. Leeb

Institut für Nachrichtentechnik und Hochfrequenztechnik,  
Technische Universität Wien,  
Gußhausstraße 25/389, A-1040 Wien, Austria

## ABSTRACT

We analyze the properties of erbium-doped fiber amplifiers (EDFAs) with respect to the gain provided for RZ input pulses of various duty cycles, but at a constant average input power. For representative data rates the EDFA is *average power limited*, i.e. the gain of the amplifier is determined by the *average* input power only, regardless of the RZ duty cycle. The time constants associated with the gain dynamics depend on the *peak* input power of the pulses, which justifies a detailed comparison between NRZ and RZ operation. We derive a dynamic model of the fiber amplifier, treating the erbium doped fiber as a basic three level laser system and neglecting the influence of amplified spontaneous emission (ASE). The results from our model are valid whenever saturation of the EDFA due to ASE can be neglected, i.e. for sufficiently large input signal levels. The numerical calculations show that average power limitation can be assumed for data rates above a certain threshold rate; depending on the pump power, threshold values for the data rate are found to be on the order of several hundred *kbit/s*. Further, the predictions of the theoretical model are confirmed by experimental verifications.

**Keywords:** EDFA, gain dynamics, average power limitation, return-to-zero (RZ), booster, fiber amplifier

## 1. INTRODUCTION

Optical transmission at data rates of some  $10\text{Gbit/s}$  is discussed for future intersatellite links due to the demand for high transmission capacity of global broadband communication networks. Since energy resources in space are extremely limited, high receiver sensitivity is of prime importance. In connection with the requirements of high reliability, robustness and good production economy, intensity modulation (on-off-keying, OOK) with optically preamplified direct detection at a wavelength of  $\lambda = 1.55\mu\text{m}$  has turned out to be a promising concept<sup>1,2</sup>. This is due to the commercial availability of components at  $1.55\mu\text{m}$ , especially that of erbium doped fiber amplifiers (EDFAs) of high performance, which are widely employed in terrestrial fiber communications.

As has been pointed out in recent studies<sup>3-6</sup>, the sensitivity of direct detection receivers can be enhanced considerably by using return-to-zero (RZ) coding instead of the more common non-return-to-zero (NRZ) modulation format. Making the optical pulses representing a mark shorter than the bit duration reduces the *average* receive (and transmit) power requirements to achieve a certain bit error probability (BEP). A sensitivity improvement of about  $3\text{dB}$  can be expected<sup>5-8</sup>, even if the receiver parameters, especially the electrical bandwidth, are the same as for the NRZ case.

However, the RZ gain comes at the cost of an increased *peak* transmit power: For a given average power, the peak optical power of the RZ pulses increases linearly with the *RZ factor*  $D$ , which we define for convenience as the inverse of the duty cycle, i.e. the ratio of the bit duration to the pulse duration. The higher the RZ factor is chosen, the higher the peak power that has to be generated at the transmitter. This, in turn, can be a possible drawback of RZ coding if the transmitter is *peak power limited*, which would be the case, e.g., for semiconductor laser sources. If, on the other hand, a laser oscillator-optical power amplifier concept is used, the higher peak power required can easily be obtained if the optical power amplifier is *average power limited*. With an eye on RZ coded systems we therefore investigated whether EDFAs are average power limited, i.e. whether their gain is determined by the average input power only.

---

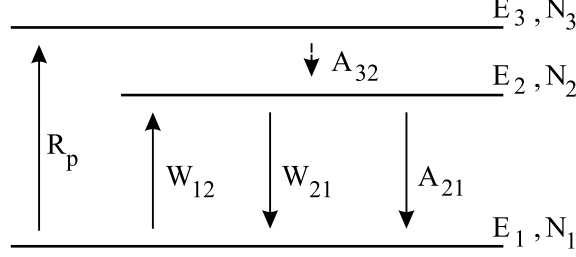
M.P.: E-mail: mpauer@nt.tuwien.ac.at; phone: +43 1 58801 38901; fax: +43 1 58801 38999

P.J.W.: E-mail: pwinzer@nt.tuwien.ac.at

W.R.L.: E-mail: Walter.Leeb@tuwien.ac.at

## 2. RATE EQUATIONS

For an analysis of the EDFA we assume that the laser medium is homogeneously broadened, which is reported to be in good agreement with the actual behaviour of the amplifier at temperatures of about  $300K$ . Further, the manifolds within each energy level due to *Stark splitting* are accounted for by employing phenomenological transition cross sections, which is also justified at room temperature as a consequence of the rapid relaxation processes within each manifold.



**Figure 1.** Energy levels and transition rates in the erbium doped fiber.

When pumped at  $\lambda_p = 980nm$ , the erbium doped fiber can then be modeled adequately as a basic three level laser system, as is depicted in Fig. 1. The energy gap between levels  $E_3$  and  $E_1$  corresponds to the pump wavelength  $\lambda_p$ , and the transition from the metastable level  $E_2$  to the ground level  $E_1$  is of almost completely radiative nature, providing gain in the signal band at wavelength  $\lambda_s$  ( $1530nm - 1560nm$ ). The predominant mechanism of decay in the pump level  $E_3$  is the spontaneous, non-radiative transition of excited erbium ions to level  $E_2$ .<sup>10,11</sup> With  $N_i$ ,  $i = 1, 2, 3$ , denoting the fractional population densities of energy levels  $E_i$ , i.e.  $N_1 + N_2 + N_3 = 1$ , the rate equations are given by

$$\frac{\partial N_1}{\partial t} = -(R_p + W_{12})N_1 + (A_{21} + W_{21})N_2, \quad (1)$$

$$\frac{\partial N_2}{\partial t} = W_{12}N_1 - (A_{21} + W_{21})N_2 + A_{32}N_3, \text{ and} \quad (2)$$

$$\frac{\partial N_3}{\partial t} = R_p N_1 - A_{32} N_3. \quad (3)$$

With  $A_{ij}$  we denote the spontaneous emission rates,  $R_p$  and  $W_{ij}$  are the stimulated transition rates. To eliminate the dependence of the erbium dopant concentration and the optical power on the angular and radial coordinates we use spatially (over the fiber cross section) averaged quantities. Then, the stimulated transition rates,  $R_p$  for the pump and  $W_{ij}$  for the signal, can be written as<sup>10</sup>

$$W_{12} = \Gamma_s \frac{\sigma_{a,s} P_s}{h\nu_s A_{\text{eff}}}, \quad W_{21} = \Gamma_s \frac{\sigma_{e,s} P_s}{h\nu_s A_{\text{eff}}}, \quad \text{and} \quad R_p = \Gamma_p \frac{\sigma_{a,p} P_p}{h\nu_p A_{\text{eff}}}, \quad (4)$$

with  $\sigma_{e,s}$  and  $\sigma_{a,s}$  being the emission and absorption cross section at the signal frequency  $\nu_s = c_0/\lambda_s$ , and  $\sigma_{a,p}$  denoting the absorption cross section for the pump at frequency  $\nu_p$ ;  $h$  is Planck's constant and  $c_0$  the speed of light in vacuum. Signal and pump power are given by  $P_s$  and  $P_p$ , and  $A_{\text{eff}}$  represents an effective doped area. The factors  $\Gamma_s$  and  $\Gamma_p$  account for the overlap of the optical intensity in the fiber with the dopant distribution.

## 3. TWO-LEVEL EDFA OF INFINITESIMAL LENGTH

To get a qualitative estimate of the fiber amplifier gain dynamics and the associated time constants, it is convenient to solve the rate equations for a step-like input signal and constant pump power at the fiber input<sup>12</sup>, since pump and signal power are known there if the EDFA is assumed to be pumped in the forward direction. For *moderate pump powers* of less than  $1W$  the population density in energy level  $E_3$  can be neglected<sup>13,15</sup>, due to the rapid transition from  $E_3$  to  $E_2$  (rate  $A_{32}$ ). In this case, the effect of pumping is to directly increase the population density  $N_2$  and

we can substitute  $R_p N_1$  for  $A_{32} N_3$  in (2); using the approximation  $N_3 \approx 0$  and  $N_1 + N_2 = 1$  yields an ordinary differential equation for the population densities\*  $N_1$  and  $N_2$ . For  $N_2$  we get

$$\frac{dN_2}{dt} + N_2 \frac{1}{\tau} (1 + p + q) - \frac{1}{\tau} \left( \frac{p}{1 + \sigma_{e,s}/\sigma_{a,s}} + q \right) = 0, \quad (5)$$

where  $\tau$  is the spontaneous lifetime in the metastable level  $E_2$  and is related to  $A_{21}$  according to  $\tau = A_{21}^{-1}$ . The quantities  $p$  and  $q$  are the signal and pump power normalized to their respective saturation powers, defined as

$$P_{sat,s} = \frac{h\nu_s A_{\text{eff}}}{\Gamma_s (\sigma_{e,s} + \sigma_{a,s}) \tau}, \quad \text{and} \quad P_{sat,p} = \frac{h\nu_p A_{\text{eff}}}{\Gamma_p \sigma_{a,p} \tau}. \quad (6)$$

The general solution of (5) is given by  $N_2(t) = C \exp(-t/\Omega) + C_1$ , which also applies for  $N_1(t)$ , but, of course, with different constants  $C$  and  $C_1$ . The gain coefficient, describing the incremental change in signal power when passing through the differential piece of the EDFA, is related to the difference between the populations  $N_2$  and  $N_1$ , according to<sup>10</sup>

$$\gamma = \Gamma_s N_t (\sigma_{e,s} N_2 - \sigma_{a,s} N_1), \quad (7)$$

with  $N_t$  being the total erbium concentration in the fiber. Thus, the gain coefficient also shows an exponential time dependence and can be written as

$$\gamma(t) = (\gamma(0) - \gamma_\infty) e^{-t/\Omega} + \gamma_\infty, \quad (8)$$

where the constant  $\gamma_\infty$  is identified as the steady state solution ( $d/dt = 0$ ) obtained for constant input conditions. Assuming that the input signal changes at time  $t = 0$ ,  $\gamma(0)$  gives the initial condition<sup>†</sup> for the gain coefficient, just before the input changes. The time constant  $\Omega$  can be calculated from (5) to be

$$\Omega_{on} = \tau_p = \frac{\tau}{1 + p + q} \quad (9)$$

if the input signal ( $p$ ) is turned on; If the input signal is turned off ( $p = 0$ ), we get

$$\Omega_{off} = \tau_q = \frac{\tau}{1 + q}. \quad (10)$$

### 3.1. Average power limitation is not obvious

Equations (9) and (10) show that the time constants associated with the amplifier gain are *not* necessarily on the order of the metastable lifetime  $\tau$ . The fastest transients will occur if both signal and pump power are strong<sup>12</sup>. This may be the case especially for RZ coded input signals with large RZ factors  $D$ , since at a *constant average* power the *peak* input power of RZ pulses increases linearly with  $D$ , resulting in a considerable increase of the emission rate  $W_{21}$  (4), and thus leading to a faster depletion of the upper energy level  $E_2$ , since  $\tau_p$  depends on the *peak* input power, as expressed in (9). On the other hand, the pump power is the same, regardless of the coding scheme, and likewise will be the time constant  $\tau_q$ , describing the recovery of the system. Hence, it is *not* obvious whether the gain coefficient and the amplifier gain are still determined by the *average* input power in case of RZ coded signals.

As an example, we will consider a periodic square wave pulse train, corresponding to a '...1010...' bit pattern, with constant average input power  $\bar{p}$  and calculate the gain coefficient in the case of NRZ coding and RZ coding for various RZ factors  $D$ ,

$$D = \frac{T_b}{T_p}, \quad (11)$$

with  $T_b$  being the bit duration and  $T_p$  being the pulse duration; an RZ factor of  $D = 1$  can be regarded as NRZ coding. For such signals, the peak input power is given by

$$\hat{p} = 2D\bar{p}. \quad (12)$$

\* Since we restrict ourselves to an infinitesimal slice of the EDFA,  $N_i$  are functions of time only.

† The system needs not to be in steady state for (8) to be valid.

When applying the pulse train for a sufficiently long time, the system will settle to a dynamic equilibrium, indicated by the oscillation of the gain coefficient  $\gamma(t)$  between constant upper and lower limits, denoted by  $\gamma_q$  and  $\gamma_p$ , with characteristic time constants given by (9) and (10). These limits can be calculated from (8): We find

$$\gamma_p = \Delta\gamma_0 e^{-T_p/\tau_p} \frac{1 - \exp(-T_q/\tau_q)}{1 - \exp(-T_p/\tau_p - T_q/\tau_q)} + \gamma_{\infty,p} \quad (13)$$

and

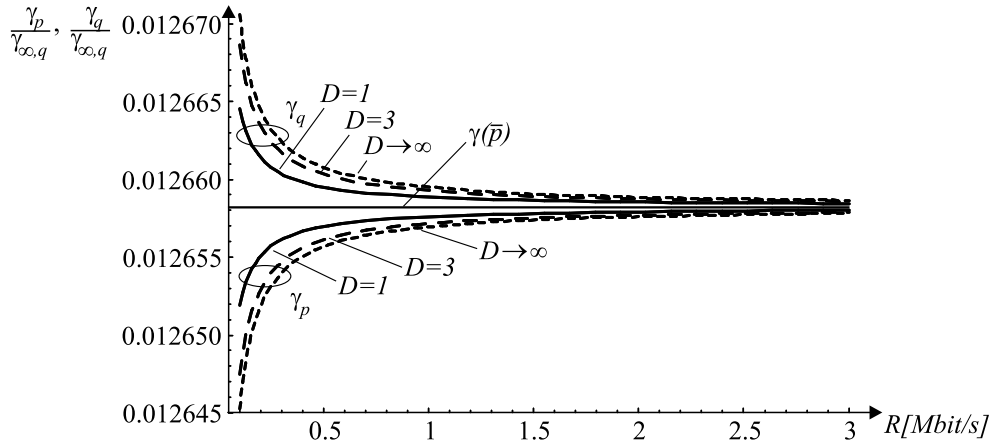
$$\gamma_q = \Delta\gamma_0 e^{-T_q/\tau_q} \frac{\exp(-T_p/\tau_p) - 1}{1 - \exp(-T_p/\tau_p - T_q/\tau_q)} + \gamma_{\infty,q} , \quad (14)$$

where  $\gamma_{\infty,p}$  and  $\gamma_{\infty,q}$  correspond to the gain coefficient in steady state for a constant input power  $p = \hat{p}$  and  $p = 0$ , respectively. The factor  $\exp(-T_p/\tau_p)$  describes the saturation of the amplifier due to amplification of one pulse. As  $T_q$  denotes the duration of the spaces between adjacent pulses,  $\exp(-T_q/\tau_q)$  accounts for the recovery of the system. Further, the abbreviation  $\Delta\gamma_0 = \gamma_{\infty,q} - \gamma_{\infty,p}$  was used. With the data rate  $R = T_b^{-1}$ , pulse duration  $T_p$  and recovery time  $T_q$  are given by

$$T_p = \frac{1}{RD} , \text{ and } T_q = \frac{2D - 1}{RD} \quad (15)$$

for the input sequence assumed.

Plots of the lower and upper limit of the gain coefficient,  $\gamma_p$  and  $\gamma_q$ , parameterized by the RZ factor  $D$ , are shown in Fig. 2 as a function of the data rate. The normalized pump power was chosen to be  $q = 77$ , the average signal power equals  $\bar{p} = 1$  and a typical value of  $\tau = 10ms$  was assumed<sup>10</sup>. The curves show an asymptotic behaviour with respect to the RZ factor  $D$ . This can be explained considering the pulse duration  $T_p$  and the time constant  $\tau_p$ : Both are inversely proportional to  $D$  and thus their ratio, indicating the saturation of the amplifier, approaches a limit for high RZ factors. Hence, despite of the high peak power in the RZ case, leading to extremely fast transients (9), the inversion of the amplifier is *not* depleted arbitrarily (until it becomes transparent, corresponding to  $\gamma_p = 0$ ), even though RZ pulses ( $D > 1$ ) end up at a lower level of  $\gamma_p$  than NRZ pulses ( $D = 1$ ). In fact, we find the differences  $\gamma_q - \gamma_p$  of the gain coefficient between the leading and the trailing edge of an input pulse to increase with the RZ factor, which, however, is distinct only at low data rates. Such differences manifest themselves as a distortion of the output pulse shape.



**Figure 2.** Plot of normalized upper and lower bounds  $\gamma_q$  and  $\gamma_p$  of the gain coefficient  $\gamma(t)$  in dynamic equilibrium as a function of the data rate  $R$  for an input pulse train corresponding to a '...1010...' bit sequence for NRZ pulses ( $D = 1$ ) and RZ pulses ( $D > 1$ ). An asymptotic behaviour with respect to the RZ factor  $D$  can be observed. As  $R$  is increased,  $\gamma_q$  and  $\gamma_p$  are determined by the average input power  $\bar{p}$  of the modulated signal, indicated by  $\gamma(\bar{p})$ , regardless of the RZ factor.

On the other hand, spaces between pulses become wider as  $D$  is increased, allowing for a longer recovery time  $T_q$  of the system. This obviously compensates for the deeper drop of  $\gamma$  in the RZ case, as the curves of  $\gamma_q$  corresponding

to  $D > 1$  lie above that for  $D = 1$ . Thus, the faster depletion of inversion due to the higher peak power of RZ pulses is perfectly balanced by an accompanying shorter pulse duration and a longer interval  $T_q$  for recovery. It can be shown analytically that  $\gamma_q$  and  $\gamma_p$ , in the limit of large data rates  $R$ , converge at the value of  $\gamma(\bar{p})$ , the steady state solution of the gain coefficient for a constant input power  $\bar{p}$ . The value  $\bar{p}$  equals the *average* power of the modulated input signal. This is true not only in the NRZ case (as could be expected), but is *independent* of the RZ factor  $D$ .

#### 4. FINITE LENGTH THREE-LEVEL EDFA

The main simplification made in the previous section which enables analytical solutions was that we did not take into account the variation of optical power along the amplifier. In a realistic case, signal power will grow and pump power will be absorbed during propagation, and thus the population densities  $N_i$  also depend on the longitudinal coordinate  $z$ . To incorporate this feature, we put up propagation equations of the form

$$\frac{\partial P_p(z)}{\partial z} = -\Gamma_p \sigma_{a,p} N_1(z) N_t P_p(z) \quad (16)$$

for the pump, and

$$\frac{\partial P_s(z)}{\partial z} = \Gamma_s [\sigma_{e,s} N_2(z) - \sigma_{a,s} N_1(z)] N_t P_s(z) \quad (17)$$

for the signal. Equations (16) and (17) only describe the evolution of pump and signal power. This means that we neglect the effect of amplifier saturation due to amplified spontaneous emission (ASE), which will occur in high gain amplifiers at low input powers<sup>14</sup> only and is thus irrelevant for optical booster amplifiers operated in the saturated output power regime.

To derive a theoretical model, we basically follow references [15]-[17], but consider the general case and treat the EDFA as a three level system. With the relation  $N_3 = 1 - N_1 - N_2$  and the explicit equations for the transition rates (4), the rate equations (1) and (2) can be combined with the propagation equations (16) and (17). With  $L$  denoting the length of the EDFA and using a time dependent path-averaged population density  $\rho_i(t)$  of laser level  $E_i$ , defined as

$$\rho_i(t) = \frac{1}{L} \int_{z=0}^L N_i(z, t) dz, \quad i = 1, 2, \quad (18)$$

which can take values between 0 and 1, we get

$$\frac{d\rho_1}{dt} = \rho_2(t) A_{21} + p_{in}(t) \frac{A_{21}}{(g_s + \alpha_s)L} (\exp[(g_s \rho_2(t) - \alpha_s \rho_1(t))L] - 1) + q_{in}(t) \frac{A_{21}}{\alpha_p L} (\exp[-\alpha_p L \rho_1(t)] - 1), \quad (19)$$

and

$$\frac{d\rho_2}{dt} = A_{32} - \rho_1(t) A_{32} - \rho_2(t) (A_{21} + A_{32}) - p_{in}(t) \frac{A_{21}}{(g_s + \alpha_s)L} (\exp[(g_s \rho_2(t) - \alpha_s \rho_1(t))L] - 1). \quad (20)$$

The parameters introduced in (19) and (20) are the small signal gain coefficient<sup>18</sup>  $g_s = \Gamma_s N_t \sigma_{e,s}$  and the absorption coefficients for signal and pump,  $\alpha_s = \Gamma_s N_t \sigma_{a,s}$  and  $\alpha_p = \Gamma_p N_t \sigma_{a,p}$ ; optical powers are normalized to their respective saturation powers (6).

For known input conditions, e.g. constant pump power  $q_{in}(t) \equiv q = \text{const.}^\dagger$  and a pulse train as the input signal  $p_{in}(t)$ , (19) and (20) can be solved numerically. Once the variables  $\rho_1(t)$  and  $\rho_2(t)$  are known, it is straightforward to calculate the corresponding output powers or the time dependent gain of the amplifier.

So far, we have not incorporated nonlinear effects in our model, which might occur for broadband pulses and high peak powers. Several studies have been carried out investigating the amplification of short pulses and rather

---

<sup>†</sup>In the following only the case of constant input pump power is considered and thus we omit the index and the time dependence of  $q$ .

high output peak powers are reported: In reference [19], a peak output power of  $105W$  for  $10ps$  pulses was obtained in a two stage EDFA with a pump power of  $90mW$ . Other than with sub-picosecond pulses, gain dispersion at pulse durations larger than  $1ps$  is generally negligible<sup>20</sup>. At a data rate of  $R = 10Gbit/s$ , a pulse duration of  $T_p = 1ps$  would correspond to an RZ factor as high as  $D = 100$ , yielding a peak power of  $200W$  for an average output power of  $1W$ . In a realistic application of RZ coded systems, where an RZ factor of about  $D = 3$  would be employed, nonlinear effects are thus negligible.

## 5. NUMERICAL CALCULATIONS

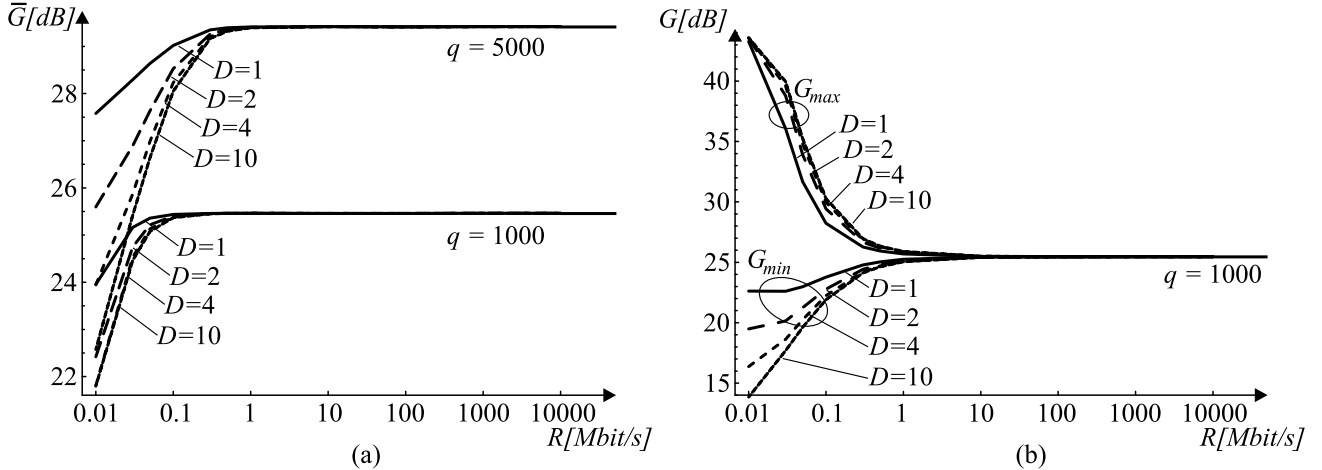
We will now use the theoretical model derived in Sect. 4 to calculate the EDFA gain in dynamic equilibrium for the case of a square wave input signal, corresponding to a '...1010...' bit pattern, and various RZ factors  $D$ . The average input power is kept constant, since we are interested in the average power limitation of the fiber amplifier.

Numerical values of the parameters underlying our calculations are  $A_{21} = 83s^{-1}$ ,  $A_{32} = 5 \cdot 10^4 s^{-1}$ ,  $g_s L = 10.8$ ,  $\alpha_s L = 11.8$ ,  $\alpha_p L = 16.11$  and  $P_{sat,s} = 563\mu W$ . A typical value for  $A_{32}$  was taken from literature<sup>10</sup>, the other parameters were derived from measurements on the EDFA used for the experiment (see Sect. 6). The normalized pump power was assumed to be either  $q = 1000$  or  $q = 5000$ , which should cover the range found in power amplifiers. The average input signal power was set to  $\overline{P}_{s,in} = 0dBm$ , which assures saturated operation.

We define an instantaneous amplifier gain  $G(t)$  as the ratio of output and input power and introduce the average gain,  $\overline{G}$ , for an input pulse as

$$\overline{G} = \frac{1}{T_p} \int_{t=0}^{T_p} G(t) dt . \quad (21)$$

We will use  $\overline{G}$  as a criterion for the dependence of EDFA gain on the RZ factor  $D$ . A plot of  $\overline{G}$  as a function of the data rate  $R$  is shown in Fig. 3 (a), where we have depicted two sets of curves, each corresponding to a different pump power  $q$ ; the parameter is the RZ factor  $D$ . Clearly, the EDFA can be said to be average power limited only at data rates *above* a certain threshold  $R_{th}$ . The value of  $R_{th}$  depends on pump power  $q$ , shifting to higher values as  $q$  is increased. Figure 3 (a) shows that  $R_{th}$  typically lies in the range of several hundred  $kbit/s$ . The RZ factor  $D$



**Figure 3.** (a) Dependence of average gain  $\overline{G}$  on data rate  $R$  for two different normalized pump powers  $q$  and an input bit sequence of '...1010...' in dynamic equilibrium (average input power  $\overline{P}_{s,in} = 0dBm$ ). Parameter of the curves is the RZ factor  $D$ . For a constant input power  $P_{s,in} = \overline{P}_{s,in}$ , the gain amounts to  $G = 29.4dB$  ( $q = 5000$ ) and  $G = 25.4dB$  ( $q = 1000$ ). The curves show a threshold  $R_{th}$  for average power limitation, depending on  $q$ . In (b), maximum ( $G_{max}$ ) and minimum ( $G_{min}$ ) gain during pulse amplification for  $q = 1000$  are shown. For  $R$  exceeding a threshold  $R_{th}$ ,  $G_{max}$  and  $G_{min}$  are determined by the average input power  $\overline{P}_{s,in}$  only, regardless of the RZ factor.

has negligible effect on the threshold  $R_{th}$ . Thus, for  $R > R_{th}$ , the average gain  $\overline{G}$  is essentially the same for NRZ and RZ pulses.

To assess RZ pulse distortion we determined the maximum gain  $G_{max}$ , available at the leading edge of a rectangular pulse, and the minimum gain  $G_{min}$ , present at the trailing edge. Figure 3 (b) presents the results for a pump power of  $q = 1000$  and different RZ factors  $D$ . The gain curves of Fig. 3 (b), valid for the entire EDFA device, nicely mirror the results of Fig. 2, valid for an infinitesimal slice at the fiber amplifiers input. In the RZ case, the shorter pulse duration  $T_p$  and the longer recovery time  $T_q$  compensate for the deeper depletion of the EDFA. Thus, as the data rate is increased, pulses are amplified, *independent of the RZ factor  $D$* , at an essentially constant gain equal to  $\bar{G}$  or  $G(\bar{P}_{s,in})$ , the gain determined by the *average* input power.

## 6. EXPERIMENTAL VERIFICATION

According to the results derived in the previous section, the EDFA is average power limited at data rates above a certain threshold  $R_{th}$ . To observe deviations from average power limitation, measurements were carried out at data rates below  $R_{th}$ .

The EDFA used in our experiments<sup>§</sup> is characterized (for numerical calculations) by the parameters given in Sect. 5.<sup>¶</sup> The normalized pump power for this device was determined to be  $q = 77$ ; it is *not a high power amplifier*. To confirm the results of the theoretical model, we measured the average gain (21) of the EDFA at data rates below  $10\text{ kbit/s}$ . Additionally, the output pulse shape  $P_{s,out}(t)$  was monitored. The input pulses corresponded to a '...100100...' bit pattern<sup>||</sup> with a constant average input power of  $\bar{P}_{s,in} = -10\text{ dBm}$ ; RZ factors of up to  $D = 4$  were used.

### 6.1. Average gain

The measured average gain  $\bar{G}$  as a function of the data rate  $R$  is in good agreement with the predicted values, calculated from the theoretical model (see Fig. 4). Solid lines indicate measured values, whereas dashed lines correspond to the theoretical curve. As expected, we find  $\bar{G}$  to drop faster for RZ pulses below about  $R = 5\text{ kbit/s}$ ; for higher data rates, differences brought by RZ modulation become negligible. The threshold for average power limitation in this case amounts to  $R_{th} \approx 5\text{ kbit/s}$ . This confirms the dependence of  $R_{th}$  on the pump power, as found in the previous section: Here we have  $q = 77$ , while for  $q = 1000$  and  $q = 5000$  we had found  $R_{th} \approx 500\text{ kbit/s}$  and  $R_{th} \approx 1\text{ Mbit/s}$ , respectively.

Differences between theory and measurement show up as the data rate approaches  $1\text{ kbit/s}$ . They can be attributed to the effect of gain saturation due to ASE, which was not incorporated in the theoretical model. For instance, a numerical evaluation at  $R = 1\text{ kbit/s}$  predicts a gain of about  $44\text{ dB}$  for the leading edge of a pulse. This high value is caused by the long recovery time  $T_q$  between pulses. On the other hand, corresponding measured gains are between  $35\text{ dB}$  and  $36.3\text{ dB}$  (depending on RZ factor). Obviously, the theoretical model is no longer valid in this case: A calculated gain of  $44\text{ dB}$  is larger than the specified small signal gain of the EDFA investigated ( $38\text{ dB}$ ) and thus a consequence of neglecting ASE. Differences are more pronounced for higher RZ factors, due to the longer intervals between pulses.

### 6.2. Output pulse shapes

The average gain  $\bar{G}$  relates the pulse energies at the input and the output of the EDFA, but does not show explicitly the gain dynamics and pulse distortion. To verify that this feature is also accounted for with reasonable accuracy by our theoretical model, we measured the shape of the output pulses at  $R = 10\text{ kbit/s}$ .

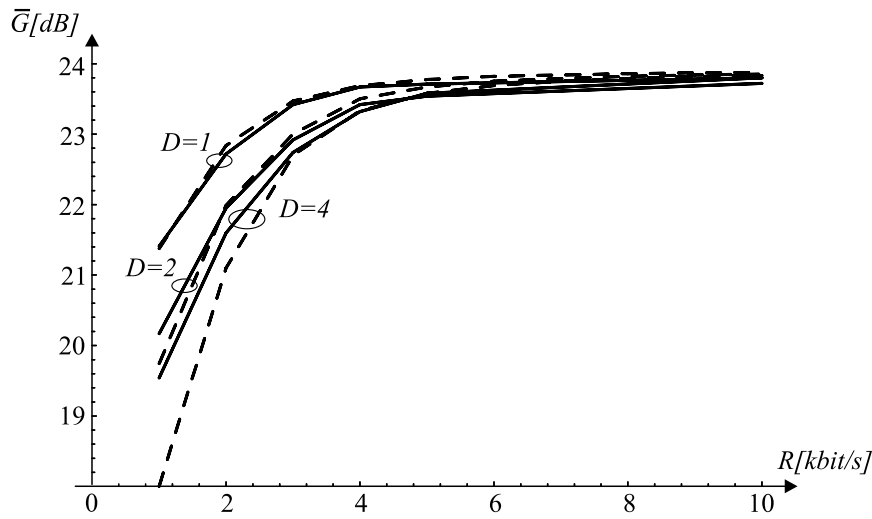
Oscilloscope traces of the output pulses  $P_{s,out}(t)$  for different RZ factors  $D = 1, 2, 4$  are given in Fig. 5 as solid lines, dashed lines are calculated pulse shapes. The theoretical results match very well the measured pulses, both regarding absolute power levels and time dependence. For a detailed comparison, the measured and calculated gain values are given in Table 1.

Slight differences can be attributed to the uncertainty of the parameters used for calculations. Other than predicted by theory, measured values of the initial gain  $G_{max}$  do not show a steady increase with increasing RZ

<sup>§</sup>OPREL OFP14W-1142S

<sup>¶</sup>To improve the fit between measurements and numerical calculations, losses at the input ( $\eta_{in} = 0.2\text{ dB}$ ) and output ( $\eta_{out} = 3\text{ dB}$ ) of the EDFA were introduced, which were also taken into account in Sect. 5.

<sup>||</sup>This sequence was chosen because the gain reduction for pulses with different RZ factor is more distinct in this case, which in turn simplifies experimental evaluation.



**Figure 4.** Average gain  $\bar{G}$  as a function of data rate  $R$  for an input bit pattern of '...100100...' in dynamic equilibrium and different RZ factors  $D$  for an average input power of  $\bar{P}_{s,in} = -10dBm$ . The gain for a constant input power  $P_{s,in} = \bar{P}_{s,in}$  was measured to be  $23.8dB$ . Measurement (solid lines) and calculated (dashed lines) results are in good agreement. Deviations at low data rates are attributed to gain saturation due to ASE.

$D$	measured		calculated	
	$G_{max}[dB]$	$G_{min}[dB]$	$G_{max}[dB]$	$G_{min}[dB]$
1	27.9	21.7	27.7	21.5
2	28.8	21.1	28.5	20.7
4	28.5	20.8	28.9	20.4

**Table 1.** Measured and calculated values of amplifier gain at  $R = 10kbit/s$ .

factor. This again can be attributed to ASE: Due to the relatively long duration between pulses in the case of  $D = 4$  ( $T_q = 275\mu s$ ), the growth of ASE power may slow down the build up of inversion, thus reducing the gain. Consequently, the agreement between calculated and measured pulse shapes for  $D = 4$  is slightly worse as compared to the case of  $D = 1$  or  $D = 2$ .

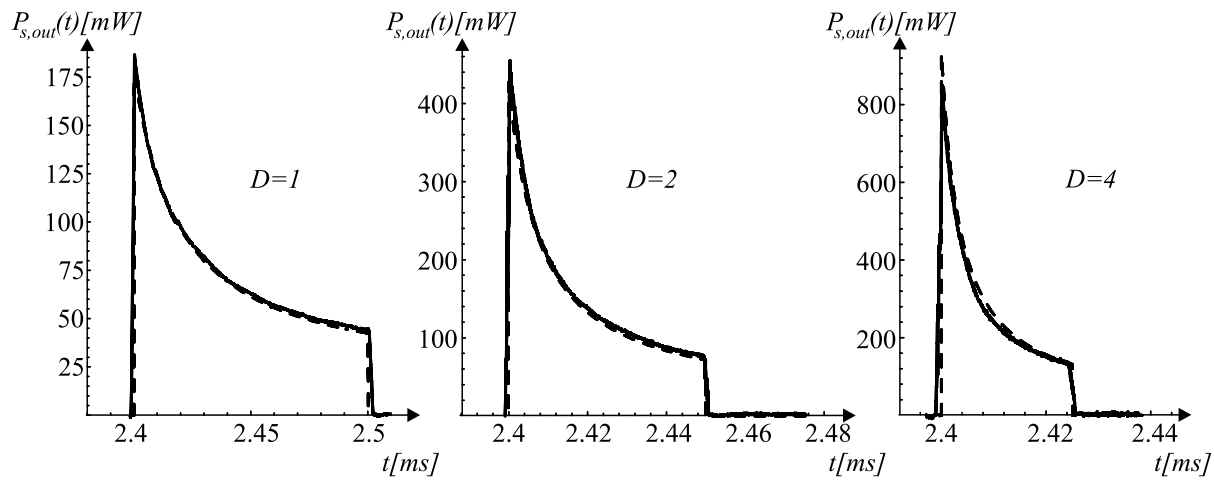
The results presented in this section are in good agreement with the theoretical model, even though it is a quite simple one. This is noteworthy, since a detailed structure of the amplifier device is not incorporated, and neither was the effect of gain saturation due to ASE. Thus, measurement results together with the conclusions based on the theoretical model show that in typical communication applications the EDFA is average power limited, regardless whether NRZ or RZ modulation is employed.

## 7. CONCLUSIONS

Considering a slice of the EDFA of infinitesimal length and simplifying the model by regarding the erbium doped fiber as a two level laser system, we could derive analytical results for the time dependence of the *gain coefficient* and the time constants associated with gain dynamics. We found that the inversion is depleted faster and deeper in the RZ case as compared to the NRZ case, but the gain coefficient shows an asymptotic behaviour with respect to the RZ factor. As the data rate is increased, distortion of the output pulses becomes negligible and the gain coefficient is determined by the average input power only, regardless of the RZ factor. This simple model facilitates a principal understanding of the dynamic behaviour found for a realistic amplifier.

The extension of our model incorporating the variation of signal and pump power along the amplifier showed that the results derived from the simple model are an excellent first estimate of the amplifier dynamics. The numerical calculations, now concerning the EDFA *gain*, show that the EDFA is *average power limited* above a certain data rate  $R_{th}$ ; this threshold notably depends on pump power, but is virtually independent of the RZ factor. Above  $R_{th}$ ,





**Figure 5.** Output pulse shapes  $P_{s,out}(t)$  for an input pulse train corresponding to a '...100100...' bit pattern in dynamic equilibrium at a data rate of  $R = 10\text{ kbit/s}$  and for various RZ factors  $D$ . Note the different ordinate scales. Solid lines represent measured pulse shapes, dashed lines correspond to calculated results. Slight differences for  $D = 4$  are attributed to the effect of ASE on the amplifier gain.

which, for booster EDFAs, is in the order of several hundred  $\text{kbit/s}$ , pulse distortion is negligible and the EDFA gain essentially is determined by the average input power.

Measurements carried out on an EDFA at low data rates were found to be in good agreement with the results from numerical calculations, except at very low data rates, where ASE, which is not accounted for in our model, is believed to cause deviations. We conclude that the theoretical model sufficiently includes the dynamic properties of the EDFA. Measurement results show that for high data rates the EDFA is sufficiently described by a *constant* gain determined by the *average* input power, for NRZ as well as for RZ coded signals.

## ACKNOWLEDGMENTS

The work presented here has been made possible by grant No. H-61/99 from the *Hochschuljubiläumsstiftung der Stadt Wien*, by funds from *Wr. Städtische Allgemeine Versicherung*, as well as by funding from the *Fonds zur Förderung der wissenschaftlichen Forschung (FWF)*, Project No. P13998TEC.

## REFERENCES

1. P.J.Winzer, A.Kalmar, W.R.Leeb, "Intersatellite laser communication at  $1.5\mu\text{m}$ : Chances and problems", *European Space Agency Contract No. 11846/96/NL/SB(SC), Final Report*, July 13, 1998
2. W.R.Leeb, P.J.Winzer, M.Pauer, "The potential of return-to-zero coding in optically amplified lasercom systems", *Proc. IEEE Lasers and Electro-Optics Society (LEOS) 1999 Annual Meeting* **1**, pp. 224-225, 1999.
3. W.Atia, R.S.Bondurant, "Demonstration of return-to-zero signaling in both OOK and DPSK formats to improve receiver sensitivity in an optically preamplified receiver", *Proc. IEEE Lasers and Electro-Optics Society (LEOS) 1999 Annual Meeting* **1**, pp. 226-227, 1999.
4. S.Tanikoshi, K.Ide, T.Onodera, Y.Arimoto, K.Araki, "High sensitivity 10Gbit/s optical receiver for space communications", *Proc. 17th AIAA International Communications Satellite Systems Conference*, pp. 178-183, 1998.
5. P.J.Winzer, A.Kalmar, "Sensitivity enhancement of optical receivers by impulsive coding", *J. Lightwave Technol.* **17**, pp. 171-177, 1999.
6. L.Boivin, M.C.Nuss, J.Shah, D.A.B.Miller, H.A.Haus, "Receiver sensitivity improvement by impulsive coding", *IEEE Photon. Technol. Lett.* **9**, pp. 684-686, 1997.
7. M.Pauer, P.J.Winzer, W.R.Leeb, "Bit error probability reduction in direct detection optical receivers using RZ coding", submitted to *J. Lightwave Technol.*

8. L.Boivin, G.J.Pendock, "Receiver sensitivity for optically amplified RZ signals with arbitrary duty cycle", *Proc. Optical Amplifiers and their Applications (OAA'99)*, ThB4, pp. 106-109, 1999.
9. J.C.Livas, E.A.Swanson, S.R.Chinn, E.S.Kintzer, "High data rate systems for space applications", *Proc. SPIE* **2381**, pp. 38-47, 1995.
10. E.Desurvire, *Erbium doped fiber amplifiers*, John Wiley and sons, inc., 1994.
11. W.J.Miniscalco, "Erbium-doped glasses for fiber amplifiers at 1500nm", *J. Lightwave Technol.* **9**, pp. 234-250, 1991.
12. E.Desurvire, "Analysis of transient gain saturation and recovery in erbium-doped fiber amplifiers", *IEEE Photon. Technol. Lett.* **1**, pp. 196-199, 1989.
13. C.R.Giles, E.Desurvire, "Modeling erbium-doped fiber amplifiers", *J. Lightwave Technol.* **9**, pp. 271-283, 1991.
14. A.A.M.Saleh, R.M.Jopson, J.D.Evankov, J.Aspell, "Modeling of gain in erbium-doped fiber amplifiers", *IEEE Photon. Technol. Lett.* **2**, pp. 714-717, 1990.
15. Y.Sun, G.Luo, J.L.Zyskind, A.A.M.Saleh, A.K.Srivastava, J.W.Sulhoff, "Model for gain dynamics in erbium-doped fibre amplifiers", *Electronics letters* **32**, pp. 1490-1491, 1996.
16. A.Bononi, L.A.Rusch, "Doped-fiber amplifier dynamics: a system perspective", *J. Lightwave Technol.* **16**, pp. 945-956, 1998.
17. S.R.Chinn, "Simplified modeling of transients in gain-clamped erbium-doped fiber amplifiers", *J. Lightwave Technol.*, **16**, pp. 1095-1100, 1998.
18. I.M.I.Habbab, A.A.M.Saleh, P.K.Runge, "Erbium-doped fiber amplifiers: linear approximations", *J. Lightwave Technol.* **13**, pp. 33-36, 1995.
19. H.Takara, A.Takada, M.Saruwatari, "A highly efficient two-stage Er-3+-doped optical fiber amplifier employing an optical gate to efficiently reduce ASE", *IEEE Photon. Technol. Lett.* **4**, pp. 241-243, 1992.
20. G.P.Agrawal, "Amplification of ultrashort solitons in erbium-doped fiber amplifiers", *IEEE Photon. Technol. Lett.* **2**, pp. 875-877, 1990.