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TIME-FREQUENCY PROJECTION FILTERS: ONLINE IMPLEMENTATION, SUBSPACE TRACKING, AND APPLICATION TO INTERFERENCE EXCISION

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ABSTRACT

We propose an *online* time-frequency (TF) projection filter with reasonable computational complexity for signals of arbitrary length. The filter uses local projections and TF subspace tracking based on an efficient iterative eigendecomposition. The high TF selectivity of the online TF projection filter is shown to be advantageous for interference excision in spread-spectrum communications.

1. INTRODUCTION

Linear, time-varying (LTV) filters or systems are important for nonstationary signal processing [1–5]. A discrete-time LTV filter $\boldsymbol{H}$ can be characterized by the time-domain input-output relation

$$y[n] = (\boldsymbol{H}x)[n] = \sum_{n'=-\infty}^{\infty} h[n,n']\,x[n'], \qquad (1)$$

where $h[n,n']$ is the filter's impulse response. In many applications, however, LTV filters are most conveniently described and designed in a joint time-frequency (TF) domain. A TF representation of a discrete-time LTV filter $\boldsymbol{H}$ is provided by the *Weyl symbol* [6–9]

$$L_{\boldsymbol{H}}(n,\theta) \triangleq 2\sum_{m=-\infty}^{\infty} g[n,m]\,e^{-j4\pi\theta m} \;\text{ with }\; g[n,m] \triangleq h[n+m,n-m], \qquad (2)$$

where θ denotes normalized frequency. The Weyl symbol has certain advantages over alternative TF representations [4, 6, 9]. On the other hand, the discrete-time Weyl symbol is a nonaliased and one-to-one representation only for *halfband filters*, i.e., LTV filters that pass only input signal components within the halfband $[\theta_0-1/4,\theta_0+1/4)$ (with θ_0 a fixed center frequency).

A TF design of LTV filters can be achieved by calculating the filter $\boldsymbol{H}$ whose Weyl symbol $L_{\boldsymbol{H}}(n,\theta)$ is closest to a specified *TF weight function* $M(n,\theta)$, possibly subject to suitable side constraints [4–6, 10, 11]. In particular, a *TF projection filter* [10, 11] results when $\boldsymbol{H}$ is constrained to be an orthogonal projection system[1]. TF projection filters implement 0/1-valued TF weightings, corresponding to TF stop and pass regions. This is appropriate for separating signal components, suppressing noise, etc. The TF cutoff characteristic of TF projection filters is sharper than that of any other type of LTV filters. The halfband restriction of the Weyl symbol in (2) is not a major problem in applications since a fullband TF projection filter can always be efficiently implemented by a halfband TF projection filter (see below). The main limitation of the conventional TF projection filter is that is allows only batch processing and involves the solution of an eigenproblem of complexity $\mathcal{O}(N^3)$, where N denotes the signal length. This implies a practical restriction of signal length.

Here, we present an efficient online algorithm for TF projection filters that has linear complexity in N and thus admits signals of arbitrary length. Specifically,

- we present a novel discrete-time version of the TF projection filter that takes into account the halfband characteristic of the Weyl symbol (Section 2);
- we propose an overlap-add online implementation of the TF projection filter involving local projections (Section 3), and an efficent TF subspace tracking scheme using Ritz-accelerated orthogonal iterations (Section 4);
- and we consider the application of online TF projection filters to interference excision in spread-spectrum systems (Section 6).

2. DISCRETE-TIME TF PROJECTION FILTER

Ideally, the TF projection filter $\boldsymbol{H}_{\mathrm{P}}^{\mathrm{opt}}$ is defined as the discrete-time LTV filter whose Weyl symbol is closest (in the least-squares sense) to a given TF weight function $M(n,\theta)$, under the constraint that $\boldsymbol{H}_{\mathrm{P}}^{\mathrm{opt}}$ be an orthogonal projection system and simultaneously a halfband filter for some given halfband center frequency θ_0. Thus,

$$\boldsymbol{H}_{\mathrm{P}}^{\mathrm{opt}} \triangleq \arg\min_{\boldsymbol{H}\in\mathscr{P}\cap\mathscr{H}} \|L_{\boldsymbol{H}}-M\|, \qquad (3)$$

where $\mathscr{P}$ denotes the set of all orthogonal projection systems and $\mathscr{H}$ denotes the linear space of all halfband filters with center frequency θ_0. Since orthogonal projection systems can only suppress or pass signal components, we use 0/1-valued TF weight functions

$$M(n,\theta) = \begin{cases} 1, & \text{for } (n,\theta)\in\mathscr{R} \\ 0, & \text{for } (n,\theta)\notin\mathscr{R}, \end{cases} \qquad (4)$$

where $\mathscr{R}$ is the filter's *TF pass region*. We assume that $\mathscr{R}$ is located within the halfband $[\theta_0-1/4,\theta_0+1/4)$, and we consider $M(n,\theta)$ to be $1/2$-periodic with respect to θ.

Because $\boldsymbol{H}_{\mathrm{P}}^{\mathrm{opt}}$ in (3) does not lend itself to an efficient online implementation, we here propose an approximation to $\boldsymbol{H}_{\mathrm{P}}^{\mathrm{opt}}$ that will be denoted by $\boldsymbol{H}_{\mathrm{P}}$. The even-indexed impulse response samples of $\boldsymbol{H}_{\mathrm{P}}$ are obtained by the following design procedure.

1. The even-indexed impulse response samples of the *Weyl filter* $\boldsymbol{H}_{\mathrm{W}}$ [4] are calculated as $h_{\mathrm{W}}[2l,2l'] = g_{\mathrm{W}}[l+l',l-l']$, where
$$g_{\mathrm{W}}[n,m] = \int_0^{1/2} M(n,\theta)\,e^{j4\pi\theta m}\,d\theta. \qquad (5)$$
(This is the inverse of (2) applied to $L_{\boldsymbol{H}_{\mathrm{W}}}(n,\theta)=M(n,\theta)$.) Note that $\boldsymbol{H}_{\mathrm{W}}$ is self-adjoint since $M(n,\theta)$ in (4) is real-valued.
2. The (real) eigenvalues λ_k and the even-indexed samples of the (orthonormal) eigenfunctions $u_k[n]$ of $\boldsymbol{H}_{\mathrm{W}}$ are calculated by solving the eigenvalue problem $\sum_{l'=-\infty}^{\infty} h_{\mathrm{W}}[2l,2l']\,u_k[2l'] = \lambda_k\,u_k[2l]$.
3. The even-indexed impulse response samples of the TF projection filter $\boldsymbol{H}_{\mathrm{P}}$ are computed according to
$$h_{\mathrm{P}}[2l,2l'] = \sum_{k\in\mathscr{I}} u_k[2l]\,u_k^*[2l'], \qquad (6)$$
where $\mathscr{I}$ denotes the set of indices k for which $\lambda_k > 1/2$.

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[1] An LTV filter $\boldsymbol{H}$ is an orthogonal projection system if it is self-adjoint ($\boldsymbol{H}^+=\boldsymbol{H}$, with $\boldsymbol{H}^+$ the adjoint of $\boldsymbol{H}$) and idempotent ($\boldsymbol{H}\boldsymbol{H}=\boldsymbol{H}$) [12].

Using $h_{\mathrm{P}}[2l,2l']$, the filtering $y[n]=(\boldsymbol{H}_{\mathrm{P}}x)[n]$ according to (1) can be performed as follows. First, the even-indexed samples of the *halfband-restricted* input signal are calculated as

$$\tilde{x}[2l] = \sum_{n=-\infty}^{\infty} \phi[n-2l]\,x[n]\,, \qquad (7)$$

where $\phi[n] \triangleq \frac{\sin(\pi n/2)}{\pi n}\, e^{j2\pi\theta_0 n}$ is the impulse response of an idealized filter with pass band $[\theta_0 - 1/4, \theta_0 + 1/4)$. Then, the even-indexed output samples of $\boldsymbol{H}_{\mathrm{P}}$ are computed as

$$y[2l] = \sum_{l'=-\infty}^{\infty} h_{\mathrm{P}}[2l,2l']\,\tilde{x}[2l']\,. \qquad (8)$$

Note that this filtering step runs at half the normal rate. Finally, the output signal samples are obtained by interpolation,

$$y[n] = \sum_{l=-\infty}^{\infty} \phi[n-2l]\,y[2l]\,. \qquad (9)$$

The LTV filter $\boldsymbol{H}_{\mathrm{P}}$ defined by the above procedure is a *halfband* TF projection filter. If a *fullband* filter is desired, we would have to interpolate the input signal by 2, implement a halfband TF projection filter at twice the original rate, and finally decimate the output signal by 2. However, all of the above design/implementation steps apart from (5) involve only even samples. Correspondingly, the interpolation of the input signal and the decimation of the output signal are reversed by (7) and (9), respectively. Thus, we simply have to replace (8) by $y[n]=\sum_{n'=-\infty}^{\infty} h_{\mathrm{P}}[n,n']\,x[n']$. The only additional effort required to obtain $h_{\mathrm{P}}[n,n']$ is then the calculation of (5) at twice the original sampling rate.

The above design/filtering procedure is not yet suited for online implementation since (i) the entire impulse response is calculated at once, using knowledge of the entire TF weight function $M(n,\theta)$ (or, equivalently, TF pass region $\mathscr{R}$); and (ii) the eigendecomposition has computational complexity $\mathscr{O}(N^3)$, where N is the signal duration (simulataneously the duration of $M(n,\theta)$). In the next two sections, we will present efficient online algorithms that use "local" TF projection filters and iterative eigendecompositions.

3. ONLINE IMPLEMENTATION

We now describe an online implementation of the design/filtering procedure discussed in Section 2. We will not discuss the online implementation of the decimation in (7) and the interpolation in (9) because they can be performed using standard methods [13].

Frequency Discretization. For an online implementation of the TF projection filter, we use a frequency-discretized (sampled) version of the TF weight function $M(n,\theta)$ that is defined as

$$M[n,k] \triangleq M\Big(n, \frac{k}{2K}\Big), \qquad k \in [0, K-1]\,.$$

The number of frequency samples K is chosen as a multiple of 4 for later convenience. (Note that K should be large enough so that $M(n,\theta)$ is sampled sufficiently densely.) Correspondingly, we replace (5) by the IDFT relation

$$g_{\mathrm{W}}[n,m] = \frac{1}{K}\sum_{k=0}^{K-1} M[n,k]\,e^{j2\pi\frac{km}{K}}, \quad m \in \Big[-\frac{K}{2}, \frac{K}{2}-1\Big]. \qquad (10)$$

Note that $g_{\mathrm{W}}[n,m]$ has finite length with respect to m.

According to the first design step in Section 2, we need to calculate $h_{\mathrm{W}}[2l,2l'] = g_{\mathrm{W}}[l+l', l-l']$. Since $l+l'$ and $l-l'$ are either both even or both odd, this means that we need only the values $g_{\mathrm{W}}[2l,2i]$ and $g_{\mathrm{W}}[2l+1,2i+1]$. Thus, (10) can be replaced by

$$g_{\mathrm{W}}[2l,2i] = \frac{2}{K}\sum_{k=0}^{K/2-1} M_{\mathrm{e}}[2l,k]\,e^{j2\pi\frac{k}{K/2}i}, \qquad (11a)$$

$$g_{\mathrm{W}}[2l{+}1,2i{+}1] = \frac{2}{K}\sum_{k=0}^{K/2-1} M_{\mathrm{o}}[2l{+}1,k]\,e^{j2\pi\frac{k}{K/2}i}, \qquad (11b)$$

with $M_{\mathrm{e}}[n,k] \triangleq M[n,k]+M[n,k+K/2]$, $M_{\mathrm{o}}[n,k] \triangleq \big(M[n,k]-M[n,k+K/2]\big)\,e^{j2\pi\frac{k}{K}}$, and $i\in[-K/4, K/4-1]$. Hence, for each n we can use a length-$K/2$ IDFT instead of a length-K IDFT.

Local TF Projection Filter. The next step is to use a "local" TF projection filter whose design requires only a local segment of $M[n,k]$, or, equivalently, of $h_{\mathrm{W}}[2l,2l']$. For a specific even time instant $n_0 = 2l_0$, consider $M[n,k]$ for $n\in[n_0-L, n_0+L-1]$, with fixed segment length $2L$. Via the relation $h_{\mathrm{W}}[2l,2l'] = g_{\mathrm{W}}[l+l', l-l']$ and (11), this segment of $M[n,k]$ corresponds to the segment of $h_{\mathrm{W}}[2l,2l']$ defined by $l,l'\in[l_0-L/2, l_0+L/2-1]$. This, in turn, can be associated to a local Weyl filter matrix $\mathbf{H}_{\mathrm{W}}$ of size $L\times L$ with entries $(\mathbf{H}_{\mathrm{W}})_{i,j} = h_{\mathrm{W}}[2(l_0-L/2+i), 2(l_0-L/2+j)] = g_{\mathrm{W}}[2l_0-L+i+j, i-j]$, where $i,j\in[0,L-1]$. Due to the finite m-support of $g_{\mathrm{W}}[n,m]$, only the K main diagonals of $\mathbf{H}_{\mathrm{W}}$ are nonzero, i.e., $\mathbf{H}_{\mathrm{W}}$ is a banded matrix.

From $\mathbf{H}_{\mathrm{W}}$, we can now construct a local TF projection filter matrix $\mathbf{H}_{\mathrm{P}}$ of size $L\times L$ according to Section 2. That is, we first compute the eigendecomposition of $\mathbf{H}_{\mathrm{W}}$

$$\mathbf{H}_{\mathrm{W}} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^H = \sum_{k=1}^{L} \lambda_k\,\mathbf{u}_k\mathbf{u}_k^H,$$

where the columns of the unitary matrix $\mathbf{U}$ are the (orthonormal) eigenvectors $\mathbf{u}_k$ of $\mathbf{H}_{\mathrm{W}}$ and $\boldsymbol{\Lambda} = \mathrm{diag}\{\lambda_1,\ldots,\lambda_L\}$ is the diagonal eigenvalue matrix (the λ_k are real and assumed to be sorted in descending order). Then, the local TF projection filter matrix $\mathbf{H}_{\mathrm{P}}$ is given by (cf. (6))

$$\mathbf{H}_{\mathrm{P}} = [\mathbf{u}_1\ldots\mathbf{u}_R][\mathbf{u}_1\ldots\mathbf{u}_R]^H = \sum_{k=1}^{R}\mathbf{u}_k\mathbf{u}_k^H, \qquad (12)$$

where R, the rank of $\mathbf{H}_{\mathrm{P}}$, is the number of eigenvalues of $\mathbf{H}_{\mathrm{W}}$ larger than $1/2$. The impulse response of the local TF projection filter is obtained as $h_{\mathrm{P}}[2(l_0-L/2+i), 2(l_0-L/2+j)] = \big(\mathbf{H}_{\mathrm{P}}\big)_{i,j}$, with $i,j\in[0,L-1]$; it is finally used to calculate the *single* output signal sample at time $n_0=2l_0$, i.e. (cf. (8)),

$$y[2l_0] = \sum_{l'=l_0-L/2}^{l_0+L/2-1} h_{\mathrm{P}}[2l_0, 2l']\,\tilde{x}[2l']\,.$$

This procedure is repeated for all even time instants $n_0=2l_0$.

Alternatively, to reduce computations, we may calculate $\mathbf{H}_{\mathrm{P}}$ not for all even time points $2l_0$ but only for multiples $r\cdot 2D$ of some increment $2D$. The local output signal of $\mathbf{H}_{\mathrm{P}}$ then has to be calculated not only for $n_0=2rD$ but in a local interval about $n_0=2rD$. That is, we form the local output signal $y_r[2(rD-L/2+j)] = (\mathbf{y}_r)_j$, $j\in[0,L-1]$, with $\mathbf{y}_r$ given by

$$\mathbf{y}_r = \mathbf{H}_{\mathrm{P},r}\tilde{\mathbf{x}}_r\,. \qquad (13)$$

Here, the local length-L input signal vector $\tilde{\mathbf{x}}_r$ is given by $(\tilde{\mathbf{x}}_r)_j = \tilde{x}[2(rD-L/2+j)]$, $j\in[0,L-1]$, and $\mathbf{H}_{\mathrm{P},r}$ denotes the local TF projection filter matrix at $n_0=2rD$. The individual local output signals $y_r[2l]$ thus obtained are then combined into the final output signal $y[2l]$ using an overlap-add scheme, i.e.,

$$y[2l] = \sum_{r=-\infty}^{\infty} w[l-rD]\,y_r[2l]\,, \qquad (14)$$

where $w[l]$ is a window satisfying $\sum_{r=-\infty}^{\infty} w[l-rD]=1$. A simple choice is the triangular (Bartlett) window given by $w[l]=1-|l|/D$ for $|l|\le D$ and $w[l]=0$ otherwise. Here, only two subsequent local output signals overlap in (14).

For L sufficiently large and D sufficiently small, the results obtained with the local TF projection filter will typically be similar to those obtained with the true TF projection filter $\boldsymbol{H}_{\mathrm{P}}$. For smaller L and/or larger D, the approximation error will be larger but the computational complexity will be smaller.

4. ITERATIVE EIGENDECOMPOSITION

The filter design based on local TF projections requires an eigendecomposition of complexity $\mathscr{O}(L^3)$ every $2D$ time instants. This complexity can be reduced significantly. Indeed, the fact that we only need a certain number of *dominant* eigenvalues/eigenvectors of $\mathbf{H}_\mathrm{W}$ (all eigenvalues above $1/2$ and the corresponding eigenvectors) suggests to use an iterative algorithm known as *orthogonal iteration* or *simultaneous vector iteration*. This algorithm calculates an arbitrary number p of dominant eigenvalue/eigenvector pairs by means of iterated matrix multiplications and reorthogonalizations [14].

Orthogonal Iteration. The ith orthogonal iteration ($i \geq 1$) consists of the following two steps:

1.) *Iteration step*: Calculate the $L \times p$ matrix

$$\mathbf{X}^{(i)} = \mathbf{H}_\mathrm{W}\mathbf{U}^{(i-1)}, \qquad (15)$$

where the $L \times p$ matrix $\mathbf{U}^{(i-1)}$ was obtained at the previous iteration. The columns of $\mathbf{U}^{(i-1)}$ are the p dominant eigenvector estimates $\mathbf{u}_k^{(i-1)}$.

2.) *Orthogonalization step*: Calculate the current eigenvector matrix $\mathbf{U}^{(i)}$ via QR factorization [14] of $\mathbf{X}^{(i)}$,

$$\mathbf{U}^{(i)}\mathbf{R}^{(i)} = \mathbf{X}^{(i)}. \qquad (16)$$

(The $p \times p$ triangular matrix $\mathbf{R}^{(i)}$ is not itself required.)

The choice of the initialization $\mathbf{U}^{(0)}$ will be discussed below. For $i \to \infty$, the $\mathbf{u}_k^{(i)}$ converge to the p dominant eigenvectors of $\mathbf{H}_\mathrm{W}$. Estimates of the p dominant eigenvalues can be obtained as

$$\lambda_k^{(i)} = \mathbf{u}_k^{(i)H}\mathbf{H}_\mathrm{W}\mathbf{u}_k^{(i)}, \qquad k \in [1,p]. \qquad (17)$$

These eigenvalue estimates converge with rate $\left|\lambda_{p+1}/\lambda_p\right|^i$ [14]. The local TF projection filter at the ith iteration then is (cf. (12))

$$\mathbf{H}_\mathrm{P}^{(i)} = \left[\mathbf{u}_1^{(i)} \cdots \mathbf{u}_R^{(i)}\right]\left[\mathbf{u}_1^{(i)} \cdots \mathbf{u}_R^{(i)}\right]^H = \sum_{k=1}^{R^{(i)}} \mathbf{u}_k^{(i)}\mathbf{u}_k^{(i)H}, \qquad (18)$$

where $R^{(i)}$, the rank of $\mathbf{H}_\mathrm{P}^{(i)}$, is given by the number of eigenvalue estimates $\lambda_k^{(i)}$ larger than $1/2$. The iteration is terminated when either the Frobenius norm $\|\mathbf{H}_\mathrm{P}^{(i)} - \mathbf{H}_\mathrm{P}^{(i-1)}\|$ falls below a small threshold or a certain number of iterations is reached.

The number p of dominant eigenvalues/eigenvectors to be used in the orthogonal iteration can be determined as follows. According to [11], the rank R of a TF projection filter approximately equals the area of the corresponding TF pass region $\mathscr{R}$ (cf. (4)). For the local TF projection filter, this area is determined by $M[n,k]$ within the local interval $[n_0 - L, n_0 + L - 1]$, i.e.,

$$R \approx A = \sum_{n=n_0-L}^{n_0+L-1} \sum_{k=0}^{K-1} M[n,k]$$

(recall that $M[n,k] = 1$ on $\mathscr{R}$ and $M[n,k] = 0$ else). This can alternatively be calculated as the trace of $\mathbf{H}_\mathrm{W}$, i.e., $A = \mathrm{tr}\{\mathbf{H}_\mathrm{W}\}$. Since $R \approx A$, we propose to iterate $p = A + s$ eigenvectors, with $s = 2 \ldots 4$ being a "subspace reserve" (see below). Note that typically the TF pass region has small area A and thus $p \ll L$.

Ritz Acceleration. The convergence rate of the orthogonal iteration algorithm can be significantly improved by means of *Ritz acceleration* [14]. This involves the following additional steps at the ith orthogonal iteration (see above):

3.) *Schur decomposition*: Form the $p \times p$ auxiliary matrix

$$\mathbf{S}^{(i)} = \mathbf{U}^{(i)H}\mathbf{H}_\mathrm{W}\mathbf{U}^{(i)}. \qquad (19)$$

Next, compute the Schur decomposition [14] of $\mathbf{S}^{(i)}$, i.e., a unitary matrix $\mathbf{Q}^{(i)}$ and a diagonal matrix $\mathbf{L}^{(i)} = \mathrm{diag}\{\tilde{\lambda}_1^{(i)}, \ldots, \tilde{\lambda}_p^{(i)}\}$ such that

$$\mathbf{Q}^{(i)}\mathbf{L}^{(i)}\mathbf{Q}^{(i)H} = \mathbf{S}^{(i)}. \qquad (20)$$

The $\tilde{\lambda}_k^{(i)}$ are used as improved eigenvalue estimates that replace the $\lambda_k^{(i)}$ in (17) which, therefore, need not be calculated.

4.) *Eigenvector rotation*: Compute the improved eigenvector matrix

$$\widetilde{\mathbf{U}}^{(i)} = \mathbf{U}^{(i)}\mathbf{Q}^{(i)}. \qquad (21)$$

The rotated eigenvectors $\tilde{\mathbf{u}}_k^{(i)}$ (the columns of $\widetilde{\mathbf{U}}^{(i)}$) are then used for constructing an estimate of the local TF projection filter according to (18). Furthermore, the next iteration step (15) uses $\widetilde{\mathbf{U}}^{(i-1)}$ instead of $\mathbf{U}^{(i-1)}$. The estimate $\tilde{\lambda}_k^{(i)}$ of the kth eigenvalue can be shown to converge with rate $\left|\lambda_{p+1}/\lambda_k\right|^i$, which typically is much faster than the convergence obtained without Ritz acceleration, $\left|\lambda_{p+1}/\lambda_p\right|^i$, especially for the dominant eigenvalues that we are interested in. Thus, using Ritz acceleration, fewer iterations are required to determine the local TF projection filter matrix with a specified accuracy. Note that a larger subspace reserve s results in a smaller λ_p and hence tends to yield faster convergence. However, choosing a larger s increases p and thus the computational load of each iteration step (see Section 5).

TF Subspace Tracking. The orthogonal iteration requires some initialization $\mathbf{U}^{(0)}$ of the iterated eigenvector matrix $\mathbf{U}^{(i)}$. At time instant $n = 2rD$, we propose to use for $\mathbf{U}^{(0)}$ the eigenvector matrix calculated at the previous time instant $2(r-1)D$. If the TF weight function $M[n,k]$ significantly changes between successive time instants $2(r-1)D$ and $2rD$, it is usually advantageous to shift the eigenvector estimates by the time increment D in order to account for the new time position. Thus, for the first $L - D$ rows of $\mathbf{U}^{(0)}$ at $2rD$ we use the last $L - D$ rows of the eigenvector matrix calculated at $2(r-1)D$, and for the remaining D rows we use suitably normalized random values. With this initialization, the Ritz-accelerated orthogonal iteration tends to converge after a small number of iterations (typically about $2 \ldots 3$). This scheme of updating the eigenvector estimates as we pass from one time instant to the next corresponds to a *tracking* of the TF subspace underlying the local TF projection filter (recall that this TF subspace is determined by the eigenvectors with associated eigenvalues $> 1/2$).

5. COMPUTATIONAL COMPLEXITY

We next discuss the computation and memory requirements of the proposed online TF projection filter.

At each time instant, we need $\mathscr{O}\big(\frac{K}{2}\log(\frac{K}{2})\big)$ operations to compute the impulse response $g_\mathrm{W}[n,m]$ according to (11). Furthermore, every $2D$ time instants, we need a Ritz-accelerated orthogonal iteration to determine the local TF projection filter matrix $\mathbf{H}_\mathrm{P}$.

The complexity of the tasks performed per iteration step of the Ritz-accelerated orthogonal iteration is summarized in Table 1 (cf. [14]). Since typically $p \ll L$, the overall complexity will be dominated by the tasks with $\mathscr{O}(L^2 p)$. For a given time instant, the orthogonal iteration requires $\mathscr{O}(IL^2 p)$ operations (here, I denotes the average number of iteration steps); this has to be compared with the complexity of a complete eigendecomposition, $\mathscr{O}(L^3)$. Note that usually $Ip \ll L$ and thus $IL^2 p \ll L^3$ (typical values are e.g. $I = 2 \ldots 3$, $p = 5 \ldots 7$, and $L = 64$).

Finally, the computation of each output signal sample according to the overlap-add scheme (13), (14) requires $\mathscr{O}(L)$ operations.

The memory requirements can be estimated as follows. The local TF projection filter requires storage of the $L \times L$ matrices $\mathbf{H}_\mathrm{W}$

Task	Complexity
matrix multiplication (15)	$\mathcal{O}(L^2 p)$
QR decomposition (16)	$\mathcal{O}(Lp^2)$
computation of $\mathbf{S}^{(i)}$ in (19)	$\mathcal{O}(L^2 p + Lp^2)$
Schur decomposition (20)	$\mathcal{O}(p^3)$
eigenvector rotation (21)	$\mathcal{O}(Lp^2)$
computation of $\mathbf{H}_{\mathrm{P}}^{(i)}$ in (18)	$\mathcal{O}(L^2 p)$
computation of $\|\mathbf{H}_{\mathrm{P}}^{(i)} - \mathbf{H}_{\mathrm{P}}^{(i-1)}\|$	$\mathcal{O}(L^2)$

Table 1: *Number of operations required per iteration step of the Ritz-accelerated orthogonal iteration algorithm.*

and $\mathbf{H}_{\mathrm{P}}^{(i)}$, the $L \times p$ matrix $\mathbf{U}^{(i)}$, the $p \times p$ matrices $\mathbf{Q}^{(i)}$ and $\mathbf{L}^{(i)}$, and a length-L segment of the input signal. This amounts to a total of $2(L^2 + p^2) + L(p+1)$ memory locations.

6. APPLICATION TO INTERFERENCE EXCISION

We finally apply our online TF projection filter to the problem of interference excision in spread-spectrum communications [15]. The received signal is modeled as

$$x[n] = s[n] + q[n] + w[n]\,,$$

where $s[n] = \sum_k b_k c[n - kN_c]$ is the transmit signal ($b_k \in \{-1, 1\}$ are the transmitted bits and $c[n]$ is the length-N_c spreading sequence), $q[n]$ is a jammer (interference) signal, and $w[n]$ is white noise.

In order to suppress the jammer signal, we propose an interference excision scheme that extends the method in [16]. Whereas in [16] separate rank-one TF projection filters for the individual (length-N_c) symbol periods were used, we use an *online* TF projection filter for the *entire* received signal and do not impose any rank constraints. The output signal of the online TF projection filter yields an estimate $\hat{q}[n] = (\boldsymbol{H}_{\mathrm{P}} x)[n]$ of the jammer signal $q[n]$; this estimate is subtracted from the received signal. The resulting signal $x[n] - \hat{q}[n] = s[n] + (q[n] - \hat{q}[n]) + w[n]$ is then passed to the spread-spectrum demodulator.

We simulated a spread-spectrum system with $N_c = 63$ and SNR $E_b/N_0 = 14.5\,\mathrm{dB}$. The signal length was 16128 samples. Two different FM jammer signals with slowly varying linear instantaneous frequency (IF) and fast varying sinusoidal IF were considered. The online TF projection filter was implemented using parameters $K = 32$, $D = 16$, $L = 48$, and $s = 2$. The TF weight function $M[n,k]$ was designed in a signal-adaptive fashion by thresholding a smoothed pseudo-Wigner distribution [17] of the received signal $x[n]$. This method has the advantage that no prior knowledge about the number and TF support regions of the jammers is required; furthermore, no parametric model of the jammers is assumed.

Fig. 1 shows the (uncoded) bit error rate (BER) vs. the signal-to-interference ratio (SIR) for the slow linear-IF jammer and the fast sinusoidal-IF jammer. We compared the cases of interference excision using our online TF projection filter, interference excision using a short-time Fourier transform (STFT) filter [4,5,18], and no interference excision. It is seen that for both jammers, the online TF projection filter yields a significantly lower BER than the STFT filter. For example, for a BER of 1%, the online TF projection filter has an SIR advantage of about 5 dB over the STFT filter. This is because the online TF projection filter achieves a much sharper TF cutoff than the STFT filter that is known to have reduced TF resolution [4,5]. The local TF projection filter matrices $\mathbf{H}_{\mathrm{P}}$ had low rank $R = 3 \ldots 6$. Only $2 \ldots 3$ Ritz-accelerated orthogonal iterations were required at each time point.

7. CONCLUSIONS

We presented an online time-frequency (TF) projection filter that uses local TF projections and TF subspace tracking based on efficient iterative eigendecomposition techniques. The novel filter overcomes the problems (i.e., computational complexity, batch processing, restricted signal length) that so far restricted practical application of the TF projection filter. Application to interference excision in spread-spectrum communications demonstrated the advantages to be gained from the filter's unrivaled TF selectivity.

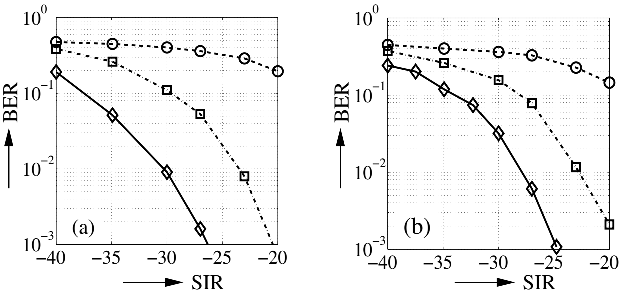

Figure 1: *BER vs. SIR obtained with TF projection filter (◇—◇), STFT filter (□··□), and direct demodulation (○- - -○) for jammers with (a) slowly varying linear and (b) fast varying sinusoidal IF.*

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