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Minimum Length of a Single-Mode Fiber Spatial Filter

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Using the concept of leaky modes, we derive the minimum length of a single-mode fiber required to act as a spatial mode filter of given quality. The degree of filter action is defined by the ratio of power carried by the fundamental mode to that carried by the leaky modes.

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1 Spatial filtering

The European Space Agency's *DARWIN*[1] mission and NASA's *TPF*[2] project are both space-based interferometers dedicated to the search and analysis of Earth-like extrasolar planets in the mid-infrared spectrum (4 to 20 microns). The high flux ratio requires a starlight rejection in the order of 10^6 to make visible the weak planetary signal. The excessive instrumental requirements on such nulling interferometers can be dramatically relaxed by spatially filtering[3] the combined wavefronts. This is illustrated by the following example of a two-arm nulling interferometer.[4] If no spatial filter is present, input wavefront tilts (telescope pointing errors) with rms fluctuations of $9 \cdot 10^{-4}\lambda/D$ or wavefront perturbations with rms fluctuations of $\lambda/4400$ can be tolerated to achieve a rejection ratio of 10^6. (Here λ stands for the wavelength and D for the telescope diameter.) These requirements are relaxed to $6.4 \cdot 10^{-2}\lambda/D$ and $\lambda/63$, respectively, when spatial filtering with a single-mode fiber.

Wavefront filters may be realized either by a pinhole or by a single-mode waveguide.[3–7] Pinholes may eliminate high spatial frequency wavefront defects of a focussed wave, but have the drawback of being highly chromatic, as for increasing wavelength the filter's spatial bandwidth reduces and the insertion loss increases. Single-mode waveguides, and in particular single-mode fibers, act as modal filters. As only the waveguide's fundamental mode propagates with negligible loss, an input wave with a perturbed wavefront is filtered at the expense of optical throughput. Single-mode waveguides can correct wavefront defects of any order and provide filter action over a wide bandwidth.

To achieve a specified degree of filter action with a modal filter, a certain minimum waveguide length, z_0, is required, at which a spatial steady state[8] is reached. For two reasons one should aim to make a fiber dedicated to spatial filtering as short as possible. Firstly, materials transmitting in the mid-infrared are absorbing, thus increasing the filter's insertion loss. Secondly, the longer the fiber the larger the possibility to induce unwanted effects during the fabrication process (e.g. scattering centers) as well as during operation (e.g. bending).

For the length z_0, values ranging from some 10^2 to some 10^5 wavelengths have been cited in the past.[5,9–11] In this paper we will derive an upper bound for z_0 for the case of a non-absorbing single-mode fiber by calculating the leaky mode attenuation.[12]

2 Filter action

Any optical field within a dielectric waveguide can be completely described by a finite number of discrete guided modes and a continuum of radiation modes. The latter are required to provide a complete orthogonal set of modes.[13] In the vicinity of the fiber core the radiation field can be approximated by strongly attenuated leaky modes.[14]

To arrive at a basic analytical result for the minimum fiber length z_0, we assume a step-index profile with a low refractive index difference between the core and the infinitely extended cladding. We also assume the fiber to be straight and to have no refractive index inhomogeneities.

We model the radiation propagating along the fiber to consist of guided modes with real propagation constants β_{guided} and of leaky modes with complex propagation constants $\beta_{leaky} = \beta^r_{leaky} + j\beta^i_{leaky}$, where $\beta^i_{leaky} > 0$, and the superscripts indicate the real and the imaginary part. The leaky modes may be regarded as guided modes below their cutoff. If $k = 2\pi/\lambda$ denotes the free-space wavenumber and n_{co} and n_{cl} the refractive index of core and cladding material, respectively, the values for the propagation constants of the guided and the leaky modes obey the condition[13]

$$\overbrace{n_{co}k \geq \beta_{guided} > n_{cl}k}^{\text{guided modes}} \underbrace{> \beta^{r}_{leaky} \geq 0}_{\text{leaky modes}} \quad . \tag{1}$$

To obtain the propagation constant β for both types of modes, the respective eigenvalue equation[14] taking into account the fiber geometry's boundary conditions is solved numerically for the case of weak guidance,[15] i.e. for $\Delta = (n_{co}^2 - n_{cl}^2)/(2n_{co}^2) \ll 1$. This equation has the same functional form, both for guided and for leaky modes, and the electric and magnetic fields within the fiber are almost purely transverse. Multiple degenerate solutions with identical propagation constant are combined to result in linearly polarized LP_{lm} modes. The result[12] is presented in Fig. 1, showing the core parameter $U = U^r + j\,U^i = \rho\sqrt{n_{co}^2 k^2 - \beta^2}$ as a function of the normalized frequency $V = k\rho n_{co}\sqrt{2\Delta}$, where ρ is the core radius. The broken lines give the (real) core parameter of the guided modes, U_{guided}, including the fundamental mode LP_{01}. The solid lines show the real part, U^r_{leaky}, and the imaginary part, U^i_{leaky}, of the leaky modes. The figure includes some representative modes with a cutoff below $V = 10$ (including the ones with lowest attenuation).

The fields of the leaky modes are characterized by a distinct change in their physical behavior for increasing distance r from the fiber axis. In the vicinity of the fiber axis they appear like bound modes, i.e. they are evanescent and therefore the fields decrease with increasing r. At larger distances from the axis, the fields are unbound and power is radiated.

The validity of the idealized fiber modelling given above will, of course, depend on the process and technology used to manufacture a fiber. Poly-crystalline silver halide seems to be a promising material for developing a single-mode fiber with good transmission in the mid-infrared. Typical diameters for core and cladding are $20\mu m$ and some $100\mu m$ in case of a relative refractive index difference of $\Delta = 0.25\%$. The large cladding diameter and a possible coating of the cladding surface with an absorbing layer of black painting should minimize reflections at the outer cladding boundary.

3 Minimum fiber length

Each leaky mode's power attenuation in axial direction z, described by

$$P_{LM}(z) = P_{LM}(0)\exp(-\alpha z) \tag{2}$$

with the attenuation coefficient $\alpha = 2\beta^i_{leaky}$, is determined by the imaginary part of its propagation constant $\beta_{leaky} = \rho^{-1}\sqrt{V^2/(2\triangle) - U^2}$. Figure 2 gives the normalized attenuation coefficients $\alpha\rho$ of the leaky modes of Fig. 1 for a relative refractive index difference of $\Delta = 0.25\%$. It is apparent that the leaky mode attenuation can be very low, especially for the leaky LP_{11} mode at a wavelength slightly above the single-mode cutoff ($V = 2.405$), as shown in the inset.

If an arbitrary field is incident onto the fiber input face, guided modes as well as leaky modes will be excited, depending on the input field distribution and the normalized frequency V. To achieve a low insertion loss, one will strive for coupling an as large as possible portion of the incident power P_{in} into the fundamental mode (power $P_{LP_{01}}$) by employing proper focussing optics. Under the assumption of negligible absorption, the fundamental mode will propagate without attenuation. The portion of the incident power which is not coupled to the fundamental mode, $P_{LM}(0)$, is distributed among the leaky modes. The leaky modes suffer attenuation due to off-axis radiation and, for large z, will exponentially lose all their power to the cladding, $\lim_{z\to\infty} P_{LM}(z) = 0$, as illustrated in Fig. 3.The leaky modes are radiated off the fiber axis at an angle given roughly by[14] $\Theta_z \approx \arccos(\beta^r_{leaky}/(kn_{cl}))$. Especially for short pieces of fiber, this angle has to be observed when measuring the guided portion of the leaky mode power. The detector's active area has to be chosen accordingly to guarantee that no radiating power is measured.

The effectiveness of a single-mode fiber spatial filter is best defined as the degree of suppressing all modes but the fundamental mode LP_{01}. The quality of filtering is determined by the insertion loss of the fundamental mode (i.e. the coupling efficiency) and the leaky mode attenuation for a given fiber length z_0.

Coupling efficiency

The fundamental mode coupling efficiency η solely determines the filter's insertion loss (as long we neglect any absorption and Fresnel reflection). It is defined[16,17] as the ratio of the power coupled into the fiber's fundamental mode to the power available at the fiber input,

$$\eta = \frac{\left|\iint E(x,y)\, F^*(x,y)\; dxdy\right|^2}{\iint |E(x,y)|^2\; dxdy} \quad , \tag{3}$$

where (x, y) are the transversal coordinates, and E and F^* denote the input field and the complex conjugate of the fundamental mode's field, respectively. The coupling efficiency η depends on the wavelength λ. However, for fixed coupling optics and given fiber, η shows only a weak dependence on λ in the vicinity of the wavelength for which η is optimum. (For a plane input field, η is roughly achromatic within one octave above the wavelength for which the coupling system is designed.)

Attenuation factor

We quantify the filter action by means of the attenuation factor A, defined as the ratio of the fundamental mode power to the power in the leaky modes,

$$A = P_{LP_{01}}/P_{LM}(z_0) \quad . \tag{4}$$

To make a worst-case estimate of the minimum fiber length required, we assume that all power not exciting the fundamental mode is coupled to that leaky mode which suffers lowest attenuation. According to Fig. 2, this is the LP_{11} mode. The minimum normalized filter length, z_0/ρ, follows from Eq. 2 and 4 as

$$\frac{z_0}{\rho} = \frac{1}{\alpha\rho} \ln \frac{A(1-\eta)}{\eta} \quad . \tag{5}$$

Of course, the value of A has always to be larger than that determined by the coupling efficiency, i.e. $A > \eta/(1-\eta)$.

4 Examples

As an example we assume a plane input wave and analyze a spatial filter consisting of a coupling lens and a single-mode fiber. The filter is optimized for maximum coupling at the fiber's cutoff wavelength ($V_c = 2.405$) and the coupling efficiency's wavelength dependence is taken into account.

Figure 4(a) shows the minimum fiber length in multiples of the core radius, z_0/ρ, as a function of the normalized frequency V for a relative refractive index difference of $\Delta = 0.25\%$ and an attenuation factor of $A = 10^6$. The attenuation factor corresponds to actual requirements for the interferometers DARWIN and TPF. The values $\alpha\rho$ for the LP_{11} mode are taken from Fig. 2. The solid line shows the case of maximum coupling, i.e. $\eta = 0.78$ for $V = V_c$. In addition, we present the case of poor coupling, i.e. $\eta = 0.1$ for $V = V_c$ (broken line). Apparently, the dependence of the required minimum filter length on coupling efficiency, i.e. insertion loss, is not severe, at least for $V \leq 2$. Figure 4(b) gives z_0/ρ as a function of A for $V = 1.5$, 2 and 2.2, where $\Delta = 0.25\%$ and $\eta(V_c) = 0.78$ is assumed.

The curves shown in Fig. 4 can be applied to all fibers with $\Delta = 0.25\%$, independent of wavelength and core radius. The minimum filter lengths for a typical silver halide fiber with $\Delta = 0.25\%$, $n_{co} = 2.18$, and a core diameter of $19\mu m$, designed to be single-mode above $4\mu m$, are likewise included in Fig. 4(a). Slightly above single-mode cutoff, e.g. at $\lambda = 4.2\mu m$, the minimum filter lengths amount to $z_0 = 1.9\,cm$ and $2.4\,cm$ for high ($\eta(V_c) = 0.78$) and poor ($\eta(V_c) = 0.1$) coupling, respectively. For larger wavelengths the leaky mode attenuation increases dramatically (see Fig. 2). This significantly reduces the minimum filter length, although much more power is coupled to the leaky modes. Because the fiber's fundamental mode extends deeply into the cladding for small values of V, unwanted reflections at the outer cladding boundary might degrade the idealized filter action.

5 Conclusion

Our analysis demonstrates that fiber lengths of only a few wavelengths are not sufficient to provide good spatial filtering. A single-mode fiber spatial filter for mid-infrared applications with a length of about $2cm$, corresponding to some thousand wavelengths, will provide appropriate filter action.

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Fig. 1. Core parameter U as a function of the normalized frequency $V = k\rho n_{co}\sqrt{2\Delta}$ for some guided (broken lines, real-valued U_{guided}) and leaky modes (solid lines, $U_{leaky} = U^r_{leaky} + jU^i_{leaky}$).

Fig. 2. Normalized attenuation coefficient $\alpha\rho$ of the six leaky modes of Fig. 1 with lowest attenuation as a function of the normalized frequency V in the single-mode regime for a relative refractive index difference of $\Delta = 0.25\%$. The inset shows an enlargement of the V range where the LP_{11} mode experiences very low attenuation.

Fig. 3. In a single-mode fiber, the incident power (P_{in}) is distributed among the fiber's fundamental mode (power $P_{LP_{01}}$) and the leaky modes (power P_{LM}). In the non-absorbing fiber the fundamental mode will propagate without attenuation, while the leaky modes are exponentially attenuated at a distance z_0.

Fig. 4. (a) Required minimum filter length z_0, given in multiples of the fiber core radius ρ, as a function of the normalized frequency V for an attenuation coefficient $A = 10^6$ and a relative refractive index difference $\Delta = 0.25\%$. The solid line shows the case of minimum insertion loss, i.e. maximum coupling with $\eta = 0.78$ for $V = V_c = 2.405$, the broken line gives the case of high insertion loss, i.e. poor coupling with $\eta = 0.1$ for $V = V_c$. A second pair of coordinate axes (at the top and at the right) is scaled in wavelengths λ and absolute values for z_0, respectively. Here, parameters of a silver halide mid-infrared fiber are assumed (core diameter: $19\mu m$, core refractive index: $n_{co} = 2.18$, relative refractive index difference: $\Delta = 0.25\%$). (b) Ratio z_0/ρ as a function of the attenuation coefficient A for $V = 1.5$, 2 and 2.2 with $\Delta = 0.25\%$ and $\eta(V_c) = 0.78$.

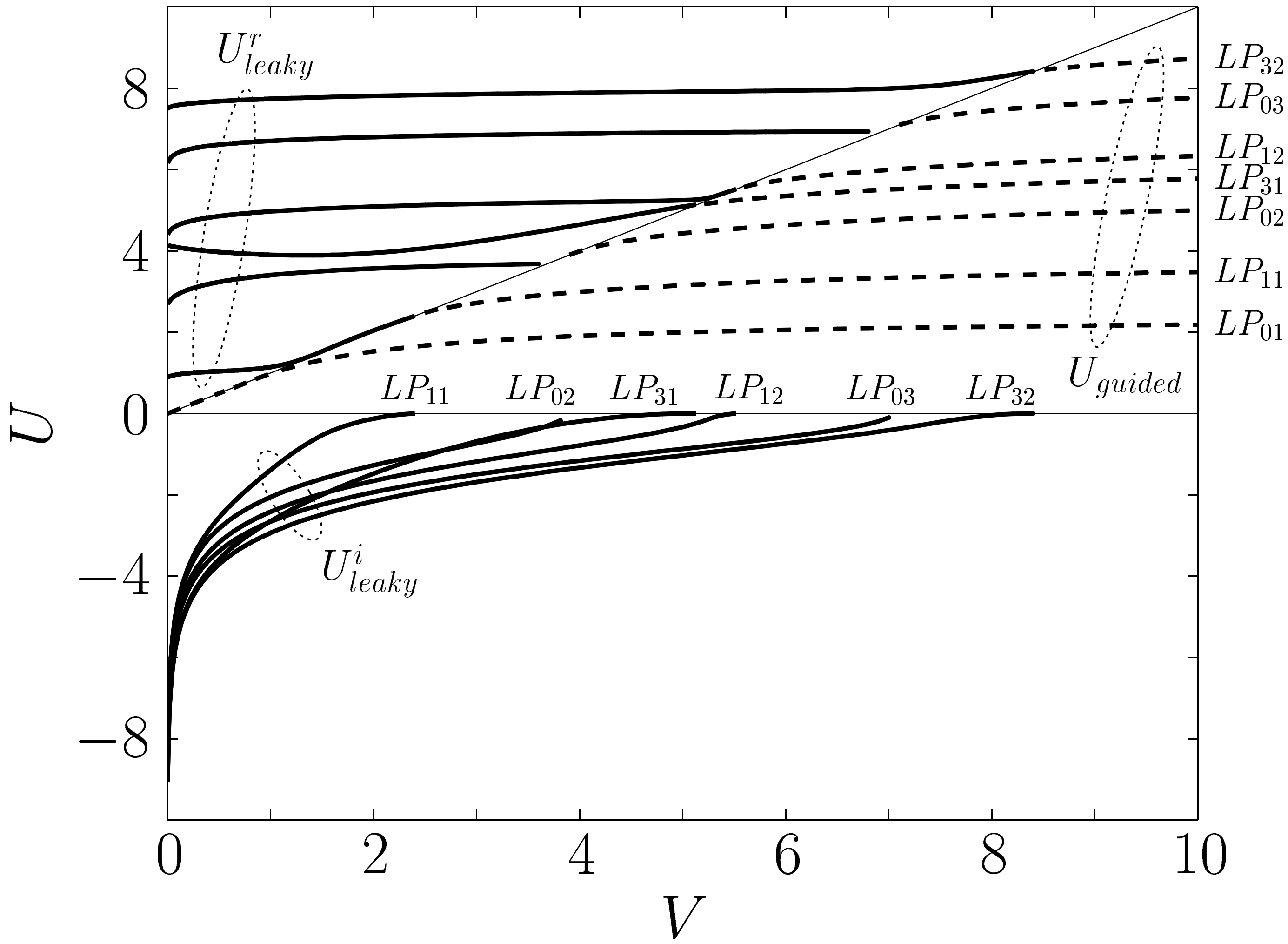

Figure 1, O. Wallner, JOSA A

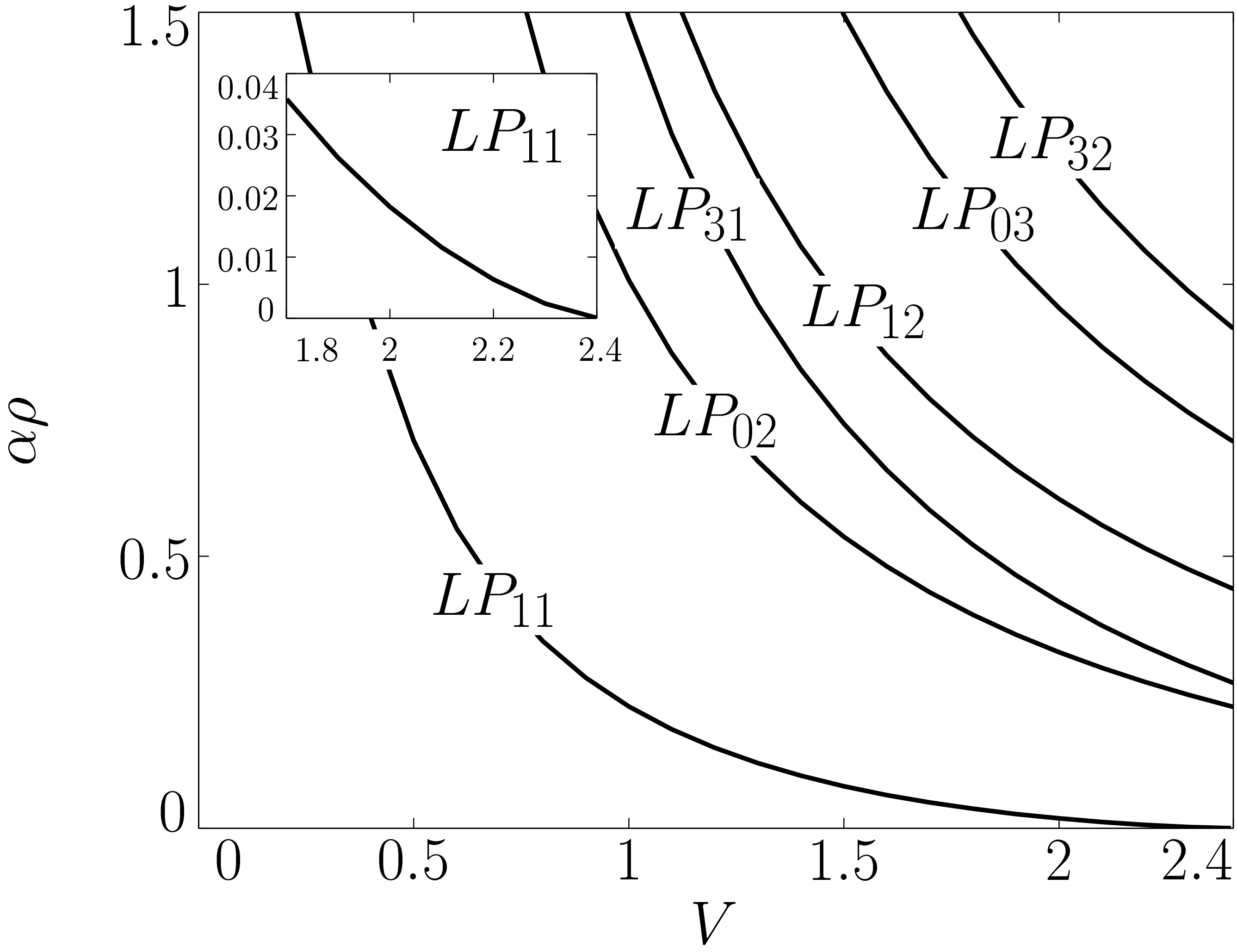

Figure 2, O. Wallner, JOSA A

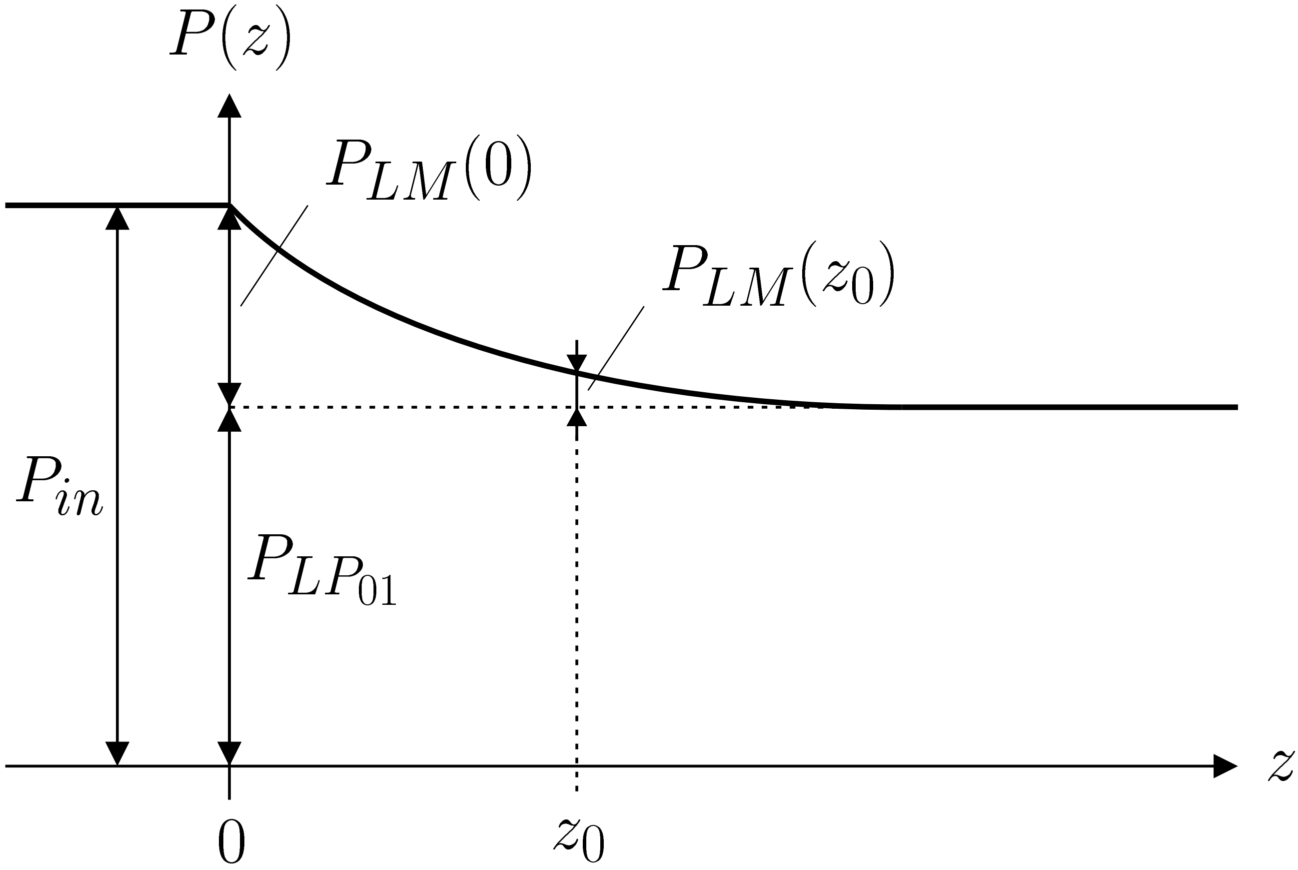

Figure 3, O. Wallner, JOSA A

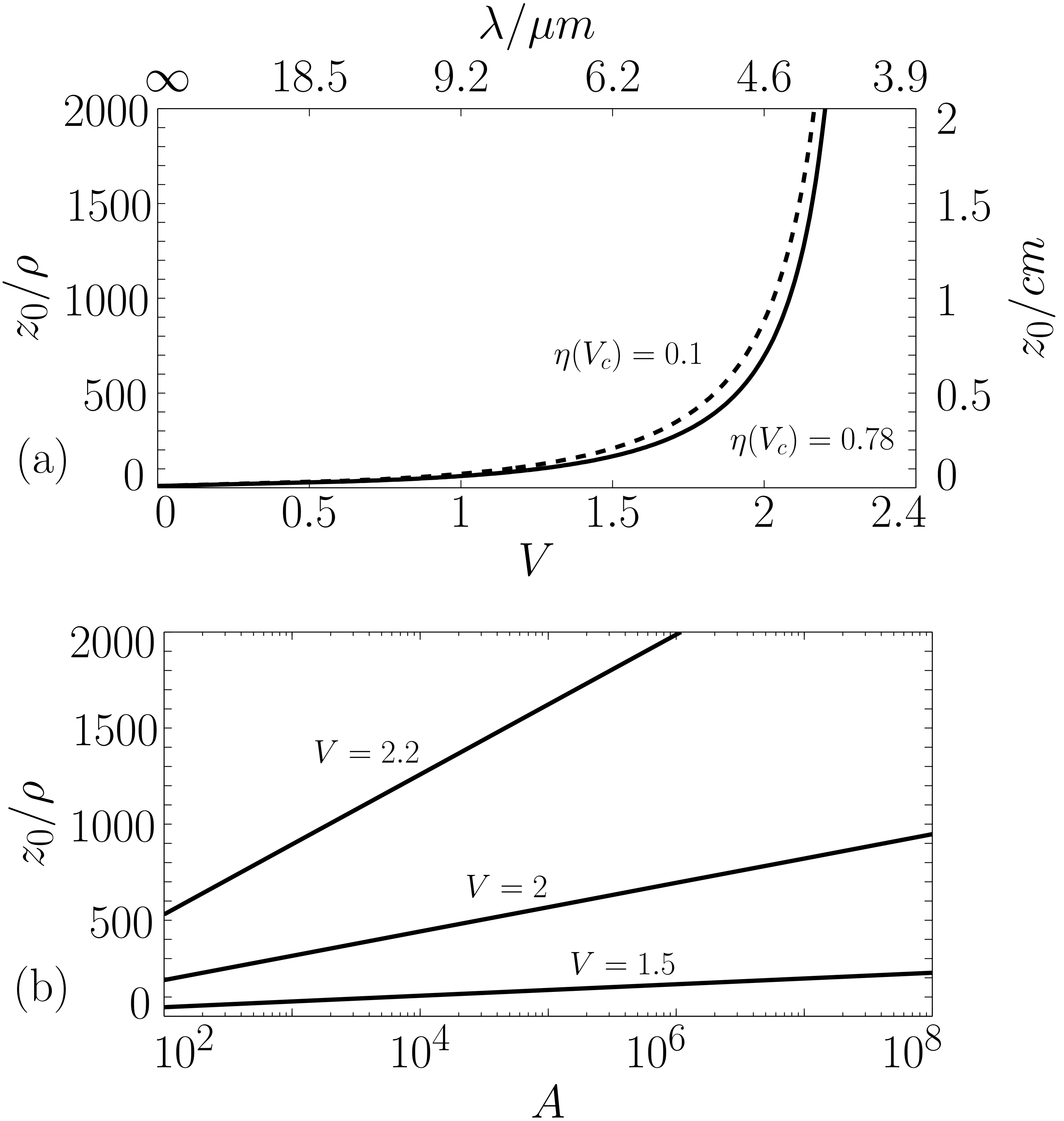

Figure 4, O. Wallner, JOSA A