Optimum Power Allocation for Transmit Diversity in Mobile Communications

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Abstract—**Spatial transmit diversity systems in mobile communications utilize either different antennas, polarizations, or beams. Whereas separated antennas carry the same power on average, the average channel gain might vary for different polarizations or beams. Thus, the optimum trade-off between channel gain and diversity gain has to be found: Which allocation of the limited transmit power over the available transmit diversity branches is optimum? As optimization criterion we apply the outage capacity for a given outage probability level. By means of numerical optimization techniques, we derive the optimum power allocation for frequency flat Rayleigh fading. The numerical results show that diversity branches with higher channel gain are always allocated more power, i.e. a poor channel gain is not compensated by the transmit power. Furthermore, the performance of an even power allocation over the optimum set of transmit diversity branches is very near to the optimum power allocation.**

Index terms — transmit diversity, polarization diversity, spatial diversity, antenna arrays, outage capacity, power allocation

I Introduction

Transmit diversity techniques have been utilized in mobile communication systems for many years in order to mitigate the detrimental effects of fading caused by multipath propagation [1]. Whereas conventional diversity techniques apply two spatially separated antennas, more innovative approaches make use of orthogonal polarizations or orthogonal beams in the angular domain. Beams can be formed by closely spaced antenna arrays, so called adaptive antennas or smart antennas. The separation of the diversity branches in space, polarization or angle has a common purpose: The signals of different diversity branches should be as uncorrelated as possible in order to maximize the achieved diversity.

Beside this common property, polarization and beam diversity systems show an important difference to spatially separated antennas. In general, the channel gain might be different for orthogonal polarizations or beams. On one hand, outdoor mobile radio channels are known to prefer the vertical polarization [2]. Thus, transmitting on a vertically polarized antenna results in a significantly larger channel gain than transmitting with horizontal polarization. An important advantage of polarization diversity compared to antenna diversity is the possibility to incorporate both polarizations in one antenna only, leading to a significantly reduced size of the antenna system.

On the other hand, beamforming by means of an antenna array is a well known technique for increasing the amount of power that is transmitted via the radio link [3]. This increase is called beamforming gain. There exists a variety of methods for creating orthogonal transmit beams, among them fixed beams and eigenbeams. The former approach utilizes a fixed predefined set of antenna weights, which form as many orthogonal beams as antenna elements. The latter technique performs an eigen-decomposition of the spatial signal covariance matrix, and utilizes the eigenvectors as transmit weights. The beamforming gain is different for each beam and depends on the spatial properties of the mobile radio channel. The most important benefit of smart antenna systems is their ability to utilize beamforming gain and diversity.

Allocating the entire transmit power to the strongest diversity branch results in a high channel gain. On the other hand, if the amount of allocated transmit power is proportional to the inverse of the respective diversity branch gain, the amount of diversity is maximized. Which is more important? Where is the optimum trade-off between channel gain and diversity? The following sections will give an answer to these questions.

II System Model

A Signal Model

We assume that M diversity branches are available at the base station. These M branches can be different antennas, polarizations or beams, and are the same for up- and downlink. The signal propagates via several multipath components from the mobile terminal to the base station. The resulting fading of the channel coefficients is assumed to be complex Gaussian distributed, i.e. Rayleigh fading. For reasons of simplicity, the coherence bandwith of the channel is assumed to be larger than the signal bandwidth. The receive signal x_m at diversity branch m can therefore be written as

$$x_m[t] = h_m^{\mathrm{UL}}[t]\, s[t]\,, \qquad (1)$$

where t and s denote the discrete time instant and the transmitted symbol, respectively. The uplink channel coefficients of the m-th branch are denoted by h_m^{UL}. By arranging all the received signals and the channel coefficients of all diversity branches in vectors, we obtain a vector signal model

$$\boldsymbol{x}[t] = \boldsymbol{h}_{\text{UL}}[t]\, s[t]\,. \tag{2}$$

The total transmit energy P is allocated over the diversity branches for transmit diversity. The portion of transmit power allocated to diversity branch m is denoted by β_m,

$$\sum_{m=1}^{M} \beta_m = P\,. \tag{3}$$

The vector $\boldsymbol{\beta}$ is defined as $\boldsymbol{\beta} = [\beta_1\ \beta_2\ \cdots\ \beta_M]^T$.

B Second Order Statistics of the Channel

The uplink channel coefficients are estimated in the base station receiver by means of training symbols. We can estimate the second order statistics of the uplink channel coefficients, i.e. the spatial covariance matrix $\boldsymbol{R}_{\text{UL}}$, by averaging over a time interval T:

$$\boldsymbol{R}_{\text{UL}} = \sum_{t=t_0}^{t_0+T-1} \boldsymbol{h}_{\text{UL}}[t]\, \boldsymbol{h}_{\text{UL}}^H[t]\,. \tag{4}$$

Since the approximation of a statistical expectation operator by means of a sum over time samples implicitly assumes ergodicity, the averaging interval T has to fulfill two conditions: It has to be long enough that the samples within the interval completely describe the underlying statistics; and it has to be short enough that the statistical properties, i.e. the spatial channel properties, do not change significantly during the averaging. The second order statistics of the channel are long-term properties in the sense of [4].

Because the duplex distance in FDD mobile communication systems is rather small compared to their carrier frequency, the statistical properties of the channel coefficients are assumed to be the same in up- and downlink. Obviously, for TDD systems this assumption is also fulfilled. Thus, we can directly approximate the downlink covariance matrix to be the same as in the uplink

$$\boldsymbol{R}_{\text{DL}} \approx \boldsymbol{R}_{\text{UL}}\,. \tag{5}$$

The elements on the main diagonal show the average channel gain of the diversity branches and will be denoted by $\mu_1, \mu_2, \ldots, \mu_M$ in the following.

C Transmit Diversity Techniques

There exists a great variety of transmit diversity techniques. Practical examples are delay transmit diversity [5] and space time codes [6, 7]. The former approach transmits a delayed replica of the transmit signal on another diversity branch. This way an artificial delay spread is introduced. The receiver in the mobile terminal can collect the energy from the different diversity branches by conventional temporal processing. The latter technique, space time codes, spreads the transmit symbols over the spatial and temporal domain in such a way, that it can be reassembled at the receiver. The most prominent and simple example of a space time code is the Alamouti scheme [8], which works only on two transmit branches.

Assuming idealized conditions, the two examples above add up the energy of the diversity branches at the receiver. This is also true for most other transmit diversity techniques. In the following, we will therefore presume that the total signal energy at the receiver is the sum of all diversity branches.

D Performance Criterion

Most of the current mobile communication systems support fixed modulation formats and data rates only. For a fixed modulation format, the received SNR (signal to noise ratio) must not fall below a certain threshold $\frac{\gamma_0}{n_0}$ in order to be able to guarantee a certain BER (bit error ratio), where γ_0 and n_0 denote the received power threshold and the noise power, respectively. The data rate and the desired BER are part of the QoS (quality of service) requirements. The QoS also defines an outage probability level F_0 that is tolerated, i.e. it is tolerated that the instantaneous receive power falls below γ_0 with probability F_0. The outage capacity at outage probability F_0 is defined as the Shannon Capacity of a virtual non-fading channel with SNR $\frac{\gamma_0}{n_0}$. The instantaneous capacity is higher than the outage capacity with probability $(1-F_0)$.

We define the required total transmit power, which is necessary to guarantee a received power larger than γ_0 with probability $(1-F_0)$, as the performance criterion for an optimum transmit diversity power allocation scheme. The less transmit power is needed the better is the scheme.

A larger channel gain reduces the required transmit power. Diversity, on the other hand, reduces the necessary fading margin and hence also the average transmit power. Thus, the proposed performance criterion will lead to a trade-off between diversity and channel gain.

III UNCORRELATED FREQUENCY-FLAT RAYLEIGH FADING CASE

Throughout this section, we assume uncorrelated diversity branches that see frequency-flat Rayleigh fading. We will extend our investigations to correlated fading conditions in the following section.

The cdf (cumulative probability density function) F_γ of the received signal power γ can be written as [9]

$$F_\gamma(\gamma) = 1 - \sum_{m=1}^{M} \frac{(\beta_m\mu_m)^{M-1} \exp\left(-\frac{\gamma}{\beta_m\mu_m}\right)}{\prod\limits_{\substack{n\neq m\\ n=1}}^{M} (\beta_m\mu_m - \beta_n\mu_n)} \tag{6}$$

for $\gamma > 0$. As already explained in Section II, μ_m and β_m denote the channel gain and the allocated transmit power of

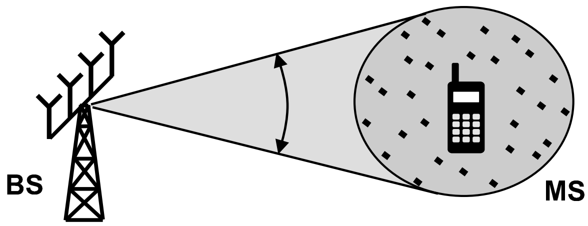

Fig. 1. Visualization of the channel model used for parametrizing the spatial channel properties. The multipath components are inside an angular window, which is centered around the broadside of the base station array.

the m-th diversity branch.

The transmit power allocation $\boldsymbol{\beta}_{\text{opt}}$ that minimizes the total transmit power P is given by

$$\begin{aligned}\boldsymbol{\beta}_{\text{opt}} &= \arg\min_{\boldsymbol{\beta}} P \\ &\text{constraint 1: } \beta_m \geq 0 \quad , \; m \in \{1, 2, \ldots, M\} \\ &\text{constraint 2: } F_\gamma(\gamma_0, \boldsymbol{\beta}) \leq F_0 \, . \end{aligned} \tag{7}$$

Unfortunately, there exists no closed form solution to this optimization problem since the cdf is a transcendental function of the transmit power coefficients $\boldsymbol{\beta}$. We therefore solved Equation (7) by numerical optimization.

A Spatial Parametrization of the Mobile Radio Channel

The optimal power allocation $\boldsymbol{\beta}_{\text{opt}}$ strongly depends on the spatial properties of the mobile radio channel. In order to be able to present the dependence of the power allocation on the channel, we want to parametrize the channel with a single scalar parameter. To this end, we assume a four element uniform linear array (with a spacing of half a wavelength) at the base station and introduce a very simple spatial channel model: Multipath components are restricted to an angular window centered around the broadside of the array. Inside this angular window, the angle of the multipath components is evenly distributed and the expected power is the same for each angle. The number of multipath components is very large. Figure 1 visualizes this channel model.

As diversity branches we apply the eigenvectors of the spatial covariance matrix, which guarantees that the branches are uncorrelated. By means of the simple channel model, we can present Figure 2. It shows the power of the diversity branches (on a linear scale) depending on the spatial properties of the radio channel, i.e. the angular window size. For small angular windows, the diversity offered by the channel is very minor. With increasing angular window size, the diversity is increasing as well up to four equally strong branches. The total channel power is always normalized to 1.

B Numerical Results

Utilizing the parametrization of Subsection A, we numerically solved Equation (7) for different outage probability levels F_0. The required receive power threshold γ_0 is always set to 1. A scaling of γ_0 just causes the required transmit power P to be scaled by exactly the same factor, and is therefore not considered any further.

The results for an outage probability $F_0 = 0.1$ are presented in Figures 3–5. Figure 3 shows the total transmit power necessary for an outage probability of 0.1. The five different curves represent different transmission strategies. For the first four curves (solid lines), the transmit power was allocated evenly to the K strongest diversity branches, where K varies from 1 to 4. For the fifth curve, labeled "optimum power allocation", the transmit power was allocated to the diversity branches according to $\boldsymbol{\beta}_{\text{opt}}$ of Equation 7.

We can draw several conclusions out of this figure: For different spatial channel conditions, a different number of transmit diversity branches is optimum. Picking the wrong number of diversity branches can cause a significant performance loss up to 6dB. The difference between the optimum power allocation and the even power allocation (on the optimum number of diversity branches) is rather minor. By means of Figure 4, we can see that the difference becomes largest at the transition regions, where the optimum diversity order changes. Still it is below 0.2dB.

Figure 5 shows how the numerical optimization allocates the powers to the transmit diversity branches. For the pre-

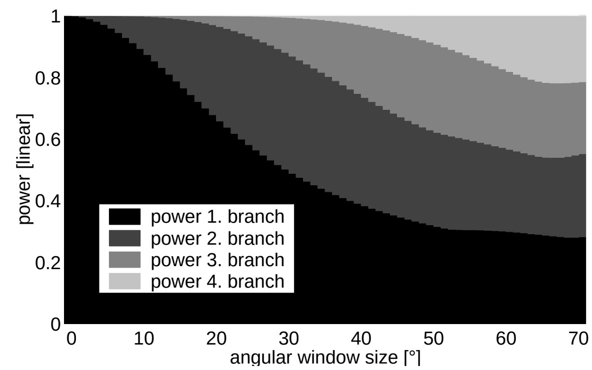

Fig. 2. Average powers of the diversity branches depending on the angular window size. In this case, the eigenvectors of the spatial covariance matrix are utilized as diversity branches.

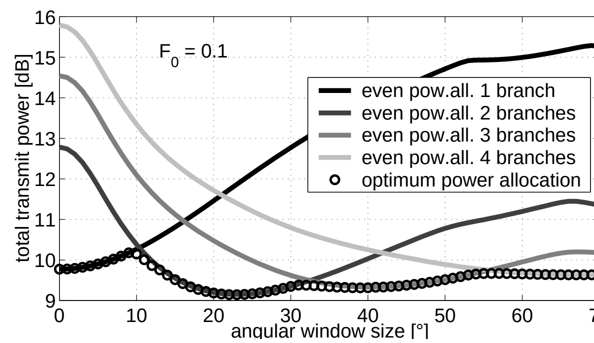

Fig. 3. Required total transmit power vs. channel parameter for a target outage probability of $F_0{=}0.1$. Different lines represent different transmit strategies.

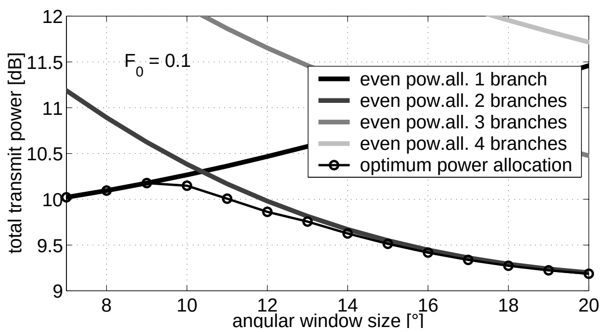

Fig. 4. Zoomed detail of Figure 3.

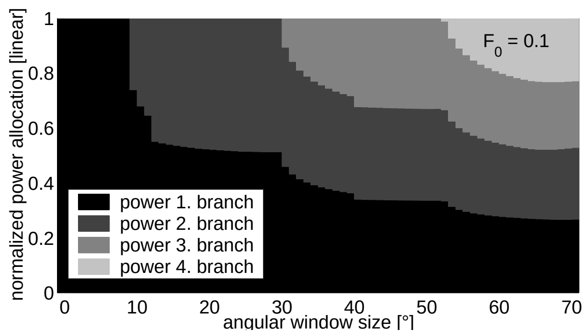

Fig. 5. Optimum power allocation (normalized to the total transmit power) vs. channel parameter for a target outage probability of F_0= 0.1.

sentation of this figure, the total transmit power was normalized to 1. The stronger the diversity branch the more power is allocated to it. At the transitions between diversity orders, the "new" diversity branch is gradually allocated more power until the entire power is allocated evenly. Between the transitions are regions where the even power allocation stays constant.

Figures 6 and 7 show the corresponding results for an outage probability level of F_0=0.001. The variations of the required transmit power for the optimum power allocation are much larger now. Significantly more transmit power is necessary for a channel with low spatial diversity. Compared to the results for F_0=0.1, the transition points between different optimum diversity orders is shifted to the left, i.e. exploiting diversity becomes more attractive. This is due to the increased need for reliability. Again, the performance difference between the even power allocation (on the optimum number of branches) is only slightly worse than the optimum power allocation scheme.

In order to stress that the even power allocation performs quasi-optimum, we present Figure 8. It illustrates the performance loss (in terms of additionally required transmit power) compared to the optimum power allocation scheme vs. the channel parameter and the outage probability. Please note the scaling at the right hand side, the maximum loss is 0.2dB and therefore negligible. We can reduce the problem of the optimum transmit diversity power allocation to the question of the optimum diversity order. The bright lines, where the loss is relatively high, mark the transitions between regions of different optimum diversity order.

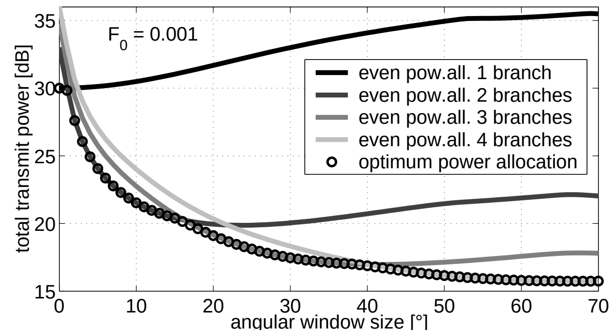

Fig. 6. Required total transmit power vs. channel parameter for a target outage probability of $F_0 = 0.001$. Different lines represent different transmit strategies.

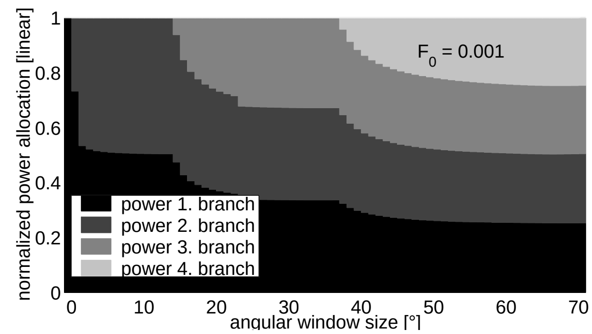

Fig. 7. Optimum power allocation (normalized to the total transmit power) vs. channel parameter for a target outage probability of F_0= 0.001.

IV Correlated Fading on the Diversity Branches

In Section III, the investigations were restricted to uncorrelated frequency-flat Rayleigh fading for reasons of simplicity. In this Section, we will treat the case of correlated fading on the diversity branches.

We assume the covariance matrix of the channel coefficients to be given by $\boldsymbol{R}_{\text{DL}}$ as defined in Section II. Furthermore, we define the matrix $\boldsymbol{B}$ as a diagonal matrix with the power allocation coefficients $\beta_1, \beta_2, \ldots, \beta_M$ as diagonal elements. At the mobile receiver, the signal from the m-th transmit antenna is given by $r_m[t] = \sqrt{\beta_m} h_m^{\text{DL}}[t] s[t]$, and the vector $\boldsymbol{r}[t] = [r_1 \; r_2 \; \cdots \; r_M]^T$ reads as

$$\boldsymbol{r}[t] = s[t] \, \boldsymbol{B}^{1/2} \, \boldsymbol{h}_{\text{DL}}[t] \, . \tag{8}$$

Using Equations (4) and (5), we can directly calculate the signal covariance matrix $\boldsymbol{R}_{\text{signal}}$ as

$$\boldsymbol{R}_{\text{signal}} = \boldsymbol{B}^{1/2} \boldsymbol{R}_{\text{DL}} \boldsymbol{B}^{1/2} \, . \tag{9}$$

Correlation reduces the benefits of diversity transmission. Given the eigen-decomposition of the signal covariance matrix

$$\boldsymbol{R}_{\text{signal}} = \boldsymbol{U} \boldsymbol{\Lambda} \boldsymbol{U}^H \, , \tag{10}$$

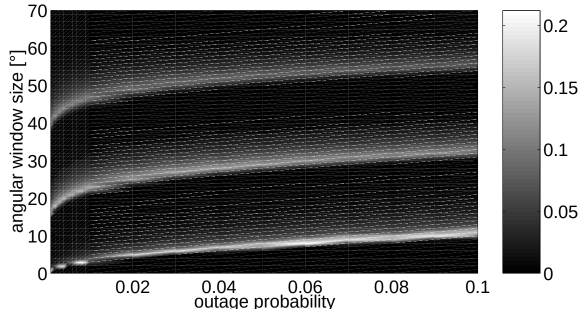

Fig. 8. Performance loss (in terms of additionally required transmit power) of even power allocation compared to optimum power allocation vs. channel parameter and target outage probability.

the elements $\lambda_1, \lambda_2, \ldots, \lambda_M$ of the diagonal matrix $\mathbf{\Lambda}$ are the powers of the uncorrelated signal parts. Thus, the cdf of the received signal power γ at the receiver can easily be calculated by exchanging all μ_m by λ_m in Equation 6:

$$F_{\gamma,\mathrm{corr}}(\gamma) = 1 - \sum_{m=1}^{M} \frac{\left(\lambda_m\left(\boldsymbol{B}\right)\right)^{M-1} \exp\left(-\frac{\gamma}{\lambda_m\left(\boldsymbol{B}\right)}\right)}{\prod\limits_{\substack{n \neq m \\ n=1}}^{M} \left(\lambda_m\left(\boldsymbol{B}\right) - \lambda_n\left(\boldsymbol{B}\right)\right)} . \quad (11)$$

The higher the correlations are the more difference between the eigenvalues can be seen, which leads to a reduced diversity gain.

Unfortunately, we cannot allocate transmit power directly to an eigenmode but only to a diversity branch. The power on the uncorrelated eigenmodes can only be influenced implicitly according to Equation (11). However, allocating the transmit power evenly over D transmit branches is the same as allocating the power evenly over the eigenmodes of these D branches. In both cases, the transmit power is evenly spread over the utilized signal sub-space. Thus, the main conclusion of Section III is still valid: The problem of optimum transmit power allocation reduces to the question of optimum diversity order.

Figure 9 shows numerical results for an exemplary transmit diversity system with two correlated diversity branches. On the abscissa, the power allocation varies from transmitting on branch 1 only (left hand side) to transmitting on branch 2 only (right hand side); in the middle of the abscissa the transmit power is allocated evenly over both diversity branches. The ordinate shows the required total transmit power P. Different curves correspond to different target values for the outage probability. The outage probability levels are varied from 10% for the bottommost line to 0.1% for the topmost line. The channel covariance matrix $\boldsymbol{R}_{\mathrm{DL}}$ for the presented system is

$$\boldsymbol{R}_{\mathrm{DL}} = \begin{bmatrix} 0.0928 & 0.1697\,\mathrm{j} \\ 0.1697\,\mathrm{j} & 0.9072 \end{bmatrix} . \quad (12)$$

For high outage levels, no diversity is required and the best performance is achieved by using the second branch

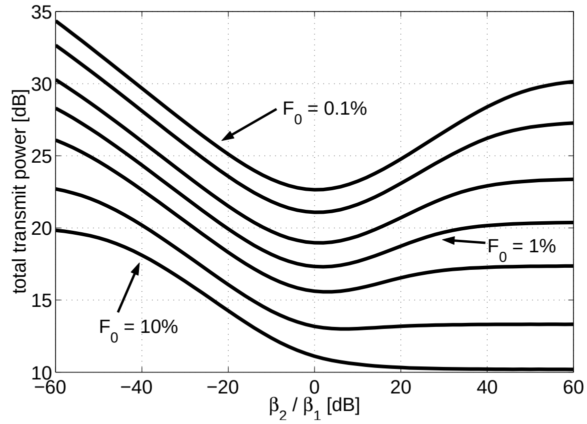

Fig. 9. Required total transmit power vs. power allocation for a system with two correlated diversity branches. Different lines represent different outage probabilities. The outage probability values are (from the bottommost line to the topmost line): 10%, 5%, 2%, 1%, 0.5%, 0.2%, and 0.1%.

only, which carries more power than the first branch according to Equation (12). By tightening the requirements concerning the outage probability, diversity becomes essential and the point of optimum power allocation moves to the middle of the abscissa. For an outage probability of 0.1%, the optimum power allocation is an even allocation over both diversity branches. Please note that in the transition region, where the optimum lies between transmitting on branch 2 only and an even power allocation, the curve of the total transmit power is very flat. This means that in this region the required transmit power is rather insensitive to deviations from the optimum allocation scheme. Thus, the example of Figure 9 confirms the statement that, also in a correlated environment, an even power allocation over the optimum number of diversity branches performs quasi-optimum.

However, in contrast to an uncorrelated system, the power of a diversity branch does not tell the whole story. If D transmit branches have to be chosen from a system with $M > D$ correlated diversity branches, the channel gains of the D branches are not a sufficient criterion for minimizing the required transmit power. Besides a high channel gain, it is important to chose D branches that are as less correlated as possible. In a correlated environment it is not sufficient to answer the question *how many* diversity branches should be used for transmission. We have to answer the question *which* diversity branches are optimum. This optimum choice can only be determined by the eigenvalues of Equation (10) in conjunction with Equation (11).

V Conclusions

We numerically analyzed the problem of finding the optimum power allocation to transmit diversity branches with different average channel gains. The optimization criterion was the required transmit power for a certain outage capacity.

For frequency-flat Rayleigh fading the order of power is not reversed by the optimum allocation, i.e. diversity branches with a higher channel gain are also allocated more power. If the average channel gain of a diversity branch is sufficiently low (depending on outage probability and channel conditions) no power is allocated to this branch at all.

The main conclusion is that allocating the transmit power evenly over the optimum set of diversity branches performs nearly as well as the optimized power allocation. The performance loss is negligible (less than 0.2dB). Thus, the problem of an optimum power allocation to diversity branches reduces to the question which set of diversity branches is optimum for transmission.

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