A Novel Stochastic MIMO Channel Model and Its Physical Interpretation

Werner Weichselberger*, Hüseyin Özcelik*, Markus Herdin*, Ernst Bonek*†

* Institut für Nachrichtentechnik und Hochfrequenztechnik, Technische Universität Wien
Gußhausstraße 25/389, A-1040 Wien, Austria, email: {weichsel,oh,ebonek}@nt.tuwien.ac.at

† ftw. Forschungszentrum Telekommunikation Wien, Donau-City-Straße 1, A-1220 Wien, Austria

Abstract *— We present a novel stochastic channel model for multiple-input multiple-output (MIMO) wireless radio channels. In contrast to state of the art stochastic MIMO channel models, we do not divide the spatial correlation properties of the channel into separate contributions from transmitter and receiver. We rather model the joint correlation properties by describing the average coupling between the eigenmodes of the two link ends. The structure of this coupling is shown to be crucial for the spatial properties of a MIMO channel. We discuss the mathematical elements of the model from a radio propagation point of view, and explain the physical restrictions on the MIMO setup imposed by the model. A comparison to the more restrictive but popular 'Kronecker' model is provided. Finally, we show that our model is capable of correctly predicting the mutual information of measured indoor MIMO channels.*

1 Introduction

Various measurements have shown that realistic MIMO channels show a significantly lower capacity than idealized i.i.d. MIMO channels [1, 2, 3], which is due to spatially correlated antenna signals at the transmitter and the receiver [4]. As a consequence, various publications deal with the modelling of the spatial correlation properties of MIMO channels [2, 5, 3, 6]. Their common approach is to model the correlation at the receiver and at the transmitter independently. However, realistic indoor MIMO channels cannot be modelled adequately by this approach, channel capacity is underestimated and the multipath structure is rendered wrongly [1]. We will, therefore, present an enhanced stochastic MIMO channel model which is able to model the spatial properties of realistic MIMO channels correctly. It does not only account for the correlation at both link ends, but it also models their mutual dependence.

2 Signal model and definitions

The presented model focuses on the *spatial* structure of MIMO channels. We will, therefore, restrict ourselves to frequency-flat and stationary MIMO channels, which can be described by a single channel transfer matrix $\boldsymbol{H}$. In the following, we will assume the channel matrix $\boldsymbol{H}$ to be multivariate complex-normal distributed with zero-mean. Denoting the number of transmit (Tx) and receive (Rx) antennas with M_{Tx} and M_{Rx} respectively, the matrix $\boldsymbol{H}$ is of size $M_{\text{Rx}} \times M_{\text{Tx}}$. The signal vector $\boldsymbol{y}$ at the M_{Rx} receive antennas reads as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}\,, \tag{1}$$

where $\boldsymbol{x}$ denotes the transmit signal vector, and $\boldsymbol{n}$ is the noise vector observed at the receiver. Throughout the entire paper, the subscripts $\bullet_{\text{Tx}}$ and $\bullet_{\text{Rx}}$ will indicate to which side of the radio link an entity is associated.

We will denote the m-th column of the channel matrix by $\boldsymbol{h}_{\text{col},m}$, and the channel matrix elements as $h_{n,m}$. The vec($\bullet$) operator stacks the columns of a matrix into one tall vector,

$$\text{vec}\,(\boldsymbol{H}) = \left[\begin{array}{cccc} \boldsymbol{h}^{\text{T}}_{\text{col},1} & \boldsymbol{h}^{\text{T}}_{\text{col},2} & \cdots & \boldsymbol{h}^{\text{T}}_{\text{col},M_{\text{Tx}}} \end{array}\right]^{\text{T}}, \tag{2}$$

where the superscript $\bullet^{\text{T}}$ denotes the matrix transpose. We will also use the superscripts $\bullet^{\text{H}}$ and $\bullet^{*}$ to indicate the conjugate matrix transpose and the complex conjugate, respectively. Finally, we define two matrix product operators. The symbol $\otimes$ denotes the Kronecker matrix product, and $\odot$ denotes the element-wise product of two matrices.

3 The general MIMO channel

In order to describe the spatial behavior of a general MIMO channel, the full correlation matrix $\boldsymbol{R_H}$, which describes the correlation between all channel matrix elements, is required.

$$\boldsymbol{R_H} = \text{E}_{\boldsymbol{H}}\left\{\text{vec}\,(\boldsymbol{H})\,\text{vec}\,(\boldsymbol{H})^{\text{H}}\right\} \tag{3}$$

According to Equation (3), $\boldsymbol{R_H}$ can be written as a block-matrix of $M_{\text{Tx}} \times M_{\text{Tx}}$ blocks, each of size $M_{\text{Rx}} \times M_{\text{Rx}}$,

$$\boldsymbol{R_H} = \left[\begin{array}{cccc} \boldsymbol{R}_{\text{Rx},1,1} & \boldsymbol{R}_{\text{Rx},1,2} & \cdots & \boldsymbol{R}_{\text{Rx},1,M_{\text{Tx}}} \\ \boldsymbol{R}_{\text{Rx},2,1} & \boldsymbol{R}_{\text{Rx},2,2} & \cdots & \boldsymbol{R}_{\text{Rx},2,M_{\text{Tx}}} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{R}_{\text{Rx},M_{\text{Tx}},1} & \boldsymbol{R}_{\text{Rx},M_{\text{Tx}},2} & \cdots & \boldsymbol{R}_{\text{Rx},M_{\text{Tx}},M_{\text{Tx}}} \end{array}\right],$$

where one block is given by $\boldsymbol{R}_{\text{Rx},m_1,m_2} \triangleq \text{E}_{\boldsymbol{H}}\left\{\boldsymbol{h}_{\text{col},m_1}\boldsymbol{h}^{\text{H}}_{\text{col},m_2}\right\}$. A diagonal block $\boldsymbol{R}_{\text{Rx},m,m}$ can be interpreted as the spatial correlation matrix at the receiver caused by the single transmit antenna m. It is the auto-correlation matrix of the vector $\boldsymbol{h}_{\text{col},m}$. The off-diagonal blocks $\boldsymbol{R}_{\text{Rx},m_1,m_2}$ can be interpreted in an analogous way. They are spatial cross-correlation matrices of the receive channel vectors $\boldsymbol{h}_{\text{col},m_1}$ and

$\boldsymbol{h}_{\text{col},m_2}$. One channel vector is generated by transmitting from Tx antenna m_1, the other one by transmitting from Tx antenna m_2.

The two ends of a MIMO channel are strongly linked by the spatial structure of the MIMO channel. As a consequence, a stochastic description of the receiver has to be parameterized with the statistical properties of the transmit signal. In case of transmit beamforming with fixed Tx weights $\boldsymbol{w}_{\text{Tx}}$, whose elements will be denoted as $w_{\text{Tx},m}$, the resulting Rx correlation matrix reads as

$$\boldsymbol{R}_{\text{Rx},\boldsymbol{w}_{\text{Tx}}} \triangleq \mathrm{E}_{\boldsymbol{H}}\left\{\boldsymbol{H}\boldsymbol{w}_{\text{Tx}}\boldsymbol{w}_{\text{Tx}}^{\text{H}}\boldsymbol{H}^{\text{H}}\right\} \tag{4}$$

$$= \sum_{m_1=1}^{M_{\text{Tx}}}\sum_{m_2=1}^{M_{\text{Tx}}} w_{\text{Tx},m_1} w_{\text{Tx},m_2}^{*} \boldsymbol{R}_{\text{Rx},m_1,m_2} \tag{5}$$

Transmitting multiple signal streams or using time-varying spatial transmit weights, the statistical description of the Tx signal is provided by the signal covariance matrix $\boldsymbol{Q}_{\text{sig}}$. By means of its eigenvectors $\boldsymbol{u}_{\text{sig},m}$ and eigenvalues $\lambda_{\text{sig},m}$, the parameterized Rx correlation matrix can be expressed as

$$\boldsymbol{R}_{\text{Rx},\boldsymbol{Q}_{\text{sig}}} \triangleq \mathrm{E}_{\boldsymbol{H}}\left\{\boldsymbol{H}\boldsymbol{Q}_{\text{sig}}\boldsymbol{H}^{\text{H}}\right\} = \sum_{m=1}^{M_{\text{Tx}}} \lambda_{\text{sig},m}\, \boldsymbol{R}_{\text{Rx},\boldsymbol{u}_{\text{sig},m}}\,. \tag{6}$$

Additionally to the parameterized correlation matrices, we want to define a one-sided correlation matrix that is *independent* of the other link end:

$$\begin{aligned}\boldsymbol{R}_{\text{Rx}} &\triangleq \mathrm{E}_{\boldsymbol{H}}\left\{\boldsymbol{H}\boldsymbol{H}^{\text{H}}\right\} = \sum_{m=1}^{M_{\text{Tx}}} \boldsymbol{R}_{\text{Rx},m,m}\\ \boldsymbol{R}_{\text{Tx}} &\triangleq \mathrm{E}_{\boldsymbol{H}}\left\{\boldsymbol{H}^{T}\boldsymbol{H}^{*}\right\} = \sum_{n=1}^{M_{\text{Rx}}} \boldsymbol{R}_{\text{Tx},n,n}\,.\end{aligned} \tag{7}$$

The trace of these correlation matrices gives the total mean energy of the channel matrix $\boldsymbol{H}$, and will be denoted by $P_{\boldsymbol{H}}$,

$$P_{\boldsymbol{H}} \triangleq \text{trace}\left(\boldsymbol{R}_{\text{Rx}}\right) = \text{trace}\left(\boldsymbol{R}_{\text{Tx}}\right) = \sum_{n=1}^{M_{\text{Rx}}}\sum_{m=1}^{M_{\text{Tx}}} \mathrm{E}_{\boldsymbol{H}}\left\{|h_{n,m}|^2\right\}$$

As the MIMO channel is *symmetric* w.r.t. transmitter and receiver, above considerations also hold for the respective properties of the reverse link. Throughout this paper, we will extensively make use of the inherent symmetry of the MIMO channel. *Results derived for one link end will be automatically extended to the other end.*

4 Novel stochastic MIMO channel model

A significant drawback of the full correlation matrix is its huge size. It requires $\frac{1}{2}\left(M_{\text{Tx}}M_{\text{Rx}}\left(1+M_{\text{Tx}}M_{\text{Rx}}\right)\right)$ complex parameters to be fully specified. In this section, we will present a MIMO channel model which is based on correlations without assuming separability of Rx and Tx correlation properties.

4.1 Modeling assumption

We postulate the following modeling assumption:
All auto- and cross-correlation matrices of one link end have the same eigenbasis.

In formal notation, this assumption reads as

$$\begin{aligned}\boldsymbol{R}_{\text{Rx},m_1,m_2} &= \boldsymbol{U}_{\text{Rx}}\boldsymbol{\Lambda}_{\text{Rx},m_1,m_2}\boldsymbol{U}_{\text{Rx}}^{\text{H}}\\ \boldsymbol{R}_{\text{Tx},n_1,n_2} &= \boldsymbol{U}_{\text{Tx}}\boldsymbol{\Lambda}_{\text{Tx},n_1,n_2}\boldsymbol{U}_{\text{Tx}}^{\text{H}}\,,\end{aligned} \tag{8}$$

where $\boldsymbol{U}_{\text{Rx}}$ denotes the eigenbasis, consisting of the orthonormal eigenvectors; and $\boldsymbol{\Lambda}_{\text{Rx},m_1,m_2}$ is a diagonal matrix whose elements are the eigenvalues of the correlation matrix. The *eigenbasis* $\boldsymbol{U}_{\text{Rx}}$ *does not depend* on the indices m_1 and m_2, it is the same for all correlation matrices. In general, however, the *eigenvalues do differ* for each correlation matrix.

Due to Equations (5), (6), and (7), we see that also all one-sided correlation matrices have the same common eigenbasis:

$$\boldsymbol{R}_{\text{Rx},\boldsymbol{w}_{\text{Tx}}} = \boldsymbol{U}_{\text{Rx}}\boldsymbol{\Lambda}_{\text{Rx},\boldsymbol{w}_{\text{Tx}}}\boldsymbol{U}_{\text{Rx}}^{\text{H}} \tag{9}$$

$$\boldsymbol{R}_{\text{Rx},\boldsymbol{Q}_{\text{sig}}} = \boldsymbol{U}_{\text{Rx}}\boldsymbol{\Lambda}_{\text{Rx},\boldsymbol{Q}_{\text{sig}}}\boldsymbol{U}_{\text{Rx}}^{\text{H}} \tag{10}$$

$$\boldsymbol{R}_{\text{Rx}} = \boldsymbol{U}_{\text{Rx}}\boldsymbol{\Lambda}_{\text{Rx}}\boldsymbol{U}_{\text{Rx}}^{\text{H}} \tag{11}$$

Note that the presented assumption is less restrictive than the separability assumption of the ‘Kronecker’ model [2]. In fact, the latter implies that all correlation matrices are identical (except a scalar multiplication). For our new model, only the eigenbasis is assumed to be identical but not the eigenvalues.

In the following, we will denote the eigenvectors of the common eigenbasis at the Rx side with $\boldsymbol{u}_{\text{Rx},m}$, and at the Tx side with $\boldsymbol{u}_{\text{Tx},m}$.

4.2 Physical interpretation of the modeling assumption

What is the physical implication of a common eigenbasis but different eigenvalues? The Tx *eigenbasis* reflects the spatial structure of the scatterers that are relevant at the receive array. This structure does not depend on the Tx weights. The *eigenvalues* reflect how the scatterers are illuminated by the radio waves propagating from the transmitter. Obviously, the pattern of illumination can change significantly with the Tx weights. Radiating in certain directions, for example, may illuminate only certain scatterers and leave others ‘dark’. Likewise, the same is true for the reverse link.

We want to emphasize that the proposed channel model is *not* restricted to homogeneous arrays. The elements of the MIMO arrays may have different radiation patterns, different polarizations, and/or different orientations. Just as the receive eigenbasis does not depend on the Tx weights, it does not depend on the individual characteristics of the Tx antenna elements either.

4.3 Formulation of channel model

By transforming the channel matrix $\boldsymbol{H}$ with the common eigenbasis at both link ends,

$$\boldsymbol{H}_{\text{eig}} \triangleq \boldsymbol{U}_{\text{Rx}}^{\text{H}} \boldsymbol{H} \boldsymbol{U}_{\text{Tx}}^{*}, \qquad (12)$$

we obtain a matrix $\boldsymbol{H}_{\text{eig}}$ whose entries $[\boldsymbol{H}_{\text{eig}}]_{n,m} = \boldsymbol{u}_{\text{Rx},n}^{\text{H}} \boldsymbol{H} \boldsymbol{u}_{\text{Tx},m}^{*}$ denote the complex amplitude of the virtual single-input single-output (SISO) channel between the m-th Tx eigenvector and the n-th Rx eigenvector. These channels are completely *uncorrelated*, which is due to the properties of the eigendecomposition and becomes obvious when we calculate the conditioned one-sided correlation matrix of $\boldsymbol{H}_{\text{eig}}$ by applying Equation (10).

$$\mathrm{E}_{\boldsymbol{H}}\left\{\boldsymbol{H}_{\text{eig}} \boldsymbol{Q}_{\text{Tx}}' \boldsymbol{H}_{\text{eig}}^{\text{H}}\right\} = \boldsymbol{\Lambda}_{\text{Rx},\boldsymbol{Q}_{\text{Tx}}} \qquad (13)$$

$$\mathrm{E}_{\boldsymbol{H}}\left\{\boldsymbol{H}_{\text{eig}}^{\text{T}} \boldsymbol{Q}_{\text{Rx}}' \boldsymbol{H}_{\text{eig}}^{*}\right\} = \boldsymbol{\Lambda}_{\text{Tx},\boldsymbol{Q}_{\text{Rx}}}, \qquad (14)$$

where $\boldsymbol{Q}_{\text{Tx}} = \boldsymbol{U}_{\text{Tx}}^{*} \boldsymbol{Q}_{\text{Tx}}' \boldsymbol{U}_{\text{Tx}}^{\text{T}}$ and $\boldsymbol{Q}_{\text{Rx}} = \boldsymbol{U}_{\text{Rx}}^{*} \boldsymbol{Q}_{\text{Rx}}' \boldsymbol{U}_{\text{Rx}}^{\text{T}}$ are signal covariance matrices according to the explanations in Section 3; and $\boldsymbol{\Lambda}_{\text{Rx},\boldsymbol{Q}_{\text{Tx}}}$ and $\boldsymbol{\Lambda}_{\text{Tx},\boldsymbol{Q}_{\text{Rx}}}$ are diagonal matrices. Equations (13) and (14) hold true for all $\boldsymbol{Q}_{\text{Tx}}$ and $\boldsymbol{Q}_{\text{Rx}}$ if and only if all elements of $\boldsymbol{H}_{\text{eig}}$ are mutually uncorrelated.

The *expected powers* of the elements of $\boldsymbol{H}_{\text{eig}}$ will, in general, differ. We will denote these expected powers between the m-th Tx eigenmode and the n-th Rx eigenmode by $\omega_{m,n}$, and define the eigenmode coupling matrix $\boldsymbol{\Omega}$ as

$$[\boldsymbol{\Omega}]_{n,m} \triangleq \omega_{n,m} = \mathrm{E}_{\boldsymbol{H}}\left\{|\boldsymbol{u}_{\text{Rx},n}^{\text{H}} \boldsymbol{H} \boldsymbol{u}_{\text{Tx},m}^{*}|^2\right\}. \qquad (15)$$

Given Equation (12), the coupling matrix $\boldsymbol{\Omega}$, and uncorrelated entries of $\boldsymbol{H}_{\text{eig}}$, *the new model reads as*

$$\boxed{\boldsymbol{H}_{\text{model}} = \boldsymbol{U}_{\text{Rx}}\left(\tilde{\boldsymbol{\Omega}} \odot \boldsymbol{G}\right) \boldsymbol{U}_{\text{Tx}}^{\text{T}}}, \qquad (16)$$

where $\boldsymbol{G}$ is a random matrix with i.i.d. zero-mean complex-normal entries with unit variance, and $\tilde{\boldsymbol{\Omega}}$ is defined as the element-wise square root of $\boldsymbol{\Omega}$. By means of different realizations of the random matrix $\boldsymbol{G}$ we can generate different realizations of the modeled channel transfer matrix.

All we need for modeling the spatial properties of a MIMO channel is:

- The spatial eigenbasis $\boldsymbol{U}_{\text{Tx}}$ at the transmitter.
- The spatial eigenbasis $\boldsymbol{U}_{\text{Rx}}$ at the receiver.
- The average energy of the virtual SISO channel between each transmit and each receive eigenmode, $\boldsymbol{\Omega}$, linking the correlation properties of both ends.

4.3.1 Full correlation matrix

How does the full correlation matrix $\boldsymbol{R}_{\boldsymbol{H}}$ read in the notation of the new model? Denoting the elements of $\boldsymbol{G}$ with $g_{n,m}$ and writing $\boldsymbol{H}_{\text{model}}$ as a sum of all eigenmodes, we can stack the channel matrix $\boldsymbol{H}_{\text{model}}$ as

$$\operatorname{vec}(\boldsymbol{H}_{\text{model}}) = \sum_{n=1}^{M_{\text{Rx}}} \sum_{m=1}^{M_{\text{Tx}}} \sqrt{\omega_{n,m}}\, g_{n,m} \cdot \left(\boldsymbol{u}_{\text{Tx},m} \otimes \boldsymbol{u}_{\text{Rx},n}\right),$$

and calculate the full correlation matrix of $\boldsymbol{H}_{\text{model}}$ as

$$\boldsymbol{R}_{\boldsymbol{H},\text{model}} = \sum_{n=1}^{M_{\text{Rx}}} \sum_{m=1}^{M_{\text{Tx}}} \omega_{n,m} \cdot \left(\boldsymbol{u}_{\text{Tx},m} \otimes \boldsymbol{u}_{\text{Rx},n}\right) \left(\boldsymbol{u}_{\text{Tx},m} \otimes \boldsymbol{u}_{\text{Rx},n}\right)^{\text{H}}. \qquad (17)$$

Equation (17) provides the *eigendecomposition of* $\boldsymbol{R}_{\boldsymbol{H}}$ *with* $(\boldsymbol{u}_{\text{Tx},n} \otimes \boldsymbol{u}_{\text{Rx},m})$ *as eigenvectors and* $\omega_{m,n}$ *as eigenvalues.*

4.4 Structure of coupling matrix

We want to point out that the eigenvalues $\lambda_{\text{Tx},m}$ and $\lambda_{\text{Rx},n}$ of the one-sided correlation matrices $\boldsymbol{R}_{\text{Tx}}$ and $\boldsymbol{R}_{\text{Rx}}$ do not directly influence the model (16). They are only given implicitly by the elements of $\boldsymbol{\Omega}$:

$$\lambda_{\text{Tx},m} = \sum_{n=1}^{M_{\text{Rx}}} \omega_{n,m} \quad \text{and} \quad \lambda_{\text{Rx},n} = \sum_{m=1}^{M_{\text{Tx}}} \omega_{n,m}. \qquad (18)$$

As already observed in [7], the structure of $\boldsymbol{\Omega}$ influences the capacity as well as the degree of diversity that is experienced on spatially multiplexed channels. It tells us how many *parallel data streams* can be multiplexed, which degree of *diversity* is present at the Tx and at the Rx side, and how much *beamforming* gain can be achieved.

If only a single column or a single row of $\boldsymbol{\Omega}$ carries significant power then we can exploit diversity at one link end, and apply beamforming at the other; spatial multiplexing is not possible. A diagonally dominated structure of $\boldsymbol{\Omega}$ allows spatial multiplexing and offers high ergodic capacity but no diversity on the data streams, whereas an evenly loaded coupling matrix shows lower ergodic capacity but maximum diversity order on each multiplexed data stream.

4.4.1 'Kronecker' model

The key assumption of the popular 'Kronecker' model [2] is the *separability of the correlation properties* of the two link ends. With this assumption, the full correlation matrix is described by the Kronecker product of the Rx and Tx correlation matrices:

$$\boldsymbol{R}_{\boldsymbol{H},\text{kron}} = \frac{1}{P_{\boldsymbol{H}}} \cdot \boldsymbol{R}_{\text{Tx}} \otimes \boldsymbol{R}_{\text{Rx}}. \qquad (19)$$

The scalar multiplication with the inverse of the total channel energy is just a normalization factor. According to the 'Kronecker' model, a realization of the channel matrix $\boldsymbol{H}$ is generated as

$$\boldsymbol{H}_{\text{kron}} = \frac{1}{P_{\boldsymbol{H}}} \cdot \boldsymbol{R}_{\text{Rx}}^{1/2} \boldsymbol{G} \left(\boldsymbol{R}_{\text{Tx}}^{1/2}\right)^{\text{T}}, \qquad (20)$$

where $\boldsymbol{G}$, again, is an i.i.d. random matrix whose entries are zero-mean complex-normal distributed. Equation (20) can be formulated along the notation of Equation (16) with a coupling matrix $\boldsymbol{\Omega}_{\text{kron}}$ of the following structure

$$[\boldsymbol{\Omega}_{\text{kron}}]_{n,m} = \frac{1}{P_{\boldsymbol{H}}} \cdot \lambda_{\text{Rx},n}\lambda_{\text{Tx},m} \,. \tag{21}$$

Evidently, the coupling matrix $\boldsymbol{\Omega}$ becomes *rank one*, and all coupling coefficients $\omega_{n,m}$ are defined by the outer product of $M_{\text{Rx}} + M_{\text{Tx}}$ eigenvalues.

In [1] it was shown that the 'Kronecker' assumption does not hold, in general, for realistic MIMO channels. As all $M_{\text{Rx}} \cdot M_{\text{Tx}}$ elements of $\boldsymbol{\Omega}_{\text{kron}}$ are determined by means of $M_{\text{Rx}} + M_{\text{Tx}}$ eigenvalues, the 'Kronecker' model lacks essential degrees of freedom w.r.t. general conditions. It cannot generate diagonally dominated coupling matrices, the elements of $\boldsymbol{\Omega}$ are as evenly distributed as possible. This leads to a systematic underestimation of channel capacity, and to a mismatch of the modeled and measured multipath structure.

4.5 Structure of eigenbases

Equation (16) factorizes the channel matrix into three components: a left orthonormal basis, a fading matrix in the middle, and a right orthonormal basis. Why is it optimum to take the eigenvectors of the respective link ends as bases? Could the model also be constructed by means of other bases? We will discuss this question in the context of another MIMO channel model, namely the *'virtual channel representation'* which was presented by Sayeed in [7].

Instead of assuming the spatial properties of the two link ends to be separable as the 'Kronecker' model does, Sayeed directly aims at modeling the *joint* spatial behavior of the MIMO channel. To this end, the angular range at the receiver (transmitter) is artificially divided into M_{Rx} (M_{Tx}) discrete angular bins. As a result, the spatial propagation environment between the two link ends is partitioned into $M_{\text{Rx}} \cdot M_{\text{Tx}}$ virtual DoD-DoA bins. Additionally, the expected power for each of these bins is specified, and the fading coefficients associated with the virtual DoA-DoD bins are assumed to be independent. Mathematically, the resulting 'virtual channel representation' can be written as

$$\boldsymbol{H}_{\text{virt}} = \boldsymbol{A}_{\text{Rx}} \left(\tilde{\boldsymbol{\Omega}}_{\text{virt}} \odot \boldsymbol{G} \right) \boldsymbol{A}_{\text{Tx}}^{\text{T}} \,, \tag{22}$$

where $\boldsymbol{G}$, again, is an i.i.d. random matrix whose entries are zero-mean complex-normal distributed. The columns of the orthonormal matrices $\boldsymbol{A}_{\text{Rx}}$ and $\boldsymbol{A}_{\text{Tx}}$ constitute steering vectors into the directions of the angular bins. The positive real-valued elements of $\boldsymbol{\Omega}_{\text{virt}}$ denote the average power of the virtual DoD-DoA bins, and $\tilde{\boldsymbol{\Omega}}_{\text{virt}}$ is defined as the element-wise square root of $\boldsymbol{\Omega}_{\text{virt}}$.

Due to the *predefined* virtual directions, i.e. the matrices $\boldsymbol{A}_{\text{Rx}}$ and $\boldsymbol{A}_{\text{Tx}}$, the assumption of uncorrelated DoA-DoD bins is problematic. In [7], the author argues that the steering vectors of the virtual directions become rather peaky with increasing number of antenna elements. As a consequence, different DoD-DoA bins see different scatterers which are, in turn, assumed to fade independently. However, two major problems remain. Regardless of the number of antenna elements, the angular pattern of a steering vector shows significant side-lobes in the neighboring bins; and scatterers will lie between the virtual directions resulting in significant contributions of a scatterer in neighboring DoD-DoA bins. As a result, the fading amplitudes of neighboring DoD-DoA bins will be correlated in general. Furthermore, in the case of a dual polarized antenna configuration, the correlation between the two polarizations would have to be defined a priori in order to be able to specify the bases $\boldsymbol{A}_{\text{Rx}}$ and $\boldsymbol{A}_{\text{Tx}}$.

The new model, Equation (16), and the 'virtual channel representation', Equation (22), differ only in the choice of basis matrices. In fact, the eigenbases of the new model can also be interpreted as a virtual partitioning of the spatial domain. However, the eigenbases provide a partitioning that is matched to the radio environment and the applied antenna array configurations. By definition, *the eigenbases are the only possible basis matrices resulting in independent fading of the bin amplitudes.*

5 Validation of MIMO channel model with measured data

In order to validate the proposed model, we compare the mutual information calculated from measured MIMO channels with the predicted values provided by the discussed channel models. First, the model parameters are extracted from the measurements, and synthetic channel realizations are created by means of the parameterized models. The size of the synthetic ensembles is the same as of the measured ensemble. Second, mutual information values and angular power spectra are calculated from the measured and from the synthetic MIMO channel impulse responses, and compared to each other.

Channel matrices were measured in the Electrical Engineering Building on the Vienna University of Technology Campus at 5.2GHz [1]. The transmitter consisted of a positionable monopole antenna on a grid of 20×10 positions with an inter-element spacing of half the wavelength. The receiver employed a uniform linear array (ULA) of eight directional printed dipoles having an inter-element spacing of 0.4 wavelengths and a 3dB beamwidth of 120°. The channel was probed at 193 equi-spaced frequency bins over 120MHz of bandwidth. The transmitter assumed a single fixed location in a hallway. The Rx array assumed many different locations in several offices connected to this hallway, as well as three possible orientations. In total, 72 scenarios, i.e. Rx positions and orientations, were measured. For each scenario, 130 spatial realizations of a 8×8 channel matrix were formed by moving a virtual eight-element ULA over the 20×10 grid, yielding a total of 130·193 (space and frequency) realizations per scenario.

For the calculation of the mutual information, we assume that the transmitter has no knowledge about the channel at all. Thus, the mutual information is given by

$$I = \mathrm{E}\left\{\log_2\left[\det\left(\boldsymbol{I}_{M_{\mathrm{Rx}}} + \frac{\mathrm{SNR}}{M_{\mathrm{Tx}}}\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}\right)\right]\right\}, \quad (23)$$

where SNR denotes the signal to noise ratio, and the average energy of the channel matrix entries $h_{n,m}$ is normalized to unity. The expectation operation is performed w.r.t. measured channel realizations, or w.r.t. fading realizations of the random matrix $\boldsymbol{G}$ when applying Equation (16), (20) or (22). For the following evaluations, the receive SNR was set to 20dB.

The correlation matrices for the 'Kronecker' model are calculated according to Equation (7). As basis matrices of the new model we utilize the eigenbases of these correlation matrices and obtain them from Equation (11) by eigendecomposition. For the 'virtual channel representation', the basis matrices $\boldsymbol{A}_{\mathrm{Rx}}$ and $\boldsymbol{A}_{\mathrm{Tx}}$ are constructed as explained in [7].

An estimate of the coupling matrix $\boldsymbol{\Omega}$ of the new model is obtained from the measured impulse responses $\boldsymbol{H}$:

$$\hat{\boldsymbol{\Omega}} = \mathrm{E}_{\boldsymbol{H}}\left\{(\boldsymbol{U}_{\mathrm{Rx}}^{\mathrm{H}}\boldsymbol{H}\boldsymbol{U}_{\mathrm{Tx}}^{*}) \odot (\boldsymbol{U}_{\mathrm{Rx}}^{\mathrm{T}}\boldsymbol{H}^{*}\boldsymbol{U}_{\mathrm{Tx}})\right\}, \quad (24)$$

which is equivalent to a calculation via the full correlation matrix $\boldsymbol{R}_{\boldsymbol{H}}$ (Equation (3)):

$$\hat{\omega}_{n,m} = \left(\boldsymbol{u}_{\mathrm{Tx},m} \otimes \boldsymbol{u}_{\mathrm{Rx},n}\right)^{\mathrm{H}} \boldsymbol{R}_{\boldsymbol{H}} \left(\boldsymbol{u}_{\mathrm{Tx},m} \otimes \boldsymbol{u}_{\mathrm{Rx},n}\right). \quad (25)$$

By utilizing the predefined basis matrices $\boldsymbol{A}_{\mathrm{Rx}}$ and $\boldsymbol{A}_{\mathrm{Tx}}$ instead of the eigenbases, the coupling matrix $\boldsymbol{\Omega}_{\mathrm{virt}}$ of the 'virtual channel representation' is calculated analogously.

In Figure 1 we compare the three discussed models: the 'Kronecker' model, the 'virtual channel representation' (labeled as 'Sayeed'), and the new model. The figure shows the modeled mutual information vs. the measured mutual information for each of the 72 scenarios by means of a scatter plot. Each data point in the figure corresponds to a specific model and a specific scenario. The identity line (dashed) indicates the points of no modeling error. Obviously, the 'Kronecker' model systematically underestimates the mutual information, and the 'virtual channel representation' tends to a significant overestimation. The new model shows a rather good match between measured and modeled mutual information.

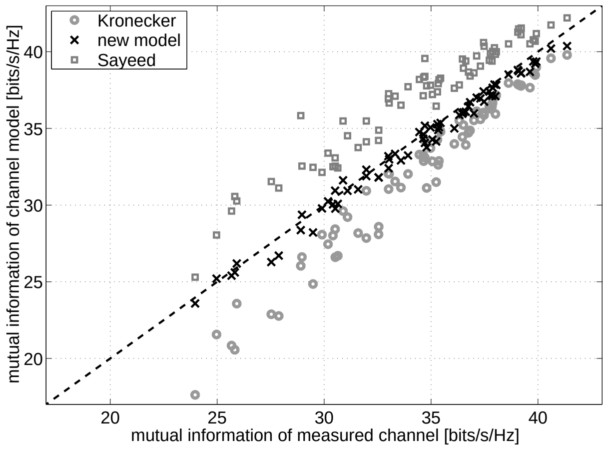

Figure 1: Mutual information, for each of the 72 measured scenarios, according to three different MIMO channel models vs. measured mutual information.

6 Conclusions

We presented a novel stochastic model for MIMO radio channels that is based on the joint correlation properties of both link ends. The necessary and sufficient condition for the proposed model to hold is that the eigenbasis at the receiver is independent of the transmit weights, and vice versa. The required model parameters are the eigenbasis at the receiver, the eigenbasis at the transmitter, and a coupling matrix which specifies how much energy is transported from each transmit eigenmode to each receive eigenmode on average. The number of elements in this coupling matrix is $M_{\mathrm{Rx}} \cdot M_{\mathrm{Tx}}$, as compared to $M_{\mathrm{Rx}} + M_{\mathrm{Tx}}$ receive and transmit eigenvalues of the 'Kronecker' model, which is not able of modeling general MIMO channels accurately. For MIMO channels whose correlation properties are truly separable into Rx and Tx components, the new model reduces to the 'Kronecker' model. By means of measured indoor MIMO channels, we have shown that the new model is capable of predicting the mutual information of real MIMO channels.

References

[1] M. Herdin, H. Özcelik, H. Hofstetter, and E. Bonek, "Variation of Measured Indoor MIMO Capacity with Receive Direction and Position at 5.2GHz," *Electronics Letters*, vol. 38, no. 21, pp. 1283–1285, Oct. 2002.

[2] J.P. Kermoal, L. Schumacher, K.I. Pedersen, P.E. Mogensen, and F. Frederiksen, "A Stochastic MIMO Radio Channel Model with Experimental Validation," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 6, pp. 1211–1226, Aug. 2002.

[3] D.P. McNamara, M.A. Beach, and P.N. Fletcher, "Spatial Correlation in Indoor MIMO Channels," in *IEEE International Symposium on Personal, Indoor and Mobile Radio Communications, 2002*, 2002, vol. 1, pp. 290–294.

[4] Da-Shan Shiu, G.J. Foschini, M.J. Gans, and J.M. Kahn, "Fading Correlation and Its Effect on the Capacity of Multielement Antenna Systems," *IEEE Transactions on Communications*, vol. 48, no. 3, pp. 502–513, Mar. 2000.

[5] D. Gesbert, H. Bölcskei, D.A. Gore, and A.J. Paulraj, "Outdoor MIMO Wireless Channels: Models and Performance Prediction," *IEEE Transactions on Communications*, vol. 50, no. 12, pp. 1926–1934, Dec. 2002.

[6] M.T. Ivrlac, W. Utschick, and J.A. Nossek, "Fading Correlations in Wireless MIMO Communication Systems," *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 5, pp. 819–828, June 2003.

[7] A.M. Sayeed, "Deconstructing Multiantenna Fading Channels," *IEEE Transactions on Signal Processing*, vol. 50, no. 10, pp. 2563–2579, Oct. 2002.