Adaptive Channel-Matched Extended Alamouti Space-Time Code Exploiting Partial Feedback

Biljana Badic, Markus Rupp, and Hans Weinrichter

Since the publication of Alamouti's famous space-time block code, various quasi-orthogonal space-time block codes (QSTBC) for multi-input multi-output (MIMO) fading channels for more than two transmit antennas have been proposed. It has been shown that these codes cannot achieve full diversity at full rate. In this paper, we present a simple feedback scheme for rich scattering (flat Rayleigh fading) MIMO channels that improves the coding gain and diversity of a QSTBC for 2^n ($n = 3, 4, \cdots$) transmit antennas. The relevant channel state information is sent back from the receiver to the transmitter quantized to one or two bits per code block. In this way, signal transmission with an improved coding gain and diversity near to the maximum diversity order is achieved. Such high diversity can be exploited with either a maximum-likelihood receiver or low-complexity zero-forcing receiver.

Keywords: Multi-input multi-output (MIMO), Alamouti code, feedback.

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I. Introduction

Since the work of Alamouti [1], several orthogonal and quasi-orthogonal space-time code designs have been investigated. The Alamouti code achieves diversity two with full data rate as it transmits two symbols in two time intervals. It has been shown in [2], that an orthogonal full-rate design, offering full diversity for any arbitrary complex symbol constellation, is limited to the case of two transmit antennas. Data-rate or decoding simplicity must be sacrificed if the number of transmit antennas is increased. To preserve the full rate at a small loss in complexity and performance, quasi-orthogonal designs have been proposed [3]-[6].

Space-time block codes (STBC) are assumed to work under rich scattering channel conditions. In contrast, line-of-sight conditions are typically more suited to beamforming methods, where apriori knowledge of the channel is required at the transmitter [7]. Most STBCs are designed under the assumption that the transmitter has no knowledge about the channel. On the other hand, it has been shown that an outage performance with perfect channel state information (CSI) available at the transmitter and at the receiver is better compared to the case when only the receiver has perfect knowledge of the channel. For instance, with complete CSI at the transmitter, data can be transmitted on the eigenvector related to the largest eigenvalue [8] providing the maximum transmit array gain.

Research on adapting the block code to partial channel knowledge has been an intensive area of research, with several design strategies coming up recently. Low coding complexity, high diversity and a better code rate of the orthogonal space-time codes can be obtained with only partial feedback of CSI to the transmitter [9], [10]. In [11] the transmit power varies dependent on the channel characteristics. In this way, the outage probability can be minimized if the receiver returns only roughly quantized amplitude information about the channel.

In [9], a simple and efficient extension of known orthogonal STBCs for more transmit antennas exploiting very small amounts of feedback from the receiver to the transmitter has been proposed, similar to the extended Alamouti STBCs (EA-STBCs) discussed in this paper. In [9], the extension of a simple Alamouti code to a full-rate STBC using four transmit antennas is worked out in detail. In this example, code orthogonality is preserved such that a simple matched filter receiver can be used for an optimal detection. Selecting one out of two code matrices according to one-bit feedback information per code block leads to full diversity and furthermore, even to some coding gain (array gain). However, the scheme also requires perfect synchronization of the transmitter and receiver based on the feedback. If such synchronization is erroneous or the feedback information is decoded incorrectly this concept easily looses diversity; in the case of four transmit and one receive antennas, the diversity drops from diversity four to diversity two.

In this paper, a simple and effective way to adapt a full-rate EA-STBC over 2^n transmit antennas to the actual channel is proposed achieving full diversity, nearly full orthogonality, and at the same time a low-bit error rate. However, there is not a single code with such performance capability but an entire family of codes derived from the first one by permutations of the transmit antennas and sign changes. A feedback system returning one or two bits per code block to the transmitter is presented. Depending on the feedback, the transmitter switches between two or four EA-STBCs members of this family. The receiver chooses the code that achieves the highest diversity and minimizes the channel-dependent interference parameter which is responsible for the non-orthogonality of the EA-STBC. With this simple scheme, a ZF (zero forcing) as well as an ML (maximum likelihood) receiver achieves nearly optimum diversity.

The paper is organized as follows. First, a short overview of the well-known Alamouti scheme is given, and the EA-STBC as one example of an efficient QSTBCs for four transmit antennas is defined. In section III, the generation of the feedback information is explained and the probability of the channel-dependent random variable responsible for the signal interference is derived. This random variable controls the feedback information. Simulation results are presented in section IV and conclusions are given in section V.

II. System Design Based on Extended Alamouti Schemes

1. Review of the 2×1 Alamouti Scheme

A very simple and effective coding scheme for two transmit antennas and a single receive antenna has been introduced by Alamouti [1]. Data block (s_1 ,s_2^*) is sent over the first antenna and block (s_2 , $-s_1^*$) over the second antenna, where * denotes complex conjugation. Assuming a flat-fading channel with transmission coefficients h_1 and h_2, the received vector $\mathbf{r}$ is formed by stacking two consecutive received data samples $\mathbf{r}=[r_1, r_2]^T$ in time, resulting in

$$\mathbf{r} = \mathbf{S}_{12}\mathbf{h} + \mathbf{v}, \tag{1}$$

where $\mathbf{h}=[h_1, h_2]^T$ is the complex channel vector and $\mathbf{v}$ is the noise vector at the receiver. Here, the symbol block $\mathbf{S}_{12}$ is defined as

$$\mathbf{S} = \begin{pmatrix} s_1 & s_2 \\ s_2^* & -s_1^* \end{pmatrix}. \tag{2}$$

Explicitly, (1) reads as

$$r_1 = s_1 h_1 + s_2 h_2 + v_1,$$
$$r_2 = s_2^* h_1 - s_1^* h_2 + v_2,$$

which is equivalent to

$$r_1 = h_1 s_1 + h_2 s_2 + v_1, \tag{3}$$
$$r_2^* = -h_2^* s_1 + h_1^* s_2 + v_2. \tag{4}$$

Vector equation (1) can thus be rewritten as

$$\begin{pmatrix} r_1 \\ r_2^* \end{pmatrix} = \begin{pmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2^* \end{pmatrix},$$

or in short notation

$$\mathbf{y} = \mathbf{H}_v \mathbf{s} + \mathbf{v}, \tag{5}$$

where vector $\mathbf{y} = [r_1, r_2^*]^T$ has been introduced. The resulting virtual (2×2) channel matrix $\mathbf{H}_v$ is orthogonal, i.e.,

$$\mathbf{H}_v^H \mathbf{H}_v = \mathbf{H}_v \mathbf{H}_v^H = h^2 \mathbf{I}_2.$$

$\mathbf{I}_2$ is the 2×2 identity matrix and h_2 is the power gain of the channel with $h^2 = |h_1|^2 + |h_2|^2$. Due to this orthogonality, the Alamouti scheme decouples the MISO channel into two virtually independent channels with channel gain h^2 and diversity $d = 2$.

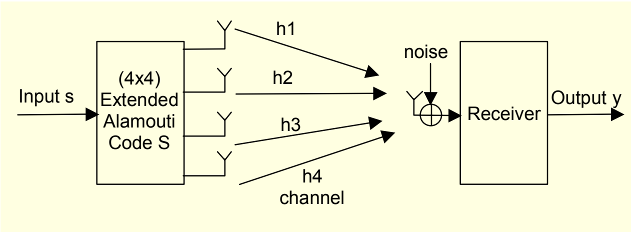

Fig. 1. Extended Alamouti scheme.

2. Extended Alamouti Scheme

Two Alamouti codes for two transmit antennas are used as building blocks to design the EA-STBC for four transmit antennas. The resulting EA-STBC extends over four time slots and is described by the following signal matrix [3], [12]:

$$\mathbf{S} = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \\ s_3^* & s_4^* & -s_1^* & -s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{pmatrix}. \qquad (6)$$

This EA-STBC results as an "alamoutisation"[1)] of basic (2×2) Alamouti codes:

$$\mathbf{S} = \begin{pmatrix} \mathbf{S}_1 & \mathbf{S}_2 \\ \mathbf{S}_2^* & -\mathbf{S}_1^* \end{pmatrix}. \qquad (7)$$

Assuming a single receive antenna, the received signals within four successive time slots are given as

$$\begin{aligned} r_1 &= s_1h_1 + s_2h_2 + s_3h_3 + s_4h_4 + \overline{v}_1 \\ r_2 &= s_2^*h_1 - s_1^*h_2 + s_4^*h_3 - s_3^*h_4 + \overline{v}_2 \\ r_3 &= s_3^*h_1 + s_4^*h_2 - s_1^*h_3 - s_2^*h_4 + \overline{v}_3 \\ r_4 &= s_4h_1 - s_3h_2 - s_2h_3 + s_1h_4 + \overline{v}_4. \end{aligned} \qquad (8)$$

With complex conjugation of the second and third equations cited above, we get

$$\begin{aligned} y_1 &= r_1, & v_1 &= \overline{v}_1 \\ y_2 &= r_2^*, & v_2 &= \overline{v}_2^* \\ y_3 &= r_3^*, & v_3 &= \overline{v}_3^* \\ y_4 &= r_4, & v_2 &= \overline{v}_4, \end{aligned}$$

resulting in the following equivalent (4×4) virtual MIMO transmission scheme with the matrix equation

$$\mathbf{y} = \mathbf{H}_v\mathbf{s} + \mathbf{v}, \qquad (9)$$

where

$$\mathbf{H}_v = \begin{pmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ -h_3^* & -h_4^* & h_1^* & h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{pmatrix} \qquad (10)$$

is a virtual (4×4) channel transmission matrix. In this way, a virtual, specifically structured MIMO channel with four transmit and four virtual receive antennas is obtained. In section III.1 ahead, it will be shown that $\mathbf{H}_v$ is "nearly orthogonal."

In this paper, we consider ZF as well as ML receivers. The particular structure of the proposed codes allows a low complexity solution for the ML receiver. More details about this can be found in [4]. However, if high order constellations are considered to be transmitted, a ZF receiver certainly exhibits lower complexity and is thus also of interest. We therefore compare the results of ML and ZF receivers in our simulations in section IV. Alternatively, an MMSE receiver can be applied as well. However, from a complexity point of view, it is very similar to the ZF receiver. Since in practical applications the noise variance is typically unknown and needs to be estimated, this type of receiver is not covered in this paper.

In a very similar way as explained in [6], "alamoutisation" can be applied iteratively to obtain larger QSTBCs for 2^n antennas with $n = 3, 4, \cdots$. The number of receive antennas can be selected freely and does not depend on the code.

In [4], transmit antenna numbers not being multiples of two are considered. While such systems are not discussed in this paper, the presented feedback approaches can be extended to them in a straight-forward manner.

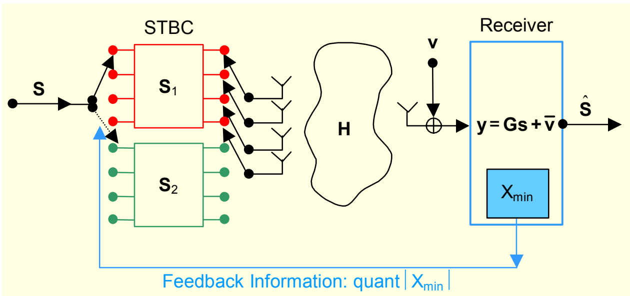

Fig. 2. Scheme with partial feedback.

III. Feedback Approaches

1. EA-STBC Scheme with One-Channel-Information Bit Per Code Block Returned to the Transmitter

The feedback scheme using one bit per code block sent from the receiver to the transmitter characterizing the channel will be

1) We use the term alamoutisation as introduced in [4] to indicate this particular method as being different from an orthogonalizing method like Gram-Schmidt.

explained first. The scheme is depicted in Fig. 2. Four transmit antennas, one receive antenna, and channel transfer vector $\mathbf{h}=[h_1, h_2, h_3, h_4]^T$ are considered. The channel transfer elements may fade in any arbitrary way but are assumed to be constant during the code block of length 4. The signal transmission can be described analogous to (1) by

$$\mathbf{r}=\mathbf{S}\mathbf{h}+\mathbf{v}, \tag{11}$$

where $\mathbf{r}$ is the (4×1) vector of the received signals from the code-block within four successive time slots. However, $\mathbf{S}$ is now either $\mathbf{S}_1$ as defined in (6) or $\mathbf{S}_2$ defined in (12) depending on the feedback bit b defined below, and $\mathbf{v}$ is the (4×1) noise vector with complex Gaussian components with zero mean and variance σ^2.

$$\mathbf{S}_2=\begin{pmatrix} -s_1 & s_2 & s_3 & s_4 \\ -s_2^* & -s_1^* & s_4^* & -s_3^* \\ -s_3^* & s_4^* & -s_1^* & -s_2^* \\ -s_4 & -s_3 & -s_2 & s_1 \end{pmatrix} \tag{12}$$

Obviously, $\mathbf{S}_1$ and $\mathbf{S}_2$ differ only in the sign of the transmitted symbols in the first column. Note that an entire family of EA-STBCs can be derived by sign changes and, alternatively, permutations of the transmit antenna order. All of the so obtained codes behave equivalently in terms of their near-orthogonality, their complexity and their BER performance for random channels. For a fixed channel, however, they behave differently, as will be shown in the following. As before, (11) can be rewritten in the form shown in (5) as

$$\mathbf{y}=\mathbf{H}_v\mathbf{s}+\mathbf{v},$$

with $\mathbf{s}=[s_1, s_2, s_3, s_4]^T$ and the virtual effective channel matrix $\mathbf{H}_v$ that is now equal to

$$\mathbf{H}_{v1}=\begin{pmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ -h_3^* & -h_4^* & h_1^* & h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{pmatrix}, \quad \text{if } \mathbf{S}=\mathbf{S}_1, \tag{13}$$

or

$$\mathbf{H}_{v2}=\begin{pmatrix} -h_1 & h_2 & h_3 & h_4 \\ -h_2^* & -h_1^* & -h_4^* & -h_3^* \\ -h_3^* & -h_4^* & -h_1^* & h_2^* \\ h_4 & -h_3 & -h_2 & -h_1 \end{pmatrix}, \quad \text{if } \mathbf{S}=\mathbf{S}_2. \tag{14}$$

In both cases we obtain [12]

$$\mathbf{G}_i=\mathbf{H}_{vi}^H\mathbf{H}_{vi}=\mathbf{H}_{vi}\mathbf{H}_{vi}^H = h^2\begin{bmatrix} 1 & 0 & 0 & X_i \\ 0 & 1 & -X_i & 0 \\ 0 & -X_i & 1 & 0 \\ X_i & 0 & 0 & 1 \end{bmatrix}, \quad \text{for } i=1,2 \tag{15}$$

with

$$h^2=|h_1|^2+|h_2|^2+|h_3|^2+|h_4|^2, \tag{16}$$

and

$$X_1=\frac{2\,\mathrm{Re}(h_1h_4^*-h_2h_3^*)}{h^2}, \quad \text{if } \mathbf{S}_1 \text{ is sent,} \tag{17}$$

and

$$X_2=\frac{2\,\mathrm{Re}(-h_1h_4^*-h_2h_3^*)}{h^2}, \quad \text{if } \mathbf{S}_2 \text{ is sent.} \tag{18}$$

It is well known that $\mathbf{G}$ should approximate a scaled identity-matrix as far as possible to achieve full diversity and an optimum BER performance [4]. If $\mathbf{G}$ is a scaled identity matrix, we have an orthogonal STBC and we could use a simple linear matrix multiplication of $\mathbf{y}$ by $\mathbf{H}_v^H$ (corresponding to a simple matched filter operation) at the receiver to decouple the channel perfectly and to get full diversity order $d=4$. Otherwise, X_i leads to a partial interference between h_1 and h_4 and between h_2 and h_3. This means X should be as small as possible. As $\mathbf{G}_i$ indicates, our scheme inherently supports full diversity $d=4$, if X_i can be made zero.

Therefore, our strategy is to transmit the code, $\mathbf{S}_1$ or $\mathbf{S}_2$, that minimizes $|X|$. By simply changing the sign of the first column of the EA-STBC we change the sign of the first term of the channel-dependent interference parameter X given in (17) and (18). In this way we obtain very small values of X due to the fact that in (17) and (18) two approximately equal valued terms are subtracted and thus at least partially compensate each other. In any case, some performance loss due to the non vanishing value of X is expected resulting from the residual interference between signal elements s_1 and s_4, or s_2 and s_3, respectively. As it is assumed that the receiver has full information of the channel, knowing h_1 to h_4, the receiver can compute X_1 and X_2 due to (17) and (18). With this information, the receiver returns the feedback bit b, informing the transmitter to select code block $\mathbf{S}_i$ ($i=1, 2$) which leads to the smaller value of X_i. With this information, the transmitter switches between EA-STBC $\mathbf{S}_1$ and $\mathbf{S}_2$ such that the resulting

$|X|$ will be min $(|X_1|,|X_2|)$. Obviously, the control information sent back to the transmitter only needs one feedback bit per code block. In our simulations it is assumed that the channel varies slowly such that the delay of the feedback information can be neglected.

2. Derivation of the PDF of Interference Parameter X

If the channel coefficients h_i are independent identically distributed (i.i.d.) complex Gaussian variables, then the probability density function of X is given in [12] as

$$f_X(x) = \begin{cases} \frac{3}{4}(1-x^2), \text{ if } |x|<1; \\ 0, \quad \text{else.} \end{cases} \tag{19}$$

To derive the probability density function of $\min(|X_1|,|X_2|)$ in the case of one feedback bit per code block, the PDFs of two random variables need to be considered. For this purpose, a new random variable is defined as

$$W = \begin{cases} X_1, \text{ if } |X_1|<|X_2|; \\ X_2, \text{ else.} \end{cases} \tag{20}$$

Due to its symmetry, only the one-sided (positive) PDF is considered:

$$\begin{aligned} f_W(w) &= f_{X1}(w) + f_{X2}(w) - f_{X1}(w)F_{X2}(w) - F_{X1}(w)f_{X2}(w) \\ &= 2f_{X1}(w)(1-F_{X1}(w)), \end{aligned} \tag{21}$$

where X_1, X_2 are assumed to be two statistically independent random variables. The final solution for $f_W(w)$ is

$$f_W(w) = \begin{cases} \frac{3}{4}(1-w^2)\left[1-\frac{3}{2}|w|(1-\frac{w^2}{3})\right], \text{if } |w|\le 1; \\ 0, \quad \text{else.} \end{cases} \tag{22}$$

Simulation results verifying this are presented in Fig. 3.

3. Two Bits Fed Back to the Transmitter

In a similar way as discussed in section III 1, we can switch between four different EA-STBCs at the transmitter. Let us discuss the case when we are allowed to send two bits b_1, b_2 as feedback information to the transmitter. Now, we let the transmitter switch between four very similar EA-STBCs, namely $\mathbf{S}_1$ and $\mathbf{S}_2$ defined in (6) and (12), and two new code matrices $\mathbf{S}_3$, and $\mathbf{S}_4$ defined as

$$\mathbf{S}_3 = \begin{pmatrix} js_1 & -js_2 & s_3 & s_4 \\ js_2^* & js_1^* & s_4^* & -s_3^* \\ js_3^* & -js_4^* & -s_1^* & -s_2^* \\ js_4 & js_3 & -s_2 & s_1 \end{pmatrix} \tag{23}$$

and

$$\mathbf{S}_4 = \begin{pmatrix} js_1 & js_2 & s_3 & s_4 \\ js_2^* & -js_1^* & s_4^* & -s_3^* \\ js_3^* & js_4^* & -s_1^* & -s_2^* \\ js_4 & -js_3 & -s_2 & s_1 \end{pmatrix}. \tag{24}$$

With $\mathbf{S}_3$ and $\mathbf{S}_4$, the corresponding channel matrix $\mathbf{H}_v$ is equal to:

$$\mathbf{H}_{v3} = \begin{pmatrix} jh_1 & -jh_2 & h_3 & h_4 \\ -jh_2^* & -jh_1^* & -h_4^* & h_3^* \\ -h_3^* & -h_4^* & -jh_1^* & jh_2^* \\ h_4 & -h_3 & jh_2 & jh_1 \end{pmatrix}, \text{ if } \mathbf{S}_3 \text{ is transmitted,} \tag{25}$$

and

$$\mathbf{H}_{v4} = \begin{pmatrix} jh_1 & jh_2 & h_3 & h_4 \\ jh_2^* & -jh_1^* & -h_4^* & h_3^* \\ -h_3^* & -h_4^* & -jh_1^* & -jh_2^* \\ h_4 & -h_3 & -jh_2 & jh_1 \end{pmatrix}, \text{ if } \mathbf{S}_4 \text{ is transmitted.} \tag{26}$$

The code matrices $\mathbf{S}_3$ and $\mathbf{S}_4$ are chosen in such a way that the resulting Grammian matrices $\mathbf{G}_3$ and $\mathbf{G}_4$ have the same quasi-orthogonal structure as in (15) with quite different values of the interference parameters X_3 and X_4. The resulting matrices $\mathbf{G}_3$ and $\mathbf{G}_4$ have exactly the same structure as $\mathbf{G}_1$ and $\mathbf{G}_2$ with exactly the same channel gain, h^2 (16). The channel-dependent interference parameter X in the case of $\mathbf{S}_3$ and $\mathbf{S}_4$ results now in

$$\begin{aligned} X_3 &= -\frac{2\,\mathrm{Im}(h_1h_4^* + h_2h_3^*)}{h^2}, \\ X_4 &= -\frac{2\,\mathrm{Im}(h_1h_4^* - h_2h_3^*)}{h^2}. \end{aligned} \tag{27}$$

Using two feedback bits, the transmitter can switch between four space-time block codes, $\mathbf{S}_i$, $i=1, 2, 3, 4$, to decrease further the influence of the interference parameter Z defined in (28) and to provide still higher diversity and a smaller bit-error ratio than in the case of relying only on two STBCs. The four EA-STBCs, $\mathbf{S}_1$ to $\mathbf{S}_4$, have been chosen in such a way that a code change can be implemented in a very simple way and that the resulting interference parameter Z gets as small as possible. In Fig. 3, the PDFs of the resulting interference parameters for all three cases: no feedback, only $\mathbf{S}_1$ used; one feedback bit, the transmitter can switch between $\mathbf{S}_1$ and $\mathbf{S}_2$; and 2 feedback bits, the transmitter can switch between $\mathbf{S}_1$ to $\mathbf{S}_4$. It turns out that the mean value of the modules of the resulting interference parameter can be reduced from 0.3 in the case of a single EA-STBC, to 0.2 if two EA-STBCs are available, and to 0.1 if four EA-STBCs are available at the transmitter.

In a similar way as described above, the feedback scheme can be applied to a family of EA-STBCs for 2^n transmit antennas with $n = 3, 4, \cdots$. More details about this generalization are reported in [6]. It turns out that this simple scheme of code switching can be applied to any QSTBC, e.g. the ABBA code [5]. This code has been investigated thoroughly in comparison to the EA-STBC, the results of which are documented in [15]. In brief, the EA-STBC and the ABBA code show the same BER performance in uncorrelated channels, but in highly correlated channels the ABBA gets dramatically worse, leading to an extremely poor performance. Applying our simple switching scheme to the ABBA code, the ABBA code shows the same performance as the EA-STBC even in highly correlated channels. In fact, applying our simple feedback scheme, the specific choice of QSTBC (or the family members) is not of further importance if we minimize the channel dependent interference parameters in this way.

4. Derivation of the PDF of the Resulting Interference Parameter in the Case of Switching between Four EA-STBCs

As explained in the last section, the main idea of our adaptive coding is to reduce the resulting interference parameter in order to improve the "quasi-orthogonality" of the virtual equivalent channel matrix. Therefore, we want to derive the corresponding probability density of the resulting random interference variable when switching between four EA-STBCs.

If we have n statistically independent, random variables X with the same PDF, $f_X(x)$, the density of the variable

$$Z = \begin{cases} X_1, & \text{if } |X_1| = \min(|X_1|,|X_2|,|X_3|,\cdots,|X_n|); \\ X_2, & \text{if } |X_2| = \min(|X_1|,|X_2|,|X_3|,\cdots,|X_n|); \\ X_3, & \text{if } |X_3| = \min(|X_1|,|X_2|,|X_3|,\cdots,|X_n|); \\ \vdots & \\ X_{n-1}, & \text{if } |X_{n-1}| = \min(|X_1|,|X_2|,|X_3|,\cdots,|X_n|); \\ X_n, & \text{else}. \end{cases} \tag{28}$$

is given by [13]

$$f_Z(z) = n[1 - F_X(z)]^{n-1} f_x(z). \tag{29}$$

With (19) and $n=4$, we get the PDF of the interference parameter, Z, as follows:

$$f_Z(z) = \begin{cases} \frac{3}{2}(1-z^2)\left[1-\frac{3}{2}|z|(1-\frac{z^2}{3})\right]^3, & \text{if } |z| \le 1; \\ 0, & \text{else}. \end{cases} \tag{30}$$

The one-sided PDFs of the three random variables, X, W and Z, are shown in Fig. 3. A comparison with Monte-Carlo simulation results, also shown in Fig. 3, exhibits excellent agreement between the simulation results and the analytical formulas given in (19), (22) and (30).

The mean absolute values of the resulting interference parameters W, with E[|W|] = 0.2, and Z, with E[|Z|] = 0.1, are substantially smaller compared to E[|X|] = 0.3, when the same code is always used.

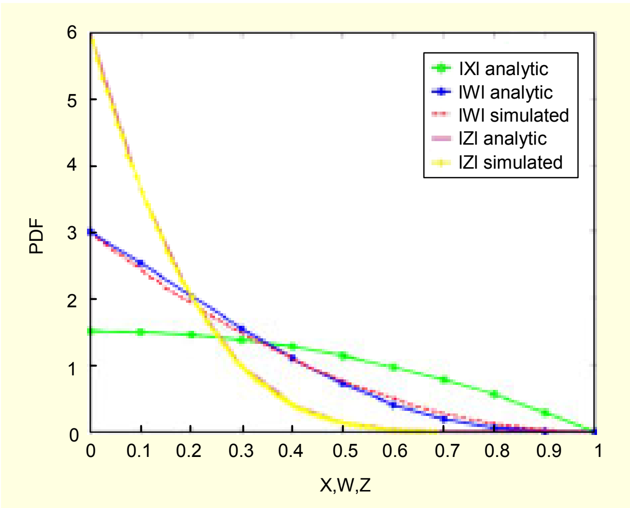

Fig. 3. One-sided PDF of the interference parameters X, W, and Z.

IV. BER Simulation Results

In our simulations, we have used a QPSK constellation. A flat Rayleigh fading channel remaining constant during the transmission of each code block has been assumed. At the receiver side, we have used ZF and ML receivers. The BER results have been averaged over 2,048 QPSK information symbols and 10^4 realizations of an i.i.d. channel matrix. We simulated MIMO systems with four and eight transmit antennas and a single receive antenna.

Figure 4 shows the resulting BER as a function of E_b/N_0 for the ZF receiver and Fig. 5 shows the results for the ML

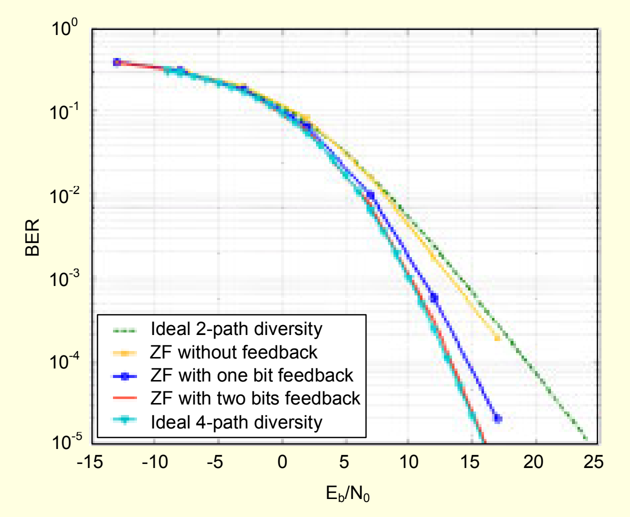

Fig. 4. BER for 4×1 extended Alamouti scheme with feedback, applying ZF receiver.

receiver in the case of four transmit antennas. Figure 6 presents the simulation results for MIMO channels with eight transmit antennas and one receiver antenna. The resulting curves are compared with ideal two, ideal four and ideal eight path diversities (interference parameter equal to zero).

Obviously, a substantial improvement of the BER can be achieved by providing only one or two feedback bits per code block enabling the transmitter to switch between two or four predefined code matrices. Note that there is only a small difference between the ZF receiver and the ML receiver performances due the reduced interference parameter X or the small amount of "non-orthogonality", respectively.

The ideal diversity curves are simulated using (2.1) in [4]:

$$\text{BER} = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{2h}{4\sigma^2}}\right)$$

for 10^6 realizations of the i.i.d. channel matrix.

In [16] and [17], where our simulation have been based on correlated MIMO channels and indoor measured MIMO channels, we have shown that QSTBCs with our simple feedback scheme are robust against channel variations, and that they perform very well even on highly correlated channels.

V. Conclusion

In this paper, a set of very simple EA-STBCs is presented, which can be used in combination with limited channel information sent back from the receiver to the transmitter. The transmitter switches between several EA-STBCs, governed by the partial channel knowledge, in order to approximate an interference-free redundant data transmission as far as possible. We have shown that this simple transmission scheme with one or two feedback bits per code block used to adapt the transmission code to the channel improves diversity and the bit-error ratio over the whole SNR range compared to the case of an open loop system without using feedback. Even with only one bit per code block fed back, the resulting system achieves a diversity which is near to the maximum.

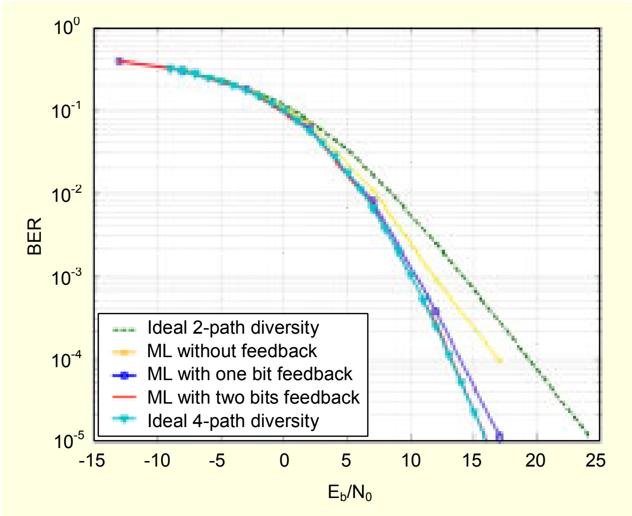

Fig. 5. BER for 4×1 extended Alamouti scheme with feedback, applying ML receiver.

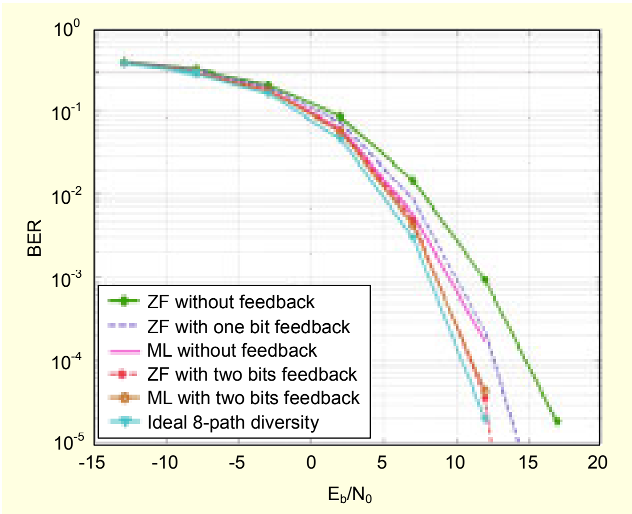

Fig. 6. BER for an 8×1 extended Alamouti scheme with feedback .

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