

Generalized Alamouti Codes for Trading Quality of Service against Data Rate in MIMO UMTS

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New space-time block coding schemes for multiple transmit and receive antennas are proposed. First, the well-known Alamouti scheme is extended to $N_T = 2^m$ transmit antennas achieving high transmit diversity. Many receiver details are worked out for four and eight transmit antennas. Further, solutions for arbitrary, even numbers ($N_T = 2k$) of transmit antennas are presented achieving decoding advantages due to orthogonalization properties while preserving high diversity. In a final step, such extended Alamouti and BLAST schemes are combined, offering a continuous trade-off between quality of service (QoS) and data rate. Due to the simplicity of the coding schemes, they are very well suited to operate under UMTS with only very moderate modifications in the existing standard. The number of supported antennas at transmitter alone is a sufficient knowledge to select the most appropriate scheme. While the proposed schemes are motivated by utilization in UMTS, they are not restricted to this standard.

Keywords and phrases: mobile communications, space-time block codes, spatial multiplexing.

1. INTRODUCTION

One of the salient features of UMTS is the provisioning of moderately high data rates for packet switched data services. In order to maximize the number of satisfied users, an efficient resource assignment to the subscribers is desired allowing flexible sharing of the radio resources. Such schemes must address the extreme variations of the link quality. Standardization of UMTS is progressing steadily, and various schemes for transmit diversity [1] and high-speed downlink packet access (HSDPA) with multiple transmit and receive antennas (MIMO) schemes [2] are currently under debate within the Third Generation Partnership Project (<http://www.3gpp.org/>).

Recently, much attention has been paid to wireless MIMO systems, (cf. [3, 4, 5]). In [6, 7], it was shown that the wireless MIMO channel potentially has a much higher capacity than was anticipated previously. In [8, 9, 10], space-time coding (STC) schemes were proposed that efficiently utilize such channels. Alamouti [11] introduced a very simple scheme allowing transmissions from two antennas with the same data rate as on a single antenna but increasing the diversity at the receiver from one to two in a flat-fading channel. While the scheme works for BPSK even with four

and eight antennas, it was proven that for QPSK, only the two-transmit-antenna scheme offers the full diversity gain [8, 12].

In order to evaluate the (single-) symbol error probability for a random channel \mathbf{H} with N_T statistically independent transmission paths with zero-mean channel coefficients h_k ($k = 1, \dots, N_T$) of equal variance,¹ known results from literature for maximum likelihood (ML) decoding of uncoded QPSK (with gray-code labelling) can be employed [13]:

$$\begin{aligned} \text{BER}_{\text{ML}} &= \frac{1}{2} E_{\mathbf{H}} \left[\text{erfc} \left(\sqrt{\frac{E_b}{2N_0} \sum_{k=1}^{N_T} |h_k|^2} \right) \right] \\ &= \frac{1}{2} E_{\alpha_{\text{ML}}} \left[\text{erfc} \left(\sqrt{\frac{\alpha_{\text{ML}}}{\sigma_V^2}} \right) \right]. \end{aligned} \quad (1)$$

Here the fading factor α_{ML} is introduced as a random variable with $\chi_{2N_T}^2$ density, the index indicating $2N_T$ degrees of freedom, that is, a diversity order of N_T . In case of independent complex Gaussian distributed variables h_k , the follow-

¹We normalize $\sum_{k=1}^{N_T} E[|h_k|^2] = 1$.

ing explicit result for QPSK modulation is obtained according to [13, Section 14.4, equations (15)]:

$$\text{BER}_{\text{ML}} = \frac{1}{2} \int_0^{\infty} \text{erfc} \left(\sqrt{x \frac{E_b}{2N_0}} \right) \frac{x^{N_T-1}}{\Gamma(N_T)} e^{-x} dx \quad (2)$$

$$= \left[\frac{1-\mu}{2} \right]^{N_T} \sum_{k=0}^{N_T-1} \binom{N_T-1+k}{k} \left(\frac{1+\mu}{2} \right)^k,$$

$$\mu = \sqrt{\frac{E_b/N_0}{N_T + E_b/N_0}}. \quad (3)$$

In contrast to this behavior, the performance for a linear zero-forcing (ZF) receiver is different. The bit error rate (BER) for a ZF receiver with N_T transmit and N_R receive antennas is given by [14]

$$\text{BER}_{\text{ZF}} = \frac{1}{2} E_{\alpha_{\text{ZF}}} \left[\text{erfc} \left(\sqrt{\frac{\alpha_{\text{ZF}}}{\sigma_v^2}} \right) \right], \quad (4)$$

with α_{ZF} being χ^2 -distributed with $2(N_T - N_R + 1)$ degrees of freedom rather than $2N_T$. A good overview of the various single symbol error performances is given in [15] and some early results on multiple symbol errors in [16]. The proposed coding schemes of this paper will be compared with these results for uncoded transmissions. In particular, selecting space-time codes will result in different degrees of freedom for the resulting fading factor α when compared to (1) and (4).

The paper is composed as follows. In Section 2, the well-known Alamouti scheme is introduced setting the notation for the remaining of the paper. In Section 3, the Alamouti space-time codes for transmission diversity is extended recursively to $M = 2^m$ antenna elements at the transmitter. While it is well known that the resulting transmission matrix for flat-fading loses its orthogonality for $m \geq 2$, it is shown that the loss in orthogonality for the new schemes is not severe when utilizing gray-coded QPSK modulation. Starting with a four-antenna scheme in Section 3, it will be demonstrated that linear receivers perform close to the theoretical bound for four-path diversity offering significant gain over the two-antenna case proposed by Alamouti. Even more interestingly, linear interference suppression can be implemented at low-complexity because the channel matrix exhibits a high degree of structure, enabling factorization in closed-form. In Section 4, this observation is generalized to extended Alamouti schemes for an arbitrary number of transmit antennas $N_T = 2^m$ preserving as much orthogonality as possible. In particular, results will be presented for the case $N_T = 8$. Transmission schemes with more than one receive antenna will be considered in Section 5 and it will be shown that even in cases with $N_T \neq 2^m$ transmit antennas, preservation of orthogonality is possible. Variable bit rate services and bursty packet arrivals are handled flexibly in UMTS by dynamically changing the spreading factor in conjunction with the transmit power, thus preserving an average E_b/N_0 , but without changing the diversity order and outage probability. A combination of BLAST and extended Alamouti schemes is proposed in Section 6 that makes use of the

existing diversity in a flexible manner, trading diversity gain against data rate and thus augmenting the diversity order and outage probability for fulfilling the quality of service (QoS) requirements. Not considered in this paper is the impact of the modulation scheme on the achieved diversity. It is well known that a certain rank criterion [8] needs to be satisfied in order to utilize full channel diversity in MIMO systems.

2. ALAMOUTI SCHEME

A very simple but effective scheme for two ($N_T = 2$) antennas achieving a diversity gain of two was introduced by Alamouti [8, 11]. It works by sending the sequence $\{s_1, s_2^*\}$ on the first antenna and $\{s_2, -s_1^*\}$ on the other. Assuming a flat-fading channel and denoting the two channel coefficients by h_1 and h_2 , the received vector \mathbf{r} is formed by stacking two consecutive data samples $[r_1, r_2]^T$ in time:

$$\mathbf{r} = \mathbf{S}\mathbf{h} + \bar{\mathbf{v}}. \quad (5)$$

Here, the symbol block \mathbf{S} and the channel vector \mathbf{h} are defined as follows:

$$\mathbf{S} = \begin{bmatrix} s_1 & s_2 \\ s_2^* & -s_1^* \end{bmatrix}, \quad \mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}. \quad (6)$$

This can be reformulated as

$$\begin{bmatrix} r_1 \\ r_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2^* \end{bmatrix} \quad (7)$$

or in short notation:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{v}, \quad (8)$$

where the vector $\mathbf{y} = [r_1, r_2^*]^T$ is introduced. The resulting channel matrix \mathbf{H} is orthogonal, that is, $\mathbf{H}^H\mathbf{H} = \mathbf{H}\mathbf{H}^H = h^2\mathbf{I}_2$, where the 2×2 identity matrix \mathbf{I}_2 as well as the gain of the channel $h^2 = |h_1|^2 + |h_2|^2$ are introduced. The transmitted symbols can be computed by the ZF approach

$$\hat{\mathbf{s}} = [\mathbf{H}^H\mathbf{H}]^{-1}\mathbf{H}^H\mathbf{y} = \frac{1}{h^2}\mathbf{H}^H\mathbf{y} = \mathbf{s} + [\mathbf{H}^H\mathbf{H}]^{-1}\mathbf{H}^H\mathbf{v}, \quad (9)$$

revealing a noise filtering. Note that due to the particular structure of \mathbf{H} , the two noise components are orthogonal. For a fixed channel matrix \mathbf{H} and complex-valued Gaussian noise \mathbf{v} , it can be concluded that they are both i.i.d. and thus are two decoupled noise components. The noise variance for each of the two symbols is given by $2\sigma_v^2/h^2$. Comparing to the optimal ML result for two-path diversity, the results are identical indicating that with a simple ZF receiver technique, the full two-path diversity of the transmission system can be obtained. Using complex-valued modulation, only for the two-antenna scheme such an improvement is possible. Only in the case of binary transmission, higher schemes with four and eight antennas exist [12]. In UMTS, QPSK is utilized on CDMA preventing perfectly orthogonal schemes with an improvement larger than a diversity of two.

3. FOUR-ANTENNA SCHEME

In UMTS with frequencies around 2 GHz, four or even eight antennas are quite possible at the base stations and two or four antennas at the mobile [17]. Since the number of antennas will vary among base stations and mobile devices, it is vital to design a flexible MIMO transmission scheme supporting various multielement antennas. As a minimum requirement, the mobile station might only be informed about the number of transmit antennas at the base station. Based on its own number of receive antennas, it can then decide which decoding algorithm to apply. Some codes offer complexity proportional to the number of receive antennas, for example, cyclic space-time codes [18]. Another example being Hadamard codes, retransmitting the symbols in a specific manner. For the case of four transmit antennas, the resulting matrix becomes $[s_1, s_2, s_3, s_4; s_1, s_2, -s_3, -s_4; s_1, -s_2, -s_3, s_4; -s_1, s_2, -s_3, s_4]$. In such schemes, the receiver can be built with very low-complexity, and higher diversity is achievable with more receiver antennas. However, by only utilizing multiple receiver antennas, the maximum possible diversity is not utilized in such systems unless transmit diversity is utilized as well.

In the following, simple block codes supporting much higher diversity in a four transmit antenna scheme for UMTS are proposed which do take advantage of additional transmit diversity.²

Proposition 1. Starting with the 2×2 -Alamouti scheme, the following recursive construction rule (similar to the construction of a complex Walsh-Hadamard code) is applied:

$$\begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix} \rightarrow \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ -h_3^* & -h_4^* & h_1^* & h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix}. \quad (10)$$

That is, the complex scalars h_1 and h_2 appearing to the left of the arrow “ \rightarrow ” are replaced by the 2×2 matrices

$$\begin{aligned} \mathbf{H}_1 &= \begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix}, \\ \mathbf{H}_2 &= \begin{bmatrix} h_3 & h_4 \\ -h_4^* & h_3^* \end{bmatrix}, \end{aligned} \quad (11)$$

and then reinserted into the Alamouti space-time channel matrix

$$\begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 \\ -\mathbf{H}_2^* & \mathbf{H}_1^* \end{bmatrix}, \quad (12)$$

where $*$ denotes complex conjugation without transposition.

This results in the following symbol block \mathbf{S} for transmitting the four symbols $\mathbf{s} = [s_1, \dots, s_4]^T$:

$$\mathbf{S} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \\ s_3^* & s_4^* & -s_1^* & -s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix}. \quad (13)$$

The received vector can be expressed in the same form as (5). Converting the received vector by complex conjugation

$$\begin{aligned} y_1 &= r_1, & v_1 &= \bar{v}_1, \\ y_2 &= r_2^*, & v_2 &= \bar{v}_2^*, \\ y_3 &= r_3^*, & v_3 &= \bar{v}_3^*, \\ y_4 &= r_4, & v_4 &= \bar{v}_4, \end{aligned} \quad (14)$$

results in the following equivalent transmission scheme:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{v}, \quad (15)$$

in which \mathbf{H} appears again as channel transmission matrix. If $\bar{\mathbf{v}}$ is a complex-valued Gaussian vector with i.i.d. elements, then so is \mathbf{v} .

3.1. ML receiver performance

While a standard ML approach is possible with correspondingly high complexity, an alternative ML approach applying matched filtering is first possible with much less complexity. After the matched filtering operation, the resulting matrix \mathbf{H}^H is

$$\mathbf{G} = \mathbf{H}^H\mathbf{H} = \mathbf{H}\mathbf{H}^H = h^2 \begin{bmatrix} \mathbf{I}_2 & X\mathbf{J}_2 \\ -X\mathbf{J}_2 & \mathbf{I}_2 \end{bmatrix}, \quad (16)$$

where the 2×2 matrix

$$\mathbf{J}_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad (17)$$

as well as the Gramian \mathbf{G} have been introduced. The gain of the channel is

$$h^2 = |h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2, \quad (18)$$

and the channel dependent real-valued random variable X is defined as follows:

$$X = \frac{2 \operatorname{Re}(h_1 h_4^* - h_2 h_3^*)}{h^2}. \quad (19)$$

By applying the matched filter \mathbf{H}^H , this results in the reception of the following vector:

$$\mathbf{z} = \mathbf{H}^H\mathbf{y} = \mathbf{H}^H\mathbf{H}\mathbf{s} + \mathbf{H}^H\mathbf{v} = h^2 \begin{bmatrix} s_1 + Xs_4 \\ s_2 - Xs_3 \\ s_3 - Xs_2 \\ s_4 + Xs_1 \end{bmatrix} + \mathbf{H}^H\mathbf{v} \quad (20)$$

²The outage capacity of this scheme was originally reported in [19].

in which the pair $\{s_1, s_4\}$ is decoupled from $\{s_2, s_3\}$ allowing for a low-complexity solution based on the newly formed receiver vector \mathbf{z} .

The ML decoder selects \mathbf{s} minimizing

$$\Lambda_1(\mathbf{s}) = \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 = \mathbf{s}^H \mathbf{G}\mathbf{s} - 2\text{Re}(\mathbf{y}^H \mathbf{H}\mathbf{s}) + \|\mathbf{y}\|^2 \quad (21)$$

for all permissible symbol vectors \mathbf{s} from the transmitter alphabet and spatially white interference plus noise was assumed. Alternatively, the matched filter can be applied to \mathbf{y} and the ML estimator can be implemented on its output \mathbf{z} given in (20) leading to

$$\Lambda_2(\mathbf{s}) = (\mathbf{z} - \mathbf{G}\mathbf{s})^H \mathbf{G}^{-1} (\mathbf{z} - \mathbf{G}\mathbf{s}). \quad (22)$$

Note that it needs to be taken into account that the noise plus interference is spatially correlated after filtering. Assuming the elements v_k of \mathbf{v} to be zero mean and spatially white with variance σ_v^2 results in $\mathbf{w} = \mathbf{H}^H \mathbf{v}$ with covariance matrix

$$\text{E}[\mathbf{w}\mathbf{w}^H] = \sigma_v^2 \mathbf{H}^H \mathbf{H} = \sigma_v^2 \mathbf{G}. \quad (23)$$

The advantage of this approach is that this partly decouples the symbols. The pair $\{s_1, s_4\}$ is decoupled from $\{s_2, s_3\}$ allowing for a low-complexity ML receiver using the partial metrics

$$\begin{aligned} \Lambda_{2a}(s_1, s_4) &= |z_1 - h^2(s_1 + Xs_4)|^2 + |z_4 - h^2(s_4 + Xs_1)|^2 \\ &\quad - 2X \text{Re} \{ [z_1 - h^2(s_1 + Xs_4)] [z_4^* - h^2(s_4^* + Xs_1^*)] \}, \\ \Lambda_{2b}(s_2, s_3) &= |z_2 - h^2(s_2 - Xs_3)|^2 + |z_3 - h^2(s_3 - Xs_2)|^2 \\ &\quad + 2X \text{Re} \{ [z_2 - h^2(s_2 - Xs_3)] [z_3^* - h^2(s_3^* - Xs_2^*)] \}. \end{aligned} \quad (24)$$

Note that the two metrics Λ_{2a} and Λ_{2b} are positive definite when $|X| < 1$. They become semidefinite for $|X| = 1$. In UMTS with QPSK modulation, this requires a search over 2×16 vector symbols rather than over 256.

3.2. Performance of linear receivers

Linear receivers typically suffer from noise enhancement. In this section, the increased noise caused by ZF and minimum mean squared error (MMSE) detectors is investigated. Both receivers can be described by the following detection principle:

$$\hat{\mathbf{s}} = (\mathbf{H}^H \mathbf{H} + \mu \mathbf{I}_4)^{-1} \mathbf{z}, \quad (25)$$

where $\mu = 0$ for ZF and $\mu = \sigma_v^2$ for MMSE. It turns out that both detection principles have essentially the same receiver complexity. The following lemmas can be stated.

Lemma 1. Given the 4×4 Alamouti scheme as described in (10), the eigenvalues of $\mathbf{H}^H \mathbf{H}/h^2$ are given by

$$\lambda_1 = \lambda_2 = 1 + X, \quad \lambda_3 = \lambda_4 = 1 - X, \quad (26)$$

where h^2 and X are defined in (18) and (19).

Proof. The Grammian $\mathbf{H}^H \mathbf{H}$ is diagonalized by $\mathbf{V}_4^T \mathbf{H}^H \mathbf{H} \mathbf{V}_4$ with the orthogonal matrix

$$\mathbf{V}_4 = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I}_2 & \mathbf{J}_2 \\ \mathbf{J}_2 & \mathbf{I}_2 \end{bmatrix}. \quad (27)$$

Some favorable properties are worth mentioning. The eigenvectors of $\mathbf{H}^H \mathbf{H}$ which are stacked in the columns of \mathbf{V}_4 do not depend on the channel; they are constant. The scaled matrix $\sqrt{2}\mathbf{V}_4$ is sparse, that is, half of its elements vanish and the nonzero entries are ± 1 .

Lemma 2. If the channel coefficients h_i ($i = 1, \dots, 4$) are i.i.d. complex Gaussian variates with zero mean and variance $1/4$, then the following properties hold:

- (1) X and h^2 are independent;
- (2) let λ_i be an eigenvalue of $\mathbf{H}^H \mathbf{H}/h^2$. The probability density of λ_i is $f_{\lambda_i}(\lambda) = (3/4)\lambda(2 - \lambda)$ for $0 < \lambda < 2$ and zero elsewhere. Likewise, $\lambda_i/2$ is beta(2,2)-distributed;
- (3) let ξ_i be an eigenvalue of $\mathbf{H}^H \mathbf{H}$. The probability density of ξ_i is $f_{\xi}(\xi) = 4\xi e^{-2\xi}$ for $\xi > 0$.

Proof. The joint distribution of X and h^2 is derived in Appendix A. The eigenvalues ξ_i of $\mathbf{H}^H \mathbf{H}$ and λ_i of $\mathbf{H}^H \mathbf{H}/h^2$ are proportional to each other, that is, $\xi_i = h^2 \lambda_i$ for $i = 1, \dots, 4$. \square

It can be concluded that $\text{E}[\lambda_i] = 1$ and $\text{Var}(\lambda_i) = 0.2$ for all i , indicating that the normalized channel matrix $\mathbf{H}^H \mathbf{H}/h^2$ is close to a unitary matrix with high probability.

Let $\gamma \geq 1$ be the following random variable which depends on the channel gain if $\mu > 0$:

$$\gamma = \frac{h^2 + \mu}{h^2} = \begin{cases} 1 & \text{for ZF,} \\ 1 + \frac{\sigma_v^2}{h^2} & \text{for MMSE.} \end{cases} \quad (28)$$

For evaluating the BER of the linear receiver for general $\mu \neq 0$,

$$\begin{aligned} &\text{tr} \left[(\mathbf{H}^H \mathbf{H} + \mu \mathbf{I}_4)^{-1} \mathbf{H}^H \mathbf{H} (\mathbf{H}^H \mathbf{H} + \mu \mathbf{I}_4)^{-1} \right] \\ &= \left(\frac{4}{h^2} \right) \frac{\gamma^2 + X^2(1 - 2\gamma)}{(\gamma^2 - X^2)^2} \end{aligned} \quad (29)$$

needs to be evaluated which is obtained via

$$[\mathbf{H}^H \mathbf{H} + \mu \mathbf{I}_4]^{-1} = \frac{1}{h^2(\gamma^2 - X^2)} \begin{pmatrix} \gamma \mathbf{I}_2 & -X \mathbf{J}_2 \\ X \mathbf{J}_2 & \gamma \mathbf{I}_2 \end{pmatrix}. \quad (30)$$

When replacing the arguments of the complementary error function with (29), two interpretations can be discussed.

Comparing the arguments of the complementary error function with the standard ML solution for multiple diversity, one recognizes the beneficial diversity term h^2 indicating four times diversity together with an additional term, say

$$\delta_4 \triangleq \frac{\gamma^2 + X^2(1 - 2\gamma)}{(\gamma^2 - X^2)^2} = \frac{1}{\gamma^2 - X^2} - 2(\gamma - 1) \frac{X^2}{(\gamma^2 - X^2)^2}. \quad (31)$$

In [Appendix A](#), it is shown that X and h^2 are statistically independent variates. Therefore, δ_4 can be interpreted as an increase in noise while h^2 causes full fourth-order diversity. Alternatively, one can interpret the whole expression $\alpha_{ZF,4} = h^2\delta_4$ as defining a new fading factor with the true diversity order without noise increase. Both interpretations can be used to describe the scheme's performance.

3.2.1. Noise enhancement

If the first interpretation is favoured, the following result is obtained.

Lemma 3. *Given the 4×4 Alamouti scheme in independent flat Rayleigh fading as described in (10), a four-times diversity is obtained at the expense of a noise enhancement of*

$$E[\delta_4] = \frac{3}{2} - 2\mu^2 + 2\mu e^{2\mu} E_1(2\mu)(2\mu^2 + \mu - 2), \quad (32)$$

where $E_n(x)$ denotes the exponential integral defined for $\text{Re}(x) > 0$ as follows:

$$E_n(x) \triangleq \int_1^\infty \frac{e^{-xt}}{t^n} dt. \quad (33)$$

Proof. The expectation $E[\delta_4]$ in (A.10) needs to be evaluated. Note that δ_4 depends on X and h^2 . It is shown in [Appendix A](#) that X and h^2 are independent if h_1, \dots, h_4 are i.i.d. complex-valued zero-mean Gaussian variates. Therefore, we can evaluate $E[\delta_4]$ via (A.11) which leads to the result (32). \square

In case of a ZF receiver, the noise is increased by a factor of $3/2$ which corresponds to 1.76 dB, a value for which the four-times diversity scheme gives much better results as long as E_b/N_0 is larger than about 3 dB. Therefore, the noise enhancement $E[\delta_4]$ is maximum for ZF receivers ($\mu = 0$) and it does not exceed 1.76 dB for MMSE. The formula

$$E[\delta_{N_T}] \triangleq \left(\frac{1}{N_T}\right)^2 E\left[\text{tr}\left((\mathbf{H}^H \mathbf{H})^{-1}\right) \text{tr}(\mathbf{H}^H \mathbf{H})\right] = 2 \frac{N_T - 1}{N_T} \quad (34)$$

seems to describe the noise enhancement for ZF receivers for the general case of N_T transmit antennas. Note that $\text{tr}(\mathbf{H}^H \mathbf{H})$ is the squared Frobenius norm of \mathbf{H} . The argument of the expectation operator is closely related to the numerical condition number κ of \mathbf{H} . Let ξ_{N_T} and ξ_1 be the largest and the smallest eigenvalue of $\mathbf{H}^H \mathbf{H}$, respectively. Then $\text{tr}((\mathbf{H}^H \mathbf{H})^{-1}) \text{tr}(\mathbf{H}^H \mathbf{H}) \geq \xi_{N_T}/\xi_1 = \kappa^2$. The noise enhancement can be lower bounded by the squared numerical condition number, that is, $E[\delta_{N_T}] \geq E[\kappa^2]$.

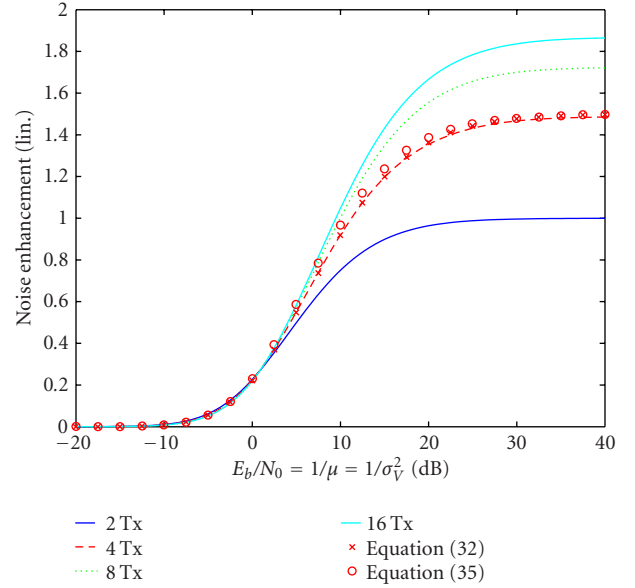


FIGURE 1: Comparison of the noise enhancement versus $E_b/N_0 = 1/\sigma_v^2$ for ZF and MMSE receivers.

The formula was explicitly validated for $N_T = 2, 4$, and 8 and with Monte Carlo simulations for larger values of N_T . Although no formal proof exists, the upper limit for the noise enhancement was found at 3 dB. The behavior of (32) versus $1/\mu$ (which equals E_b/N_0 for the MMSE) is shown in [Figure 1](#) indicated by crosses labeled “x.”

Additional insight into the behavior of (32) is gained by regarding the channel gain h^2 as approximately constant, an assumption that holds asymptotically true for $N_T \rightarrow \infty$. This assumption enables us to replace the joint expectation over X and γ in (32) by a conditional one, that is, conditioned on h^2 ,

$$E[\delta_4|h^2] = \frac{9}{2}\gamma - 3 + \left(\frac{9}{4}\gamma^2 - \frac{3}{2}\gamma - \frac{3}{4}\right) \log \frac{\gamma - 1}{\gamma + 1}. \quad (35)$$

This approximation is compared with the exact expression of (32) in [Figure 1](#) where the approximation obtained from (35) is plotted versus $E[1/(\gamma - 1)] = E_b/N_0$. The values are indicated by circles labeled “o.” The horizontal shift in E_b/N_0 between (32) and (35) is generally less than 1 dB. This approximation becomes exact for the case of ZF receiver where $\mu \rightarrow 0$, that is, the limit for $\gamma \rightarrow 1$ of (35) is $3/2$.

3.2.2. True diversity

The second interpretation of (29) leads to a refined diversity order. In this case, the term in γ and X purely modifies the diversity but leaves the noise part unchanged. The BER performance can be computed explicitly. We restrict ourselves to the ZF case for which $\gamma = 1$ and $\delta_4 = 1/[1 - X^2]$. In this case, δ_4 and h^2 are statistically independent. We obtain

$$\text{BER}_{\text{ZF}} = \int_h \int_\delta \text{erfc}\left(\frac{h^2 \delta}{2\sigma_v^2}\right) \frac{h^3 e^{-h}}{2\Gamma(4)} f_\delta(\delta) dh d\delta. \quad (36)$$

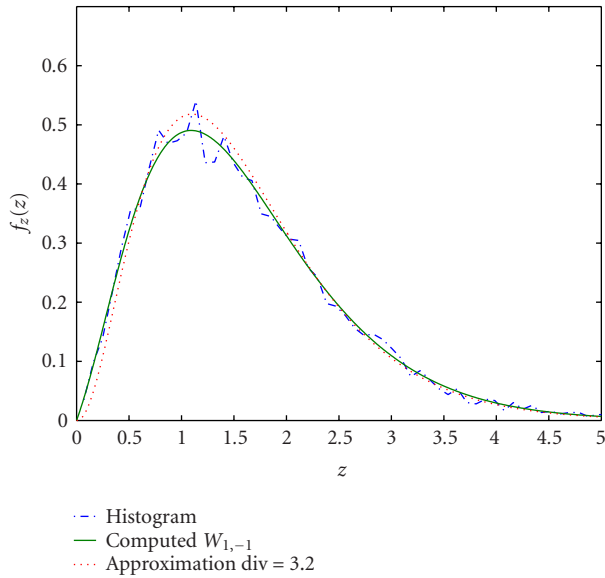


FIGURE 2: Histogram of a sample of z defined in (40) and its density $f_z(z)$ in (43).

Using the result from [13], (2) is obtained correspondingly, however, with a different solution for a random variable μ :

$$\mu(X) = \sqrt{\frac{E_b/N_0}{2(1-X^2) + E_b/N_0}}, \quad (37)$$

leading to rather involved terms. A much simpler method is to interpret the term $h^2\delta$ as a new fading factor $\alpha_{ZF,4}$ with χ -statistics. Since δ is a fractional number, the new factor $\alpha_{ZF,4} = h^2\delta$ cannot be expected to have an integer number of freedoms. Comparing with a Nakagami- m density, the mean value of $h^2\delta$ corresponds to the number of degrees of freedom m for this density. Computing $E[h^2\delta_4] = m = 3.2$ is obtained. Figure 2 displays a histogram of $\alpha_{ZF,4}$ from 5,000 runs. Furthermore, the exact density function is shown and a close fit obtained by the squared Nakagami- m distribution with $m = 3.2$, or equivalent χ^2 with 6.4 degrees of freedom. This result contradicts the general belief that ZF receivers obtain only $2(N_R - N_T + 1) = 2$ degrees of freedom. The result is different here due to the channel structuring.

An exact derivation of the probability density for this random variable is lengthy and is only sketched here. The random variable $(1 - X^2)h^2$ can be constructed from two independent variables $\mathbf{u}^H\mathbf{u}$ and $\mathbf{v}^H\mathbf{v}$ which are each χ^2 -distributed with four degrees of freedom (diversity order two). Substitute

$$\mathbf{x}^T = [h_1, h_2], \quad \mathbf{y}^T = [h_4, -h_3]. \quad (38)$$

Then $X = (\mathbf{x}^H\mathbf{y} + \mathbf{y}^H\mathbf{x})/(\mathbf{x}^H\mathbf{x} + \mathbf{y}^H\mathbf{y})$. Using $\mathbf{u} = [\mathbf{x} - \mathbf{y}]/\sqrt{2}$ and $\mathbf{v} = [\mathbf{x} + \mathbf{y}]/\sqrt{2}$, the following result is obtained:

$$(1 - X^2)h^2 = 4 \frac{\mathbf{u}^H\mathbf{u}\mathbf{v}^H\mathbf{v}}{\mathbf{u}^H\mathbf{u} + \mathbf{v}^H\mathbf{v}} = \frac{4}{1/\mathbf{u}^H\mathbf{u} + 1/\mathbf{v}^H\mathbf{v}}. \quad (39)$$

The joint density $p_{w,z}(w, z)$ of this expression can be computed via the transformation

$$z = \frac{1}{1/\mathbf{u}^H\mathbf{u} + 1/\mathbf{v}^H\mathbf{v}}, \quad w = \mathbf{v}^H\mathbf{v}, \quad (40)$$

achieving

$$p_{w,z}(w, z) = \frac{w^3 z}{(w - z)^3} \exp\left(-\frac{w^2}{w - z}\right). \quad (41)$$

The density of z is found by marginalizing the joint density $p(w, z)$. The density can be expressed using a Whittaker function (see [20]):

$$\begin{aligned} f_z(z) &= 2^9 z \int_u^\infty \frac{t^{3/2} \exp(-4t)}{\sqrt{t - z}} dt \\ &= 2^6 z^{3/2} \Gamma\left(\frac{1}{2}\right) \exp(-2z) W_{1,-1}(4z) \approx \frac{4^{3.2} z^{2.2} e^{4z}}{\Gamma(3.2)}. \end{aligned} \quad (42)$$

This last approximation is also shown in Figure 2, obviously a good fit.

3.3. Simulation results

Figure 3 displays the simulated behavior of the uncoded BER transmitting QPSK (gray coded) of the linear MMSE receiver and zero fading correlation between the four transmit paths. The BER results were averaged over 16,000 symbols and 3,200 selections of channel matrices \mathbf{H} for each simulated E_b/N_0 . For comparison, the BER from the ZF receiver and the cases of ideal two- and four-path diversity are also shown. The values marked by circles “o” labeled “expected theory” are the same as for four-path diversity, but shifted by the noise enhancement (n.e.) of 1.76 dB. Compared to the ZF receiver performance, there is just a little improvement for MMSE.

For practical considerations, it is of interest to investigate the performance when the four paths are correlated, as can be expected in a typical transmission environment. Figure 4 displays the situation when the antenna elements are correlated by a factor of $\{0.5, 0.75, 0.95\}$. As the figure reveals, no further loss is shown until the value exceeds 0.5. Only with very strong correlation (0.95), a degradation of 4 dB was noticed.

3.4. Diversity cumulating property of receive antennas

An interesting property is worth mentioning coming with the 4×1 extended Alamouti scheme when using more than one receive antenna. Typically adding more receive antennas gives rise to expect a higher diversity order in the transmission system, however, available only at the expense of more complexity in the receiver algorithms. In the extended Alamouti scheme, the behavior is slightly different as stated in the following lemma.

Lemma 4. When utilizing an arbitrary number N_R of receive antennas, the extended Alamouti scheme can obtain an N_R -fold

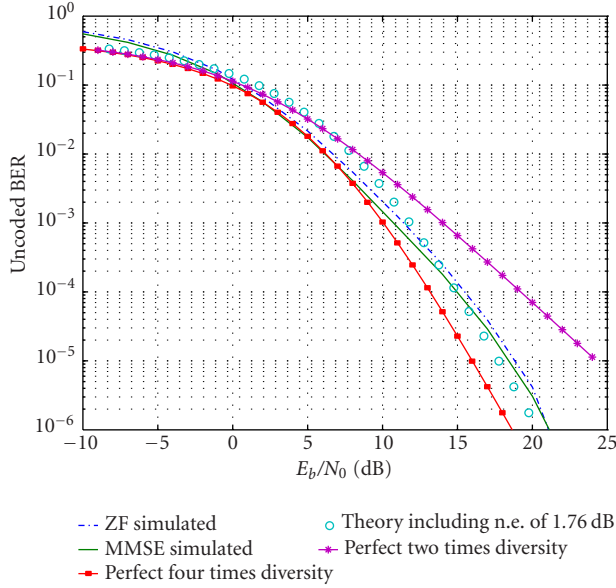


FIGURE 3: BER for four-antenna scheme with linear MMSE receiver and zero correlation between antennas.

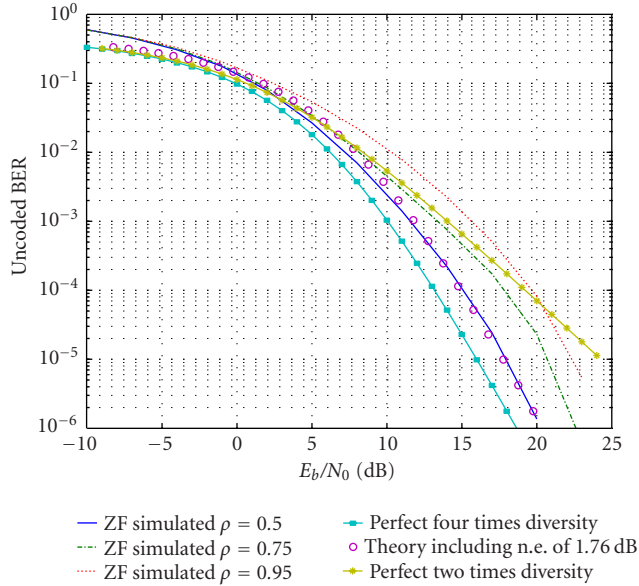


FIGURE 4: BER for four-antenna scheme with ZF receiver, fading correlation between adjacent antenna elements is $\{0.5, 0.75, 0.95\}$.

diversity compared to the single receive antenna case requiring only an asymptotically linear complexity $O(N_R)$ for ML as well as linear receivers.

Proof. The proof will be shown for two receive antennas. Extending it to more than two is a straight forward exercise:

$$\mathbf{r}_1 = \mathbf{H}_1 \mathbf{s} + \mathbf{v}_1; \quad \mathbf{r}_2 = \mathbf{H}_2 \mathbf{s} + \mathbf{v}_2. \quad (44)$$

Matched filtering can be applied and the corresponding

terms are summed up to obtain

$$\begin{aligned} \mathbf{H}_1^H \mathbf{r}_1 + \mathbf{H}_2^H \mathbf{r}_2 &= [\mathbf{H}_1^H \mathbf{H}_1 + \mathbf{H}_2^H \mathbf{H}_2] \mathbf{s} + \mathbf{H}_1^H \mathbf{v}_1 + \mathbf{H}_2^H \mathbf{v}_2 \\ &= [\mathbf{H}_1^H \mathbf{H}_1 + \mathbf{H}_2^H \mathbf{H}_2] \mathbf{s} + \tilde{\mathbf{v}}. \end{aligned} \quad (45)$$

Note that the new matrix $[\mathbf{H}_1^H \mathbf{H}_1 + \mathbf{H}_2^H \mathbf{H}_2]$ preserves the form (16):

$$\mathbf{H}_1^H \mathbf{H}_1 + \mathbf{H}_2^H \mathbf{H}_2 = (h_1^2 + h_2^2) \begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & -X & 0 \\ 0 & -X & 1 & 0 \\ X & 0 & 0 & 1 \end{bmatrix}, \quad (46)$$

with $X = [X_1 h_1^2 + X_2 h_2^2] / [h_1^2 + h_2^2]$. Thus, the matrix maintains its form and therefore, complexity of ML or a linear receiver remains identical to the one antenna case. Only the matched filtering needs to be performed additionally for as many receive antennas are present. The leading term $h_1^2 + h_2^2$ describes the diversity order, being twice as high as before. For N_R receiver antennas, a sum of all terms h_k^2 , $k = 1, \dots, N_R$, will appear in this position indicating an N_R -fold increase in capacity. \square

Note that N_R receiver antennas can be purely virtual and do not necessarily require a larger RF front end effort. For example, UMTS's WCDMA scheme enables RAKE techniques to be utilized. Thus, at tap delays τ_k where large energies occur, a finger of the RAKE receiver is positioned. Correspondingly, the channel matrix \mathbf{H} consists in this case of several components, all located at K different delay times. The received values can be structured in one vector as well and $\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{v}$ is obtained again, however now with \mathbf{y} is of dimension $4K \times 1$ and \mathbf{H} of dimension $4K \times 4$, while \mathbf{s} remains of dimension 4×1 as before. The previously discussed schemes can be applied as well and each term h^2 now consists of K times as many components as before, thus increasing diversity by a factor of K . In conclusion, such techniques work as well in a scenario with interchip interference as in flat Rayleigh fading with the additional benefit of having even more diversity and thus a better QoS, provided the cross-correlation between different users remains limited.

4. EIGHT AND MORE ANTENNA SCHEMES

Applying (10) several times ($m - 1$ times), solutions for $N_T = 2^m \times 1$ antenna schemes can be obtained. The obtained matrices exhibit certain properties that will be utilized in the following. They are listed in the following lemma and proven in Appendix B.

Lemma 5. Applying rule (10) $m - 1$ times results in matrices \mathbf{H} of dimension $N_T \times N_T$, $N_T = 2^m$, with the following properties:

- (1) all entries of $\mathbf{H}^H \mathbf{H}$ are real-valued;
- (2) the matrix $\mathbf{H}^H \mathbf{H}$ is of the form

$$\mathbf{H}^H \mathbf{H} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ -\mathbf{B} & \mathbf{A} \end{bmatrix} \quad (47)$$

and the inverse of $\mathbf{H}^H \mathbf{H}$ is of block matrix form

$$[\mathbf{H}^H \mathbf{H}]^{-1} = \begin{bmatrix} \mathbf{A} & -\mathbf{B} \\ \mathbf{B} & \mathbf{A} \end{bmatrix} \begin{bmatrix} (\mathbf{A}^2 + \mathbf{B}^2)^{-1} & \emptyset \\ \emptyset & (\mathbf{A}^2 + \mathbf{B}^2)^{-1} \end{bmatrix}. \quad (48)$$

Due to the form (47), all eigenvalues are double;³

(3) each nondiagonal entry X_i of $\mathbf{H}^H \mathbf{H}/\text{tr}[\mathbf{H}^H \mathbf{H}]$ is either zero, or X_i follows the distribution

$$f_X(\xi) = \frac{1}{2^{N_T-2} \text{B}(N_T/2, N_T/2)} (1 - \xi^2)^{N_T/2-1}, \quad |\xi| \leq 1. \quad (49)$$

Applying rule (10) two times in succession results in the 8×8 scheme. It can immediately be verified that the matrix $\mathbf{H}^H \mathbf{H}$ is given by

$$\mathbf{H}^H \mathbf{H} = h^2 \begin{bmatrix} \mathbf{I}_2 & X\mathbf{J}_2 & -Z\mathbf{J}_2 & Y\mathbf{I}_2 \\ -X\mathbf{J}_2 & \mathbf{I}_2 & -Y\mathbf{I}_2 & -Z\mathbf{J}_2 \\ Z\mathbf{J}_2 & -Y\mathbf{I}_2 & \mathbf{I}_2 & X\mathbf{J}_2 \\ Y\mathbf{I}_2 & Z\mathbf{J}_2 & -X\mathbf{J}_2 & \mathbf{I}_2 \end{bmatrix}, \quad (50)$$

with

$$\begin{aligned} h^2 &= \sum_{k=1}^8 |h_k|^2, \\ X &= \frac{2 \text{Re}(h_1 h_4^* - h_2 h_3^* + h_5 h_8^* - h_6 h_7^*)}{h^2}, \\ Y &= \frac{2 \text{Re}(h_1 h_7^* - h_3 h_5^* + h_2 h_8^* - h_4 h_6^*)}{h^2}, \\ Z &= \frac{2 \text{Re}(h_2 h_5^* - h_1 h_6^* + h_4 h_7^* - h_3 h_8^*)}{h^2}. \end{aligned} \quad (51)$$

According to property (2), the block structure of this matrix can be recognized. Note that $\mathbf{A}^2 + \mathbf{B}^2 = \alpha \mathbf{I}_4 + \beta \mathbf{J}_4$, with

$$\begin{aligned} \mathbf{J}_4 &= \begin{bmatrix} \emptyset & \mathbf{J}_2 \\ -\mathbf{J}_2 & \emptyset \end{bmatrix}, \\ \alpha &= X^2 - Y^2 - Z^2 + 1, \quad \beta = 2(X - YZ), \end{aligned} \quad (52)$$

and the inverse can also be expressed by a combination of \mathbf{I}_4 and \mathbf{J}_4 :

$$[\mathbf{A}^2 + \mathbf{B}^2]^{-1} = \frac{1}{\alpha^2 - \beta^2} (\alpha \mathbf{I}_4 - \beta \mathbf{J}_4) \quad (53)$$

if $|\alpha| \neq |\beta|$ which enables a computationally efficient implementation.

The ML receiver decouples into two 4×4 schemes by exploiting the structure of these matrices, (cf. Section 3.1). For UMTS with QPSK modulation, this leads to a search over 2×256 vector symbols rather than $4^8 = 65\,536$.

³The proof of the latter statement is simple: if an eigenvector $[\mathbf{x}, \mathbf{y}]$ exists for an eigenvalue λ , then also $[\mathbf{y}, -\mathbf{x}]$ must be an eigenvector, linear independent of the first one, and thus the eigenvalues must be double.

4.1. Performance of linear receivers

Proceeding analogously to Section 3.2, the noise enhancement $E[\delta_8]$ for the eight-antenna scheme is governed by $\text{tr}[(\mathbf{H}^H \mathbf{H} + \mu \mathbf{I}_8)^{-1} \mathbf{H}^H \mathbf{H} (\mathbf{H}^H \mathbf{H} + \mu \mathbf{I}_8)^{-1}] = 8\delta_8/h^2$, where $\gamma = 1 + \mu/h^2$ and

$$\delta_8 \triangleq \frac{1}{8} \sum_{i=1}^8 \frac{\lambda_i}{(\gamma + \lambda_i - 1)^2}. \quad (54)$$

Lemma 6. All eigenvalues λ_i of $\mathbf{H}^H \mathbf{H}/h^2$ in (50) are given by

$$\begin{aligned} \lambda_1 &= \lambda_2 = (1 - X) + (Y - Z), \\ \lambda_3 &= \lambda_4 = (1 + X) - (Y + Z), \\ \lambda_5 &= \lambda_6 = (1 + X) + (Y + Z), \\ \lambda_7 &= \lambda_8 = (1 - X) - (Y - Z). \end{aligned} \quad (55)$$

Proof. The Grammian $\mathbf{H}^H \mathbf{H}$ is diagonalized by $\mathbf{V}_8^T \mathbf{H}^H \mathbf{H} \mathbf{V}_8$ with the orthogonal matrix

$$\mathbf{V}_8 = \frac{1}{2} \begin{bmatrix} \mathbf{I}_2 & \mathbf{J}_2 & \mathbf{J}_2 & \mathbf{I}_2 \\ \mathbf{J}_2 & \mathbf{I}_2 & -\mathbf{I}_2 & -\mathbf{J}_2 \\ \mathbf{J}_2 & \mathbf{I}_2 & \mathbf{I}_2 & \mathbf{J}_2 \\ -\mathbf{I}_2 & -\mathbf{J}_2 & \mathbf{J}_2 & \mathbf{I}_2 \end{bmatrix} \quad (56)$$

resulting in the above given eigenvalues. \square

Lemma 7. If the channel coefficients h_i ($i = 1, \dots, 8$) are i.i.d. complex-valued Gaussian variates with zero mean and variance $1/8$, then the following properties hold:

- (1) let λ_i be an eigenvalue of $\mathbf{H}^H \mathbf{H}/h^2$. The probability density of λ_i is $f_{\lambda,8}(\lambda) = (21/8192)\lambda(4 - \lambda)^5$ for $0 < \lambda < 4$ and zero elsewhere. Likewise, $\lambda_i/4$ is beta(2,6)-distributed;
- (2) let ξ_i be an eigenvalue of $\mathbf{H}^H \mathbf{H}$. The probability density of ξ_i is $f_{\xi}(\xi) = 4\xi e^{-2\xi}$ for $\xi > 0$ and zero elsewhere.

Proof. It is sufficient to give the proof for one eigenvalue, say λ_5 . The proof for the remaining eigenvalues proceeds similarly. By completing the squares (as in Appendix A), $h^2 \lambda_5/4$ can be regarded as the sum of two χ_n^2 -distributed variables with $n = 2$ degrees of freedom each, that is,

$$\left| \frac{h_1 + h_4 - h_6 + h_7}{2} \right|^2 + \left| \frac{h_2 - h_3 + h_5 + h_8}{2} \right|^2. \quad (57)$$

By introducing an orthogonal transformation via the matrix \mathbf{V}_8^T from (56), the proof is completed following the procedure in Appendices A and B. \square

The noise enhancement for the eight-antenna case and a ZF receiver ($\mu = 0$) is evaluated by using the eigenvalue statistics from Lemma 7:

$$E[\delta_8] = \int_0^4 \lambda^{-1} f_{\lambda,8}(\lambda) d\lambda = \frac{7}{4} = 1.75 \quad (58)$$

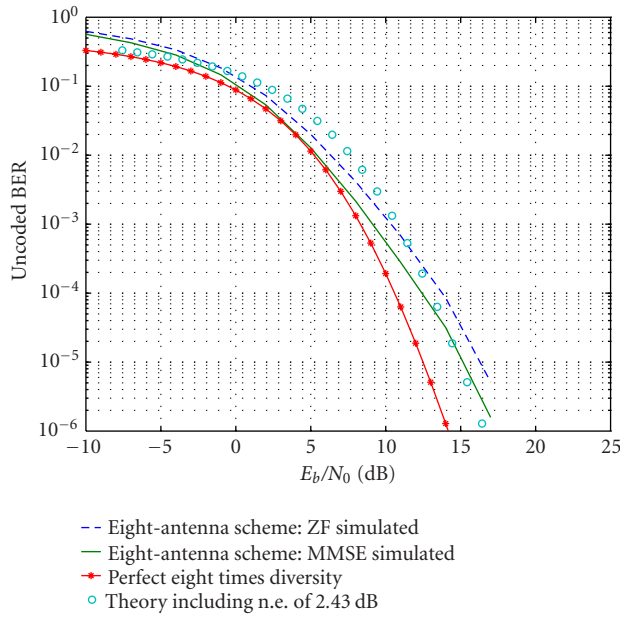


FIGURE 5: BER for eight-antenna scheme for ZF and MMSE receivers compared to theory.

or around 2.43 dB. The noise enhancement for the general linear receiver ($\mu \geq 0$) is obtained similarly to the four-antenna scheme; the result is

$$E[\delta_8] = \frac{7}{4} + 2\mu - \mu^2 + \mu e^{2\mu} E_1(2\mu)(2\mu^2 - 3\mu - 6). \quad (59)$$

Thus, the noise enhancement of the MMSE receiver is always smaller than 2.43 dB. Figure 1 compares the noise enhancement versus SNR for the ZF and MMSE receivers and for Alamouti's two-, and the proposed four-, and eight-antenna schemes. The noise enhancement for each scheme is calculated numerically by averaging over 4000 realizations of the channel matrix \mathbf{H} . For each realization, the eigenvalues λ_i of $\mathbf{H}^H \mathbf{H}$ are numerically computed and subsequently averaged over $(h^2/N_T) \sum_{i=1}^{N_T} \lambda_i / (\lambda_i + \mu)^2$, where $N_T = 2, 4, 8$, or 16. The resulting averaged curves are shown in Figure 1 labeled "2 Tx," "4 Tx," and so forth.

The theoretical values marked by small crosses, labeled "x," are calculated according to (32) versus $E_b/N_0 = 1/\sigma_V^2 = 1/\mu$ for the MMSE case. The values marked by small circles, labeled "o," are calculated according to the approximation in (35) versus $E_b/N_0 = E[1/(\gamma - 1)]$.

4.2. Simulation results

Figure 5 displays the simulated behavior of the uncoded BER for QPSK modulation and zero-fading correlation between the eight transmit paths. The BER results were averaged over 12,800 symbols and 4,000 selections of channel matrices \mathbf{H} for each simulated E_b/N_0 . The results are shown for a significance level of 99.7%. In other words, the scheme assumes a tolerated outage probability of 0.3%. Outage is assumed to occur if the numerical condition of $\mathbf{H}^H \mathbf{H}$ which is the ratio of the largest to the smallest eigenvalue exceeds $100 \approx 2^7$. In-

verting these rare but adverse (nearly singular) channel matrices $\mathbf{H}^H \mathbf{H}$ lead to the loss of at least seven bits of numerical accuracy in the receiver. The values marked by little circles "o" labeled "expected theory" are the same as for eight-path diversity, but shifted by the noise variance increase of 2.43 dB.

5. ALAMOUTIZATION

So far, mostly $N_T \times 1$ antenna schemes have been considered. However, in the future several antennas are likely to occur at the receiver as well. A cellular phone can carry two and a laptop as many as four antennas [17]. The proposed schemes can be applied, however, it remains unclear how to combine the received signals in an optimal fashion. In the following, an interesting approach is presented allowing an increase in diversity when the number of receiver antennas is more than one but typically less than the number of transmit antennas. The proposed STC schemes preserve a large part of the orthogonality so that the receivers can be implemented with low-complexity. The diversity is exploited in full and the noise enhancement remains small.

Proposition 2. Assume that a block matrix form of the channel matrix \mathbf{H} is given by

$$\mathbf{H} = [\mathbf{H}_1 \mathbf{H}_2], \quad (60)$$

where the matrices $\{\mathbf{H}_1, \mathbf{H}_2\}$ are not necessarily quadratic. Then, the scheme can be Alamouted by performing the following operation:

$$\mathbf{G} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 \\ -\mathbf{H}_2^* & \mathbf{H}_1^* \\ \mathbf{H}_2^* & \mathbf{H}_1^* \\ \mathbf{H}_1 & -\mathbf{H}_2 \end{bmatrix}. \quad (61)$$

At the receiver, a ZF operation is performed, obtaining the corresponding term $\mathbf{G}^H \mathbf{G}$ with the property

$$\mathbf{G}^H \mathbf{G} = 2 \begin{bmatrix} \mathbf{H}_1^H \mathbf{H}_1 + \mathbf{H}_2^H \mathbf{H}_2 & \emptyset \\ \emptyset & \mathbf{H}_1^T \mathbf{H}_1^* + \mathbf{H}_2^H \mathbf{H}_2 \end{bmatrix}. \quad (62)$$

Thus perfect orthogonality on the nondiagonal block entries is achieved indicating little noise enhancement while the diagonal block terms indicate high diversity values.⁴

Example 1. A two-transmit-two-receive antenna system is considered:

$$\mathbf{H}_1 = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}, \quad \mathbf{H}_2 = \begin{bmatrix} h_3 \\ h_4 \end{bmatrix}. \quad (63)$$

The matrix $\mathbf{G}^H \mathbf{G}$ becomes

$$\mathbf{G}^H \mathbf{G} = 2 \left(|h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2 \right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (64)$$

⁴This was proposed in [4] in a simpler form.

Thus, the full four times diversity can be explored, without a matrix inverse computation. Note that in this case, the transmit sequence at the two antennas reads

$$\left\{ \begin{matrix} s_1 & s_2 & -s_3^* & -s_4^* & s_3^* & s_4^* & s_1 & s_2 \\ s_3 & s_4 & s_1^* & s_2^* & s_1^* & s_2^* & -s_3 & -s_4 \end{matrix} \right\}. \quad (65)$$

Note also that during eight time periods, only four symbols are transmitted, that is, this particular scheme has the drawback of offering only half the symbol rate!

Example 2. Consider a 4×2 transmission scheme. The matrices are identified to

$$\mathbf{H}_1 = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}, \quad \mathbf{H}_2 = \begin{bmatrix} h_{13} & h_{14} \\ h_{23} & h_{24} \end{bmatrix}. \quad (66)$$

The matrix $\mathbf{G}^H \mathbf{G}$ consists of two block matrices of size 2×2 on the diagonal. Thus, the scheme is still rather simple since only a 2×2 matrix has to be inverted although a four-path diversity is achieved. A comparison of the noise enhancement shows that for this 4×2 antenna system, 3 dB is gained compared to the 4×1 antenna system. Note that now the data rate is at full speed!

Example 3. The previously discussed 4×1 antenna system can be obtained when setting

$$\mathbf{H}_1 = [h_1 \ h_2], \quad \mathbf{H}_2 = [h_3 \ h_4]. \quad (67)$$

The reader may also try schemes in which the number of receive antennas is not given by $N_R = 2^n$. As long as N_R is even, the scheme can be separated in two matrices \mathbf{H}_1 and \mathbf{H}_2 of same size allowing the Alamoutization rule (Proposition 2) to be applied.

6. COMBINING BLAST AND ALAMOUTI SCHEMES

Although the proposed extended Alamouti schemes allow for utilizing the channel diversity without sacrificing the receiver complexity, not much has been said on data rates yet. In the case of $N_T \times 1$ antenna schemes, the N_T symbols were repeated N_T times in a different and specific order guaranteeing a data rate of one. Thus, the data rates in the proposed schemes typically remain constant (equal to one) when the schemes are quadratic and can be lower when the receive antenna number is smaller than the transmit antennas as pointed out in the previous section. In BLAST transmissions, this is different. In its simplest form, the V-BLAST coding [21], N_T new symbols are offered to the N_T transmit antennas at every symbol time instant thus achieving data rates N_T times higher than in the Alamouti schemes. A combination of schemes can be achieved by simply transmitting more or less of the different repetitive transmissions. By utilizing the obtained transmission matrix structures, the diversity inherent in the transmission scheme can be exploited differently offering a trade-off between data rate and diversity order. In order to clarify this statement, an example is presented.

TABLE 1

Antenna	$n = 1$	$n = 2$
1	s_1	s_2^*
2	s_2	$-s_1^*$
3	s_3	s_4^*
4	s_4	$-s_3^*$

Example 4. A 4×2 antenna scheme is considered for transmission. In a flat-fading channel system, eight Rayleigh coefficients are available describing the transmissions from the four transmit to the two receive antennas, the transmission matrix being

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \end{bmatrix}. \quad (68)$$

It should thus be possible to transmit either four times the symbol data rate with diversity gain two, or two times the data rate with diversity four, or only at the symbol data rate but with diversity gain eight. In the first case, the 4×1 scheme as proposed in Section 3 will be used, repeating the four symbols four times, resulting in the reception of eight symbols. When assigning two paths each to one 2×2 matrix \mathbf{H}_i , $i = 1, \dots, 4$, the following transmission matrix is obtained:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 \\ -\mathbf{H}_2^* & \mathbf{H}_1^* \\ \mathbf{H}_3 & \mathbf{H}_4 \\ -\mathbf{H}_4^* & \mathbf{H}_3^* \end{bmatrix}. \quad (69)$$

Computing $\mathbf{H}^H \mathbf{H}$, a 4×4 matrix is obtained in a similar way to the 4×1 antenna case, however with twice the diversity. Thus in this case, a diversity of eight is achieved with a data rate of one.

On the other hand, by transmitting the sequences only twice, according to Table 1, the received signals at the two antennas can be formed to

$$\begin{bmatrix} y_{11} \\ y_{12} \\ y_{21} \\ y_{22} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ -h_{12}^* & h_{11}^* & -h_{14}^* & h_{13}^* \\ h_{21} & h_{22} & h_{23} & h_{24} \\ -h_{22}^* & h_{21}^* & -h_{24}^* & h_{23}^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} = \mathbf{H} \mathbf{s}. \quad (70)$$

Thus, computing $\mathbf{H}^H \mathbf{H}$ results simply in the following block matrix:

$$\mathbf{H}^H \mathbf{H} = \begin{bmatrix} \gamma_1 \mathbf{I} & \mathbf{B} \\ \mathbf{B}^H & \gamma_2 \mathbf{I} \end{bmatrix} \quad (71)$$

with $\gamma_1 = |h_{11}|^2 + |h_{12}|^2 + |h_{13}|^2 + |h_{14}|^2$ and $\gamma_2 = |h_{21}|^2 + |h_{22}|^2 + |h_{23}|^2 + |h_{24}|^2$. Due to the condition $\mathbf{B}^H \mathbf{B} = \mathbf{B} \mathbf{B}^H$, such matrices can be inverted with a 2×2 matrix inversion rather than a 4×4 :

$$[\mathbf{H}^H \mathbf{H}]^{-1} = \begin{bmatrix} \gamma_2 \mathbf{I} & -\mathbf{B} \\ -\mathbf{B}^H & \gamma_1 \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{C} & \emptyset \\ \emptyset & \mathbf{C} \end{bmatrix} \quad (72)$$

with $\mathbf{C} = [\gamma_1 \gamma_2 \mathbf{I} - \mathbf{B} \mathbf{B}^H]^{-1}$. Thus, the underlying Alamouti scheme gives us the advantage of lower complexity while the

BLAST scheme offers higher data rate. This specific scheme was investigated in [22, 23], where a diversity factor of six was found to closely match the diversity gain and the corresponding unitary matrices to diagonalize the scheme are presented.

Finally, the third transmission mode would send only one set of four symbols to the four transmit antennas. The corresponding matrix $\mathbf{H}^H \mathbf{H}$ is not of full rank and therefore, cannot be inverted. The entries on its diagonal consist of two times diversity terms like $|h_{11}|^2 + |h_{12}|^2$. The decoding can be performed either in MMSE mode or with an ML decoder [24] allowing only for diversity of two but with a data rate of four. Gaining such insight, the following conjecture can be made.

Conjecture 1. *Given a wireless communications system with N_T transmit and N_R receive antennas in a flat Rayleigh fading environment with maximum diversity $N_R N_T$ (see also [25] for definition), an Alamoutization scheme can be found with diversity order D and data rate R , if $D \in \mathbb{N}$ and $R \in \mathbb{N}$ approximately factorizing the maximum diversity, that is, $DR \approx N_T N_R$.*

Note that this statement was not formulated in terms of a lemma since it may not be exactly true in the sense that exactly a diversity of say eight is obtained when actually only 6.4 is achieved. It is thus to apply with some care. On non-flat-fading channels, the UMTS transmission allows the diversity to increase by assigning a number of fingers to each major energy contribution in the impulse response. In this case, all finger values are combined in a correspondingly larger matrix \mathbf{H} . However, $\mathbf{H}^H \mathbf{H}$ remains of the same size as before. The various fingers only contribute to higher diversity gain allowing to utilize BLAST schemes in which $\mathbf{H}^H \mathbf{H}$ would not be of full rank in a flat Rayleigh scenario.

7. CONCLUSION

In this paper, several extensions to the Alamouti space-time block code supporting very high transmit and receiver diversity have been proposed and their performance is evaluated. By combining conventional BLAST and new extended Alamouti schemes, a trade-off between diversity gain (and thus QoS) and supported data rate is offered allowing very high flexibility while the receiver complexity is kept approximately proportional to the transmitted data rate.

Not considered in this paper is the influence of the modulation scheme on the diversity. It is well known [8] that a rank criterion on the modulation scheme needs to be satisfied in order to achieve the full diversity. In QPSK transmission, this rank criterion is, for example, not satisfied in the four- and eight-antenna transmission schemes. In other words, for some transmitted symbols, the full diversity will not be achieved. One can exclude such symbols or use different modulation schemes. In [26, 27], the possibility to use offset QPSK was proposed. This can be implemented in UMTS without sacrificing much of the existent hardware solutions. Another possibility very suitable for UMTS is to work with feedback schemes. In [28, 29], it is shown for the 4×1 as well as the 8×1 antenna scheme that a very simple feedback

scheme retransmitting only one or two bits can reinstall the full diversity.

APPENDICES

A. PROOFS FOR LEMMAS 2 AND 3

Starting from the definition of X in (19), it is observed that the squares in the denominator can be appended:

$$X + 1 = \frac{|h_1 + h_4|^2 + |h_2 - h_3|^2}{|h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2}. \quad (\text{A.1})$$

In the case of i.i.d. complex-valued Gaussian distributed variables h_i , the two variates $h_1 + h_4$ and $h_2 + h_3$ become complex Gaussian distributed and independent of each other. They depend, however, on the variates in the nominator. Now, a linear orthogonal coordinate transformation is defined:

$$\begin{aligned} u &= \frac{(h_1 + h_4)}{\sqrt{2}}, & v &= \frac{(h_2 - h_3)}{\sqrt{2}}, \\ u' &= \frac{(h_1 - h_4)}{\sqrt{2}}, & v' &= \frac{(h_2 + h_3)}{\sqrt{2}}, \end{aligned} \quad (\text{A.2})$$

such that $\sum_{i=1}^4 |h_i|^2 = |u|^2 + |v|^2 + |u'|^2 + |v'|^2$ and

$$\frac{X + 1}{2} = \frac{|u|^2 + |v|^2}{|u|^2 + |v|^2 + |u'|^2 + |v'|^2} = \frac{X_1^2}{X_1^2 + X_2^2}. \quad (\text{A.3})$$

Generally, if X_1^2 and X_2^2 are independent random variables following chi-square distributions with ν_1 and ν_2 degrees of freedom, respectively, then $X_1^2/(X_1^2 + X_2^2)$ is said to follow a beta(p, q) distribution with $\nu_1 = 2p$ and $\nu_2 = 2q$ degrees of freedom and the probability density is given by $(1/B(p, q))\xi^{p-1}(1 - \xi)^{q-1}$, with $p = \nu_1/2$, $q = \nu_2/2$. This matches our case (A.3) for $\nu_1 = \nu_2 = 4$ and the probability density specializes to $6\xi(1 - \xi)$. Transforming this back to X gives the probability density

$$f_X(x) = \begin{cases} \frac{3}{4}(1 - x^2) & \text{for } |x| < 1, \\ 0 & \text{elsewhere.} \end{cases} \quad (\text{A.4})$$

The independency of X and $\eta = h^2$ can be established by transformation of variables starting from the two independent variates $Z_i = X_i^2$ defined above which are (up to a scaling) χ_4^2 distributed, that is, their joint probability density is given by

$$f_{Z_1 Z_2}(z_1, z_2) = \frac{1}{16} z_1 z_2 e^{-(z_1 + z_2)/2} \quad \text{for } z_1 > 0, z_2 > 0. \quad (\text{A.5})$$

The 2×2 transformation between the variates X , η and Z_1 , Z_2 is derived from (A.3):

$$\left\{ \begin{array}{l} X = \frac{Z_1 - Z_2}{Z_1 + Z_2} \\ \eta = Z_1 + Z_2 \end{array} \right\}, \quad \left\{ \begin{array}{l} Z_1 = \frac{1+X}{2} \eta \\ Z_2 = \frac{1-X}{2} \eta \end{array} \right\}. \quad (\text{A.6})$$

The rules for transformation of variates result in the follow-

ing joint probability density for X, η :

$$\begin{aligned} f_{X,\eta}(x, \eta) &= \frac{1}{64} (1-x^2) \eta^3 e^{-\eta/2} \\ &= f_X(x) f_\eta(\eta), \quad \text{for } |x| < 1, \eta > 0, \end{aligned} \quad (\text{A.7})$$

where f_X is given above and f_η is the χ_8^2 -density rescaled to unit mean, that is,

$$f_\eta(\eta) = \begin{cases} \frac{128}{3} \eta^3 e^{-4\eta} & \text{for } \eta > 0, \\ 0 & \text{elsewhere.} \end{cases} \quad (\text{A.8})$$

For the ZF receiver (where $\mu = 0$), it follows that the noise is increased by a factor of

$$E[\delta_4] = E\left[\frac{1}{1-X^2}\right] = \int_0^2 \lambda^{-1} f_X(\lambda-1) d\lambda = \frac{3}{2}. \quad (\text{A.9})$$

For the general linear receiver with $\mu > 0$ (including the MMSE),

$$E[\delta_4] = \frac{1}{2} E\left[\frac{1+X}{(1+\mu/h^2+X)^2} + \frac{1-X}{(1+\mu/h^2-X)^2}\right] \quad (\text{A.10})$$

is evaluated by exploiting independency of X and $\eta = h^2$:

$$E[\delta_4] = \int_{\eta=0}^{\infty} \int_{x=-1}^1 \frac{1+x}{(1+\mu/\eta+x)^2} f_X(x) f_\eta(\eta) dx d\eta. \quad (\text{A.11})$$

The integration over x is straightforward. The remaining integral

$$\begin{aligned} E[\delta_4] &= 32 \int_0^{\infty} \left(2\eta^2 + 6\mu\eta - \mu(3\mu + 4\eta) \log(2\eta + \mu) \right. \\ &\quad \left. + \mu(3\mu + 4\eta) \log \mu \right) \eta e^{-4\eta} d\eta \end{aligned} \quad (\text{A.12})$$

is evaluated in terms of the exponential integral which leads to (32).

B. SOME PROPERTIES OF $\mathbf{H}\mathbf{H}^H$

In the following it will be shown that all entries of $\mathbf{H}^H\mathbf{H}$ are real valued. The proof is performed by induction. Using block matrix notation $\mathbf{H}^H\mathbf{H}$ at a certain level m equals

$$\mathbf{H}^H\mathbf{H} = \begin{bmatrix} \mathbf{H}_1^H\mathbf{H}_1 + \mathbf{H}_2^H\mathbf{H}_2 & \mathbf{H}_1^H\mathbf{H}_2 - \mathbf{H}_2^T\mathbf{H}_1^* \\ \mathbf{H}_2^H\mathbf{H}_1 - \mathbf{H}_1^T\mathbf{H}_2^* & \mathbf{H}_1^H\mathbf{H}_1 + \mathbf{H}_2^H\mathbf{H}_2 \end{bmatrix}. \quad (\text{B.1})$$

Thus, if the property is given at the lower level scheme, $\mathbf{H}_1^H\mathbf{H}_1$ and $\mathbf{H}_2^H\mathbf{H}_2$ are real valued and so are the diagonal block matrices. In the next step, the recursion for the nondiagonal block matrix $\mathbf{H}_1^H\mathbf{H}_2 - \mathbf{H}_2^T\mathbf{H}_1^*$ is investigated. Assuming \mathbf{H}_1 is constructed by \mathbf{H}_{11} and \mathbf{H}_{12} and \mathbf{H}_2 in a similar manner by the matrices \mathbf{H}_{21} and \mathbf{H}_{22} , then the term $\mathbf{H}_1^H\mathbf{H}_2$ is given by

$$\mathbf{H}_1^H\mathbf{H}_2 = \begin{bmatrix} \mathbf{H}_{11}^H\mathbf{H}_{21} + \mathbf{H}_{12}^T\mathbf{H}_{22}^* & \mathbf{H}_{11}^H\mathbf{H}_{22} - \mathbf{H}_{12}^T\mathbf{H}_{21}^* \\ \mathbf{H}_{12}^H\mathbf{H}_{21} - \mathbf{H}_{11}^T\mathbf{H}_{22}^* & \mathbf{H}_{12}^H\mathbf{H}_{22} + \mathbf{H}_{11}^T\mathbf{H}_{21}^* \end{bmatrix} \quad (\text{B.2})$$

and the nondiagonal block matrix is obtained by such value minus its transposed form $\mathbf{H}_2^T\mathbf{H}_1^*$. Thus, every term $\mathbf{H}_1^H\mathbf{H}_2 - \mathbf{H}_2^T\mathbf{H}_1^*$ is replaced by a sum of terms of the form $\mathbf{H}_{kl}^H\mathbf{H}_{mn} - \mathbf{H}_{mn}^T\mathbf{H}_{kl}^*$. If the property holds for the level below, it also holds for the current level. To complete the induction argument, it has to be shown that the property also holds for the first level ($m = 1$). In this case, the diagonal elements are $|h_1|^2 + |h_2|^2$ and the nondiagonal values are $h_1^*h_2 \pm h_2h_1^*$, that is, either zero or $2\Re\{h_1^*h_2\}$. Thus, all entries are real valued. Note that due to the different signs occurring, it cannot be concluded that the terms become zero.

The second property is shown in [30]. For the third property, it is observed that every nondiagonal term X consists of either elements $(h_i h_k^* + h_i^* h_k)/h^2$ or $-(h_i h_m^* + h_i^* h_m)/h^2$. Thus building $X+1$ allows to consider $(h^2 + h_i h_k^* + h_i^* h_k)/h^2$ and $(h^2 - h_i h_m^* + h_i^* h_m)/h^2$, further allowing to reorganize the terms into $(h_i + h_k)(h_i + h_k)^*/h^2$ and $(h_l + h_m)(h_l + h_m)^*/h^2$. By applying the same transformation as in Appendix A:

$$\begin{aligned} u &= \frac{(h_i + h_k)}{\sqrt{2}}, & v &= \frac{(h_l - h_m)}{\sqrt{2}}, \\ u' &= \frac{(h_i - h_k)}{\sqrt{2}}, & v' &= \frac{(h_l + h_m)}{\sqrt{2}}, \end{aligned} \quad (\text{B.3})$$

the terms $X+1$ can be written in terms of independent Gaussian variables and the same rules as before apply. The resulting term then reads $(X+1)/2 = X_1^2/(X_1^2 + X_2^2)$ with X_1 and X_2 being χ^2 -distributed with $\nu = 2^m = N_T$ degrees of freedom each and

$$\frac{1}{B(N_T/2, N_T/2)} \xi^{N_T/2-1} (1-\xi)^{N_T/2-1} \quad (\text{B.4})$$

is obtained, resulting in the density (49) for X .

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