A MIMO Correlation Matrix based Metric for Characterizing Non-Stationarity

Markus Herdin and Ernst Bonek

Institut für Nachrichtentechnik und Hochfrequenztechnik, Technische Universität Wien
Gußhausstrasse 25/389, A-1040 Wien, Vienna, Austria, Email: {markus.herdin, ernst.bonek}@nt.tuwien.ac.at

Abstract— **A metric for characterizing spatially non-stationary channels is introduced. It is based on MIMO correlation matrices and measures the distance between the correlation matrices estimated at different times to characterize how strong the spatial structure of the channel has changed. By analyzing synthetic and measured MIMO data it is shown that the introduced metric is useful for characterization of spatial changes in non-stationary channels. This will be important for spatial based algorithms that are sensitive to changes in the spatial structure of the channel.**

I. INTRODUCTION

Wide-sense stationarity and uncorrelated scatterers (WSSUS) are often assumed to be valid for mobile radio channels. If this assumption is valid (and if the mobile radio channel can be fully described by second order statistics), the statistics of the mobile radio channel does not change with time or with frequency. As we all know, the statistics of the mobile radio channel *do change* due to shadowing, path drift and Doppler drift. However, if the statistics stay constant long enough, it is still possible to make use of them, as long as they can be estimated much faster than they change.

For multiple-input multiple-output (MIMO) channels, WSSUS in the Bello sense [1] is not sufficient any more. Additionally, there is the spatial domain at both receive and transmit sides. A straightforward approach to include the spatial domain is to extend the Bello system functions and their correlation functions to the spatial domain [2], and consider stationarity in all dimensions. This approach increases complexity considerably. Also, stationarity regions derived from this concept may not be useful for algorithms that mainly depend on the spatial domain. Therefore, stationarity definitions that consider the *spatial domain only* may be useful.

Stationarity of the mobile radio channel for single-input single-output (SISO) systems was already investigated by different authors. Steinbauer [3] [4] defined a local region of stationarity based on the correlation between consecutive power delay profiles. Kattenbach [5] [6] analyzed different terms of stationarity and their validity in general terms. Matz [7] [8] [9] introduced a time-frequency-dependent scattering function specifically for characterization of non-stationary mobile radio channels.

For single-input multiple-output (SIMO) or multiple-input single-output (MISO) systems there exist some investigations regarding stationarity also. Hugl [10] defines a time-frequency array correlation function that measures the correlation between time and frequency separated array responses vectors for characterizing the temporal evolution of frequency division duplex channels. Viering [11] introduced a metric for measuring the distance between temporally separated covariance matrices. This metric measures which part of the received energy can be collected by the strongest eigenvector(s) when using an out-dated version of the covariance matrix instead of the prevalent one. This metric is especially useful when eigen-beamforming [12] with only one or two eigenbeams is employed. For a MIMO system in which all eigenmodes are used, it becomes useless since then we have no beamforming gain any more. In this paper we will introduce a MIMO correlation matrix based metric for characterizing the spatial non-stationarity of the MIMO channel that is useful for MIMO systems, irrespective of how many eigenmodes are used. This metric measures the distance between the correlation matrices estimated at different times to characterize how strong the spatial structure of the channel has changed. First we will test this metric in synthetic scenarios to show its meaningfulness to describe spatial changes. Then, we analyze two comprehensive measurement campaigns with this metric. We consider both the distance between correlation matrices gathered from different measurement scenarios and the temporal evolution of the correlation matrix distance for a moving mobile within a room.

II. DEFINITION

We assume the $n \times 1$ time-variant signal vector $\mathbf{x}(t)$ to be a zero-mean stochastic vector process, where the spatial statistics (the element correlations) are fully characterized by the time-dependent element-correlation matrix

$$\mathbf{R(t)} = E\left\{\mathbf{x}(t)\mathbf{x}(t)^H\right\}. \quad (1)$$

We take the correlation matrix for $t = t_1$ and $t = t_2$ and consider the inner product between them, which fulfills

$$\begin{aligned} \langle \mathbf{R}(t_1)\mathbf{R}(t_2)\rangle &= \operatorname{tr}\left\{\mathbf{R}(t_1)\mathbf{R}(t_2)\right\} && (2)\\ &\leq ||\mathbf{R}(t_1)||_2||\mathbf{R}(t_2)||_2. && (3)\end{aligned}$$

where $\operatorname{tr}\{\cdot\}$ is the trace operator and $||\cdot||_2$ denotes the Frobenius norm. Based on the inner product we can now define the *correlation matrix distance* as

$$d_{corr,RX}(t_1,t_2) = 1 - \frac{\mathrm{tr}\left\{\mathbf{R}(t_1)\mathbf{R}(t_2)\right\}}{||\mathbf{R}(t_1)||_2||\mathbf{R}(t_2)||_2} \in [0,1] \quad (4)$$

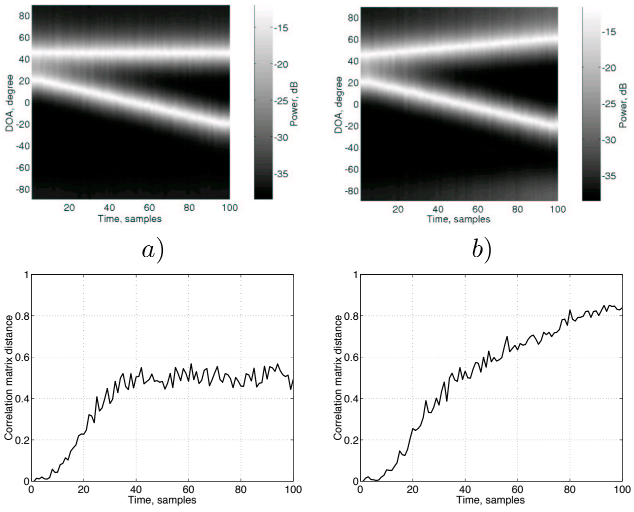
Note that $1 - d_{corr,RX}(t_1,t_2)$ can also be interpreted as the angle between the (vectorized) correlation matrices in the n^2 dimensional space. The correlation matrix distance becomes zero for equal correlation matrices and unity if they differ maximally.

The correlation matrix distance can be calculated both for transmit and receive side but also for the full channel correlation matrix that is given by

$$\mathbf{R}_H = E\left\{\mathrm{vec}\{\mathbf{H}\}\mathrm{vec}\{\mathbf{H}\}^H\right\}. \quad (5)$$

III. ANALYSIS OF SYNTHETIC SCENARIOS

In this section, we show how the introduced metric behaves in synthetic - and therefore well-known - scenarios. We consider the receive correlation matrix for a channel with changing directions-of-arrival (DOAs) where each path is modeled by 10 Laplace distributed sub-paths resulting in an rms angular spread of about 5°. For each time instant we create 100 realizations of the receive vector to get an accurate estimate of the receive correlation matrix.



a) *b)*

c) *d)*

Fig. 1. Capon receive spectrum (top view) for scenario 1 (a) and scenario 2 (b) and corresponding correlation matrix distance between $\mathbf{R}(0)$ and $\mathbf{R}(t)$ (c and d)

In Scenario 1 (Figure 1, a and c), we consider two equipowered DOAs. One is constant at 45° and the other one changes over time from 20° to −20°. For the correlation matrix distance we compare the correlation matrix at time t with the correlation matrix at time zero. The change in the spatial structure can clearly be seen in the correlation matrix distance, which reaches a maximum value of about 0.5 after 40 time samples. This reflects the evolution of the spatial structure very well since one DOA stays constant and only one changes. Hence, only 'half' of the arriving power experiences spatial changes.

Scenario 2 (Figure 1, b and d) shows the case with two changing DOAs. Again, both have equal power but now they change from 45° to 60° (path 1) and from 20° to −20° (path 2). The result is that the correlation matrix distance reaches a value of more than 0.8, but now after 100 time samples. This reflects nicely the slower change of the first path (only by 15°) within the considered time interval whereas the second path changes by 40°.

IV. ANALYSIS OF MEASURED SCENARIOS

A. Measurement Equipment and Scenario

We consider two different measurement campaigns carried out at the *Institut für Nachrichtentechnik und Hochfrequenztechnik, Technische Universität Wien.* The first measurement campaign was performed with Medav equipment. With the Medav RUSK ATM channel sounder [13] we measured the MIMO channel using a fixed transmitter in the middle of a corridor and the receiver positioned at a large number of different positions in the connected office rooms (Figure 2), looking in one of three receive directions. For a detailed measurement description see [14]. Due to the use of a virtual transmit array we were limited to static scenarios. The Medav RUSK ATM was operated at 5.2GHz with a (flat) measurement bandwidth of 120MHz. At the transmitter we used an omni-directional monopole-like antenna that was mounted on an xy-positioning table to form the virtual array of 20×10 antennas. At the receiver, an 8-element uniform linear array (ULA) of printed dipoles was utilized. Each single dipole had a 3dB beamwidth of 120°. The antenna spacing was 0.5λ at 5.26GHz at the transmitter and 0.4λ at 5.2GHz at the receiver. Within the 120MHz bandwidth, 193 frequency samples of the MIMO channel matrix were recorded. From this data we created 130 spatial realizations of an 8×8 system by moving a virtual 8-element TX ULA over all possible TX antennas. This means we have in total 130×193 realizations of an 8×8 MIMO system for each measured RX position and direction (for each of the 72 different measurement scenarios).

The second measurement campaign was performed with the Elektrobit PropSound channel sounder [15]. The channel sounder was operated at 2.45GHz with a null-to-null measurement bandwidth of 200MHz. We used switched antenna arrays at both link ends and could therefore measure the time-variant MIMO channel with a fixed transmitter (again in the middle of the corridor) but a moving receiver (Figure 3). At the transmitter we had an 8-element uniform circular array with one center element (7 on the circle, one in the middle) that is horizontally omni-directional. At the receiver, a dual-polarized (+/- 45°) 4×4 patch array with the patches arranged in a vertical plane, was used. The element spacing is 0.5λ at 2.55GHz for both transmit and receive antenna. For the evaluations we used all 8 TX antennas but only the first 8 RX antennas with polarization −45° to get an 8×8 MIMO system. The MIMO snapshots were measured continuously with a sampling interval of 0.0377s. Each snapshot consisted of 510 frequency samples of the MIMO channel matrix within the 200MHz bandwidth. For our evaluations we used only 306 frequency bins corresponding to a bandwidth of 120MHz centered in the 200MHz measurement bandwidth. During the measurements, the transmitter was always fixed and the receiver was moved along a specific path with the antenna oriented into one of the directions shown (Figure 3).

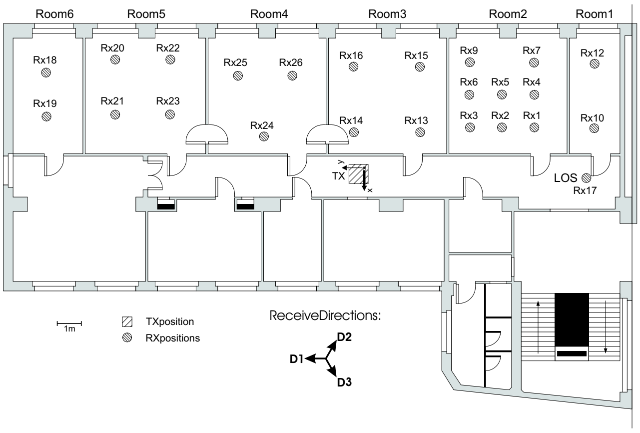


Fig. 2. Transmit and receive positions and directions for the measurements with the Medav RUSK ATM channel sounder

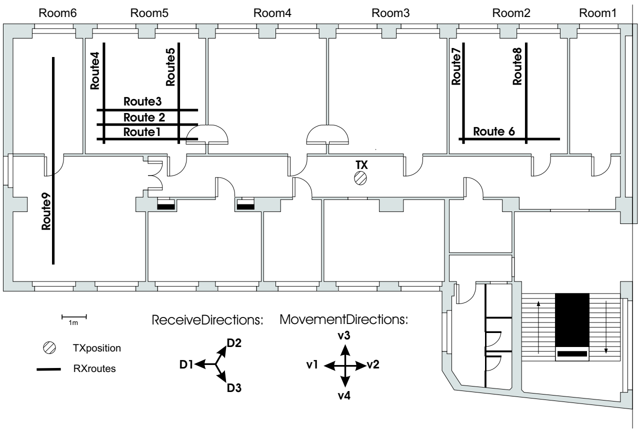


Fig. 3. Transmit position and receiver routes and directions for the measurements with the Elektrobit PropSound channel sounder

B. Evaluation

We consider the transmit and receive correlation matrices that were estimated by

$$\mathbf{R}_{TX} = E\left\{\mathbf{H}^T\mathbf{H}^*\right\}, \tag{6}$$

$$\mathbf{R}_{RX} = E\left\{\mathbf{H}\mathbf{H}^H\right\} \tag{7}$$

respectively. For the Medav RUSK ATM measurements we used all available spatial and frequency realizations of the MIMO channel for one considered scenario (RX position and direction) to average over (see Section IV-A). As a result we have 72 different RX and TX correlation matrices. For the Elektrobit PropSound measurements, we use a temporal window of 5 snapshots (about 0.19s) and all available frequency samples to estimate the correlation matrices. This turned out to give reliable estimates.

C. Results

In Figure 4 the correlation matrix distances between the correlation matrices, gathered from different measurement scenarios, are shown (Medav measurements, Figure 2). The results for the transmit side are shown in (a) and the results for the receive side in (b). The Figures show the correlation matrix distance between all combinations of transmit (receive) correlation matrices estimated for all 72 scenarios. The results are ordered by room and direction such that 1-3 corresponds to RX position 1, direction D1, D2 and D3, 4-6 to RX position 2, direction D1, D2 and D3 etc.

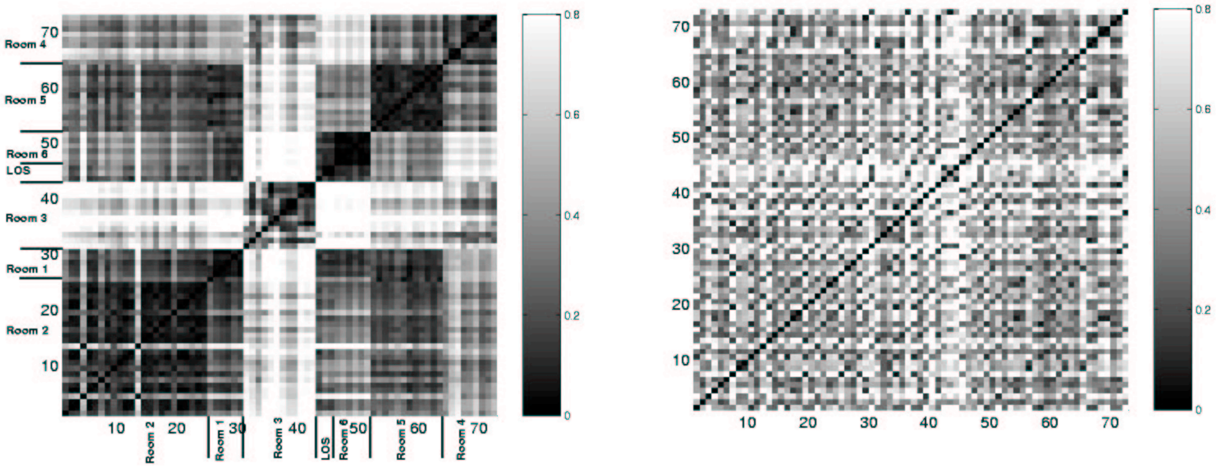
The clustering seen for the *transmit side* corresponds exactly to the room structure. This means that, regardless where the receiver is placed within an office room, the transmit correlation matrix does not change dramatically. However, if the receiver is placed in a different room, the transmit correlation matrix changes significantly. This can also be seen from the values of the correlation matrix distance. Positions within the same office show a correlation matrix distance of typically below 0.3 (with some exceptions) but positions in a different rooms have transmit correlation matrix distance values of up to 0.9, which means nearly maximum difference between the matrices.

For the *receive correlation matrix* we have to keep in mind that the measurements are ordered by the room and then by receive direction, which means that consecutive measurements have different directions. Since consecutive measurements show large values of the correlation matrix distance, we can conclude that different receive directions result in largely changed receive correlation matrices. However, there is also a slight structure superimposed as was seen for the transmit side. This means, there is a noticeable change in the receive correlation matrix also, when we move from room to room.

The results show that large changes in the long-term statistics at transmit side occur only if a user moves from one room into another room. If a user stays within the same room, the transmit correlation matrix does not change strongly.

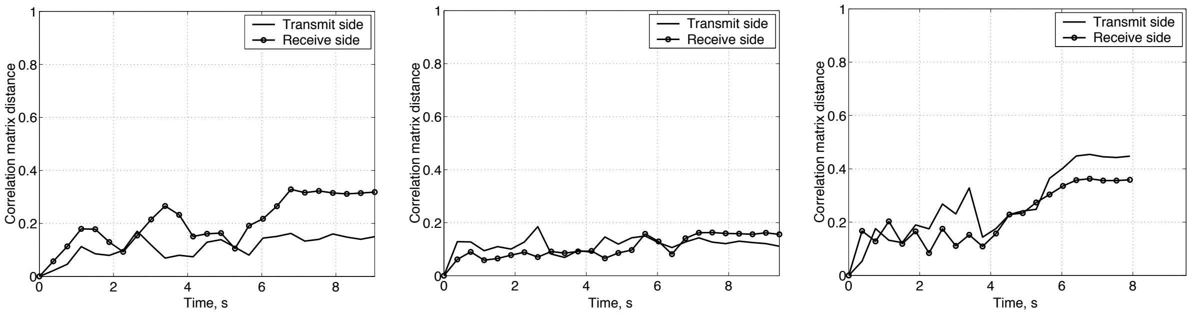
Figure 5 shows the results for the Elektrobit PropSound measurements. Here, we considered the temporal evolution of the correlation matrix distance when comparing transmit and receive correlation matrix at different times. The reference time was always $t = 0$, so that we show $d_{corr,TX/RX}(0,t)$. Out of the measured scenarios, we selected three representative scenarios.

In Figure 5a (RX route 3, movement into direction v1, receive direction D3, compare Figure 3), the result for a rather typical scenario is shown. The correlation matrix



a) *b)*

Fig. 4. Correlation matrix distance between all transmit correlation matrices (a) / all receive correlation matrices (b) for different measurement scenarios



a) *b)* *c)*

Fig. 5. Temporal evolution of the correlation matrix distance $d_{corr,TX/RX}(0,t)$ for the Elektrobit PropSound measurements

distance of the RX correlation matrices is larger than that of the TX correlation matrix and the TX correlation matrix distance stays below a value of 0.2, which can be considered as significance threshold. Figure 5b (RX route 4, movement into direction v3, receive direction D2) shows a scenario where both correlation matrices stay fairly constant. Both figures are rather representative for the considered indoor scenario, which also fits to the results from the Medav RUSK ATM measurements, where the TX correlation matrix distance between different scenarios is low when the corresponding receive positions are in the same room, and high if they are in different rooms.

Nevertheless, there are also positions where different RX positions within the same room lead to a significantly changed TX (and RX) correlation matrix, hence large values of the correlation matrix distance. This is shown in Figure 5c (RX route 7, movement into direction v4, receive direction D1). Here, both the receive and transmit correlation matrix change strongly which results in a large correlation matrix distance when comparing time t with time t=0.

V. CONCLUSIONS

A metric for characterizing spatially non-stationary channels was introduced and analyzed using synthetic and measured MIMO data. The synthetic scenarios showed that this metric provides meaningful measures when compared to the actual changes in the spatial structure. Analyzing the measurements of an office environment with an access point in the corridor, we found the transmit correlation matrices to be receive position dependent. Receive positions within the *same* office lead to very similar correlation matrices but receive positions within *different* offices lead to significantly different transmit correlation matrices. This result is in line with expectations, so we conclude that the newly introduced metric reflects non-stationarity well.

Analyzing measurements with a moving receiver we found that the receive correlation matrix changes typically faster than the transmit correlation matrix. Although this finding seems obvious for non-stationary receivers, we note with interest that the new metric reflects that as well. Movements within one office show typically no large variation in the transmit correlation matrix but there exist remarkable movement routes within some offices where significant changes occur.

What is a meaningful threshold for the correlation matrix distance to distinguish between significant and non-significant changes? Considering the synthesized scenarios, a sensible choice for this threshold could be 0.2.

The introduced metric seems to be useful for characterization of spatial non-stationary channels in conjunction with spatial based algorithms that are sensitive to changes in the spatial structure of the channel.

VI. ACKNOWLEDGMENTS

We would like to thank Helmut Hofstetter (Forschungszentrum Telekommunikation Wien, ftw.) for help with the Medav RUSK ATM measurements and T-Systems Nova GmbH for providing an eight element uniform linear array of printed dipoles. Also we would like to thank Hüseyin Özcelik for planning and doing the measurements with the Elektrobit PropSound channel sounder that were made together with Andreas Stucki and others from Elektrobit. Also we would like to thank Werner Weichselberger and Gerald Matz for fruitful discussions and comments.

REFERENCES

[1] P. Bello, “Characterization of randomly time-variant linear channels,” *IEEE Trans. Commun. Syst.*, vol. CS-11, pp. 360–393, Dec 1963.

[2] R. Kattenbach, “Statistical modeling of small-scale fading in directional radio channels,” *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 3, pp. 584–592, April 2002.

[3] M. Steinbauer, *PhD thesis: The Radio Propagation Channel - A Non-Directional, Directional and Double-Directional Point-of-View*, Technische Universität Wien, downloadable from http://www.nt.tuwien.ac.at/mobile, 2001.

[4] A. Gehring, M. Steinbauer, I. Gaspard, and M. Grigat, “Empirical channel stationarity in urban environments,” in *Proc. European Personal Mobile Communications Conference, EPMCC*, Vienna, Austria, February 2001.

[5] R. Kattenbach, “On different terms of stationarity and their validity for mobile radio channels,” *METAMORP (SMT4-CT96-2093) internal report*, October 1997.

[6] ——, “Considerations about the validity of WSSUS for indoor radio channels, td(97)70,” *COST 259, 3rd Management Committee Meeting*, September 1997.

[7] G. Matz, “Characterization of non-WSSUS fading dispersive channels,” in *Proc. IEEE International Conference on Communications*, Anchorage, AK, May 2003, pp. 2480–2484.

[8] ——, “Doubly underspread non-WSSUS channels: Analysis and estimation of channel statistics,” in *Proc. IEEE Signal Processing Advances in Wireless Communications Conference*, Rome, Italy, June 2003.

[9] ——, “On Non-WSSUS wireless fading channels,” *submitted to transactions on wireless communications.*

[10] K. Hugl, *PhD thesis: Spatial Channel Characteristics for Adaptive Antenna Downline Transmission*, Technische Universität Wien, downloadable from http://www.nt.tuwien.ac.at/mobile, 2002.

[11] W. U. Ingo Viering, Helmut Hofstetter, “Validity of spatial covariance matrices over time and frequency,” in *Proc. IEEE Globecom Conference*, vol. 1, Taipei, Taiwan, November 2002, pp. 851–855.

[12] C. Brunner, W. Utschick, and J.A. Nossek, “Exploiting the Short-Term and Long-Term Channel Properties in Space and Time: Eigenbeamforming Concepts for the BS in WCDMA,” *European Transactions on Telecommunications*, vol. 12, no. 5, pp. 365–378, 2001.

[13] R. Thomä, D. Hampicke, A. Richter, G. Sommerkorn, A. Schneider, U. Trautwein, and W. Wirnitzer, “Identification of time-variant directional mobile radio channels,” *IEEE Transactions on Instrumentation and Measurement*, vol. 49, pp. 357–364, April 2000.

[14] H. Özcelik, M. Herdin, R. Prestros, and E. Bonek, “How MIMO capacity is linked with multipath distribution,” in *Proc. International Conference on Electromagnetics in Advanced Applications*, Torino, Italy, September 2003, pp. 775–778.

[15] E. Zollinger, “A novel architecture for a flexible integrated wideband vector channel sounder for MIMO system investigation,” in *Proc. URSI, General Assembly of the International Union of Radio Science*, Maastricht, Netherlands, 2002.