A FAST ALGORITHM FOR DIGITAL PRE-DISTORTION OF NONLINEAR POWER AMPLIFIERS

Ernst Aschbacher, Holger Arthaber[†]*, and Markus Rupp*

Institute of Communications and Radio-Frequency Eng.
Vienna University of Technology
Gusshausstrasse 25/389, A-1040, Vienna, Austria
phone: +43 (0)1 58801 38934, fax: +43 (0)1 58801 38999
email: ernst.aschbacher@tuwien.ac.at
web: www.nt.tuwien.ac.at/rapid_prototyping

[†]Institute of Electrical Measurements and Circuit Design
Vienna University of Technology
Gusshausstrasse 25/354, A-1040, Vienna, Austria
phone: +43 (0)1 58801 35420, fax: +43 (0)1 58801 35499
email: holger.arthaber@tuwien.ac.at
web: www.emst.tuwien.ac.at/mw

ABSTRACT

A fast iterative algorithm for pre-equalisation of a nonlinear dynamic (with memory) system is presented . The algorithm is based on the secant method for root-finding. Within a few iterations the nonlinear and dynamic effects of the system, modelled, e.g., with a Volterra or Wiener model, can be compensated very well. The algorithm is then used to linearise a nonlinear power amplifier, modelled as a Wiener system. Measurement results prove the excellent performance of the presented method.

1. INTRODUCTION

Nonlinear systems arise frequently in applications, e.g., loudspeakers are nonlinear and power amplifiers in wireless communications are driven near saturation due to efficiency reasons. Pre-equalisation of such (often weakly) nonlinear systems can be used to linearise the whole signal path. Digital pre-distortion is a promising technique for pre-compensating nonlinear systems. The principle is to pre-distort the signal in the digital domain in such a way that the interconnection of the pre-distortion unit and the nonlinear system is as linear as possible. Since pre-distortion works entirely in the digital domain it is especially suited for Software Defined Radio (SDR) systems. Undesired spectral broadening of the transmitted signal in wireless communication systems and in-band distortions can be reduced significantly.

In 3G communication systems dynamic effects (memory effects) have impact due to increased signal bandwidths [1]. This makes the task of pre-distortion complicated, since a nonlinear dynamic system has to be pre-equalised. Methods based on look-up-tables, which are a tabulated inverse of the nonlinear system, cannot be applied due to the extreme complexity. Further, analytic solutions for the pre-inverse of the nonlinear system are not known in most cases.

This paper is focused on an iterative method, based on the secant method for root-finding, which solves the pre-distortion task in an approximate way. The method is tested via measurements on a microwave power amplifier. It is shown that significant reduction of the nonlinear distortion can be achieved, see Sec. 4.1 further ahead.

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2. ITERATIVE EQUALISATION WITH THE SECANT METHOD

The equalisation problem is represented graphically in Fig. 1. A pre-equaliser $\mathbb{P}$ equalises the nonlinear dynamic system $\mathbb{N}$.

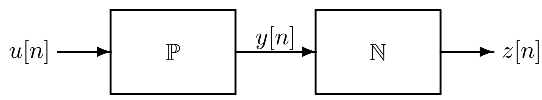

Figure 1: Nonlinear equalisation problem

Mathematically, the problem can be formulated as

$$\mathbb{N}\left(\mathbb{P}\left(u[n]\right)\right) = \mathbb{L}\left(u[n]\right) \ , \qquad (1)$$

where $\mathbb{L}$ is a linear operator and $u[n]$ denotes the input signal. If $\mathbb{L} = \mathbb{I}$, the identity operator, the filter $\mathbb{P}$ is the pre-inverse of the nonlinear system. Even for static nonlinearities, e.g., polynomials, analytic solutions for the pre-inverse are not known in the majority of cases. If the nonlinear system is dynamic, the inverse is even harder to find. Typical examples of such systems are the often used Volterra series [2, 3] and simplifications, such as Wiener systems (linear filter in front of a static nonlinearity) or Hammerstein systems (static nonlinearity in front of a linear filter). Since analytic solutions for the pre-filter $\mathbb{P}$ are seldom known, an iterative method is proposed here which converges very fast, within four to six iterations for the investigated cases. Other approaches exist which determine the parameters of a certain pre-filter $\mathbb{P}$, e.g., the inverse modelling approach [4]. In this case, the post-inverse is approximated and then used as a pre-inverse. Depending on the nonlinear filter $\mathbb{N}$ this might give poor results.

Reformulation of (1) gives

$$\mathbb{N}\left(y[n]\right) - \mathbb{L}\left(u[n]\right) = \mathbb{S}(y[n]) = 0 \ , \qquad (2)$$

with $\mathbb{P}(u[n]) = y[n]$. The solution to the above equation is the signal after the pre-filter for a specific nonlinear filter $\mathbb{N}$ and a targeted linear operator $\mathbb{L}$, which in the pre-distortion context is a simple linear amplification, $\mathbb{L}(u[n]) = g \cdot u[n]$. A fast iterative method for solving (2) for the unknown signal $y[n]$ (which is equivalent to determining the pre-filter $\mathbb{P}$) is the Newton method [5]

$$y_i[n] = y_{i-1}[n] - (\mathbb{S}^{'}(y_{i-1}[n]))^{-1}\mathbb{S}(y_{i-1}[n]) \ . \qquad (3)$$

The convergence rate of the Newton method, if it converges, is quadratic, meaning that

$$\|y_i[n]-y[n]\| \leq c\|y_{i-1}[n]-y[n]\|^2\,, \tag{4}$$

with some constant c and $y[n]$ being a possible solution for (2). This method has the drawback that the operator $\mathbb{S}$ must be analytic, otherwise the Jacobian $\mathbb{S}'$ is meaningless. The equivalent complex baseband models for the power amplifier used here are not analytic, cf. (7) and (8). Hence, the Newton method cannot be applied [6]. Further, the Jacobian $\mathbb{S}'$ has to be determined and evaluated at each iteration which increases the complexity.

Here, the secant method is proposed for solving (2) for $y[n]$. This method converges nearly as fast as the Newton method – the convergence rate is equal to the golden ratio $\phi = 1.618\ldots$ – but does not require that $\mathbb{S}$ is analytic, therefore allowing for a larger class of nonlinear systems $\mathbb{N}$. The iteration rule is

$$y_i[n] = y_{i-1}[n] - \mu_n \mathbb{S}_n(y_{i-1}[n]) \tag{5}$$

with the approximate Jacobian

$$\mu_n = \frac{y_{i-1}[n]-y_{i-2}[n]}{\mathbb{N}_n(y_{i-1}[n])-\mathbb{N}_n(y_{i-2}[n])}\,. \tag{6}$$

For determining one sample of $y[n]$, (5) has to be evaluated a certain number of times $i = 1,2,\ldots,I$, therefore the need for an algorithm with high convergence rate. Generally, $\mathbb{N}$ is an operator with memory. If an approximate solution $y_I[n-m], m \geq 1$, I denoting the total number of iterations, has been found, these values parameterise the operators $\mathbb{N}$ and $\mathbb{S}$. The operators become simple, parameterised functions which are time-variant and marked therefore with the time index n. It has to be noted that the convergence of the method (5) is not guaranteed – it depends on the operator $\mathbb{S}$ and therefore on the model $\mathbb{N}$ and the targeted linear operator $\mathbb{L}$, as well as on the range of the input signal $u[n]$ whether a solution $y[n]$ can be found or not. Since the non-linearities are not very hard, the algorithm in (5) does not experience converge problems and three to four iterations are sufficient for practical applications.

Another approach is successive approximation, proposed in [7] for general Volterra systems and applied in [8] for memoryless systems, where (1) is reformulated as a fixed-point equation. This method converges with linear order and is thus much slower than (5), cf. [6] for a comparison with the Newton method.

3. DERIVATION OF BLACK-BOX MODELS

In this section two mathematical black-box models [9], a Volterra model and a Wiener model, are used to model a multi-stage microwave high-power amplifier (HPA).

3.1 Measurement Setup

In Fig. 2 the measurement setup is presented. The test-signal $u[n]$, a multi-tone signal with a bandwidth of 1 MHz and 101 tones, equally spaced and with random phases, is generated in a PC. The complex digital baseband signal is converted to analog with the Rhode&Schwartz I/Q modulation generator AMIQ. The analog in-phase and quadrature-phase signals are then used to modulate a carrier at 1,9 GHz, using the Rhode&Schwartz Vector Signal Generator SMIQ. This test signal has a relatively high crest-factor of 8,5 dB. A single-stage driver amplifier (Minicircuits ZHL-42W) with a minimum gain of 30 dB and a three-stage high-power LDMOS EDGE amplifier follow. After attenuation, the output signal is down-converted and demodulated with a PSA signal analyser form Agilent, which delivers the complex baseband output signal $d[n]$ to the PC. The power meter is used to control precisely the total output power, see Sec. 4.1 further ahead.

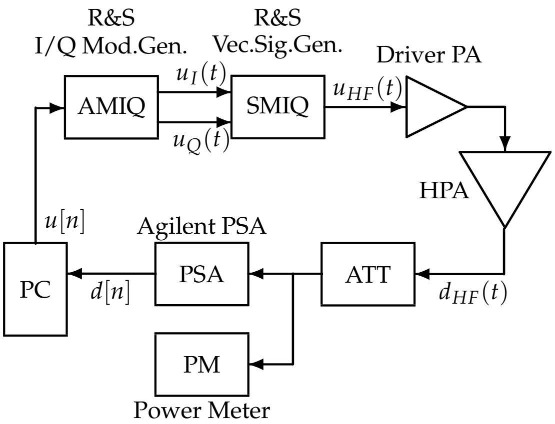

Figure 2: Measurement setup

3.2 Modeling Results

The first step is to derive a model $\mathbb{N}$ of the power amplifier chain, based on the recorded input/output data $\{u[n], d[n]\}$. Since the bandwidth is limited by the signal analyser to 8 MHz, an input signal with a maximum bandwidth of 1 MHz is used. With this, all harmonics up to the seventh order can be observed.

Two models are compared: a Volterra model and a Wiener model. The equivalent complex baseband Volterra model reads

$$z[n] = \sum_{p=0}^{P-1} \sum_{\mathbf{n}_{2p+1}=\mathbf{0}}^{\mathbf{N}_{2p+1}} h[\mathbf{n}_{2p+1}] \prod_{i=1}^{p+1} u[n-n_i] \prod_{i=p+2}^{2p+1} u^*[n-n_i]\,, \tag{7}$$

where for shorter notation the indices are summarised in $\mathbf{n}_{2p+1} = [n_1; n_2; \ldots; n_{2p+1}]^T$ and, correspondingly, the memory-lengths of the kernels are specified in $\mathbf{N}_{2p+1} = [N_1; N_3; \ldots; N_{2p+1}]$. A vector-valued summation index with $2p+1$ elements corresponds to a $2p+1$-fold summation. The total nonlinear order is $2P+1$, the * denotes complex conjugation.

The Wiener model on the other hand is

$$z[n] = \mathbb{H}(u[n]) \sum_{p=0}^{P} \theta_p p_{2p}(|\mathbb{H}(u[n])|)\,, \tag{8}$$

where $\mathbb{H}(\cdot) = \sum_{i=0}^{N} h_i q^{-i}$ is the linear filter and $p_{2p}(\cdot)$ are orthogonal polynomials having only even powers. Here, the even Hermite polynomials are used. For the estimation of the parameters the LS-method is used. For the Volterra model, the LS-method can be applied directly since the model is linear-in-parameters. The Wiener model is only linear with respect to the parameters θ_p of the static nonlinearity. Here, a two step estimation is performed: in the first step, the measured input/output data is used to estimate the parameters h_i of the linear filter with the LS-method. The input signal is then passed through this estimated linear filter giving a new signal which is the input signal for the LS-estimation of the parameters of the static nonlinearity in the second step. In Tab. 1 the achieved normalised MSE is tabulated. The normalised MSE is

case	$P_{out}\,[dBm]$	$\text{NMSE}_{Wiener}\,[dB]$	$\text{NMSE}_{Volterra}\,[dB]$
1	42,3	-37,3	-38,9
2	43,9	-35,5	-37,1
3	45,4	-33,8	-35,3

Table 1: Normalised MSE of Wiener and Volterra models

$$\text{NMSE}\,[dB] = 20\log\frac{\|d[n]-z[n]\|_2}{\|d[n]\|_2}\,, \tag{9}$$

where $d[n]$ is the measured output signal and $z[n]$ is the output signal of the model. The amplifier input power is increased in three steps, corresponding to cases 1 to 3 in Tab. 1, driving the power amplifier more and more into saturation.

The Wiener model is a four tap FIR filter with a seventh order nonlinear function, whereby the even order Hermite polynomials up to the sixth order are used, cf. (8). The Volterra model contains all nonlinear parts up to the seventh order with the memory lengths of the kernels $\mathbf{N}_7 = [5,3,2,1]^T$. Increasing the memory lengths does not reduce the modelling error. It can be observed that with increasing saturation the modelling quality decreases. The modelling error of the Wiener model is only slightly larger (max. 1,6 dB) than the modelling error of the Volterra model – but the Wiener model requires only eight parameters, whereas the Volterra model requires 36 parameters. Therefore, the Wiener model is used to calculate the signal after the pre-distortion filter using the algorithm (5).

Fig. 3 shows the measured spectra of the output signal of the system and the output signal of the Wiener model for case 1, Tab. 1. The power is normalised to 1 W, the sampling frequency $f_s = 10,24$ MHz. It can be seen that the third order nonlinear distortion is dominating, the distortion due to the fifth order intermodulation products is already more than 50 dB smaller than the in-band signal. The estimation of the third-order intermodulation products is accurate, the fifth- and seventh order products are underestimated due to the very low signal level (more than 50 dB smaller than the in-band signal).

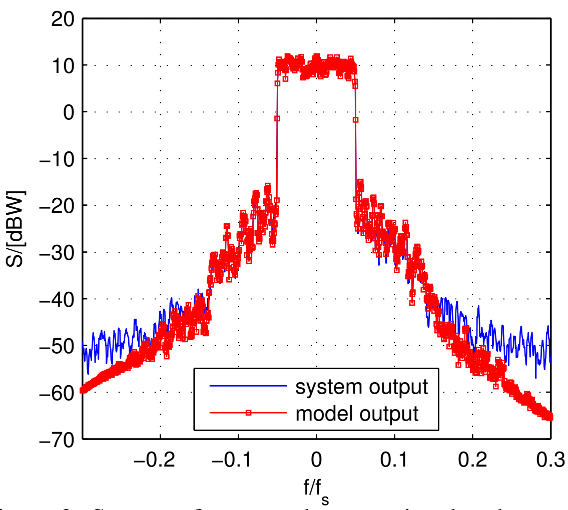

Figure 3: Spectra of measured output signal and output signal of the Wiener model for case 1, see Tab. 1.

4. EQUALISATION

After the modelling the calculation of the pre-distorted signal using (5) follows. Once this is done, the signal $y[n]$ is used to excite the power amplifier chain. With a power amplifier driven in saturation the overall gain with pre-distortion will be smaller than the gain without pre-distortion if the crest-factor of the input signal remains the same. Therefore, since the output power with pre-distortion decreases, it must be compared with a backed-off power amplifier – the output power with and without pre-distortion must be the same.

In the following, a measure for the performance gain of the proposed pre-distortion method is the total power in the out-of-band third harmonic zone using a backed-off (OBO stands for output back-off) power amplifier vs. the power in this zone using pre-distortion,

$$g_3[dB] = 10\log\left(\frac{P_{3,OBO}}{P_{3,PD}}\right)\,. \tag{10}$$

Here, $P_{3,OBO}$ is the signal power in the out-of-band third order harmonic zone (see Fig. 5 for a detailed view) with output back-off, whereas $P_{3,PD}$ is the power in this zone with pre-distortion (PD). Therefore, g_3 denotes the gain by using pre-distortion vs. a simple back-off. The out-of-band third order zone extends over the intervals $I_{L,3} = [-0,15;-0,05]$ and $I_{U,3} = [0,05;0,15]$ of the normalised frequency. It has to be noted that in this zone also higher order intermodulation products are present. The power spectral density in 90% of this zone, resulting in a 5% guard interval to the neighbouring zone, is accumulated, giving the signal power in this zone. In the out-of-band fifth order zone, which extends over the intervals $I_{L,5} = [-0,25;-0,15]$ and $I_{U,5} = [0,15;0,25]$ of the normalised frequency f/f_s, no reduction of the signal power could be achieved. From the Figs. 3 and 4 it can be seen that the distortion due to the third order intermodulation products is dominating, the distortion due to higher order intermodulation products is very low, more than 50 dB smaller than the in-band signal. The modelling of this higher order (fifth and seventh) intermodulation products is not accurate enough to achieve a performance gain with pre-distortion.

The other performance measure is the total deviation from the targeted linear amplification. Here, the MSE between the actual measured output signal $z_{OBO}[n]$ and $z_{PD}[n]$, either with output back-off or pre-distortion, and the targeted linearly amplified input signal $d[n] = g \cdot u[n]$, g denoting the gain, is computed and compared against each other, which gives

$$g_{MSE}[dB] = 20\log\left(\frac{\|z_{OBO}[n]-d[n]\|_2}{\|z_{PD}[n]-d[n]\|_2}\right) \tag{11}$$

as a performance measure. With this measure the total distortion is taken into account, also the in-band distortion.

4.1 Equalisation Results

In Fig. 4 the spectra of the output signal with and without pre-distortion (but with OBO) are represented. The total power is again normalised to 1 W. For the actual measured output power cf. Tab. 2. For the calculation of the output signal of the pre-distortion filter $y[n]$ six iterations of the algorithm in (5), using the initial values $y_0[n] = 0, \forall n$ and $y_1[n] = 10^{-3}, \forall n$, are performed.

Fig. 4 should be compared to the Fig. 3. It can be seen that an output back-off of only 1,4 dB does not result in a noticeable reduction of the nonlinear distortions, whereas pre-distortion reduces the distortions significantly. In Fig. 5 a detailed view of the out-of-band third-order harmonic zone is given, where the performance gain by using pre-distortion vs. back-off is clearly visible.

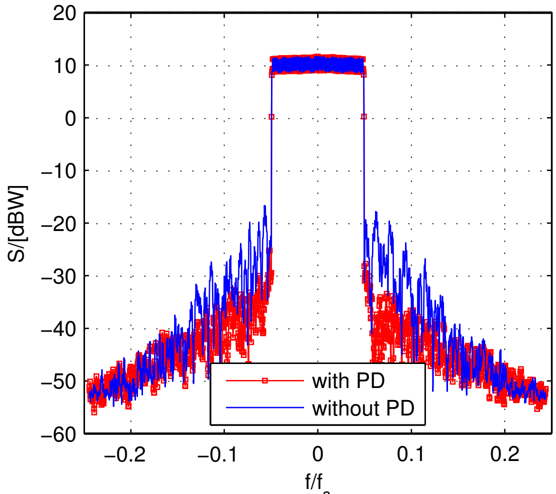

Figure 4: Spectra of measured output signal with and without pre-distortion (PD) for case 1, cf. Tab. 2, using a Wiener model for the power amplifier.

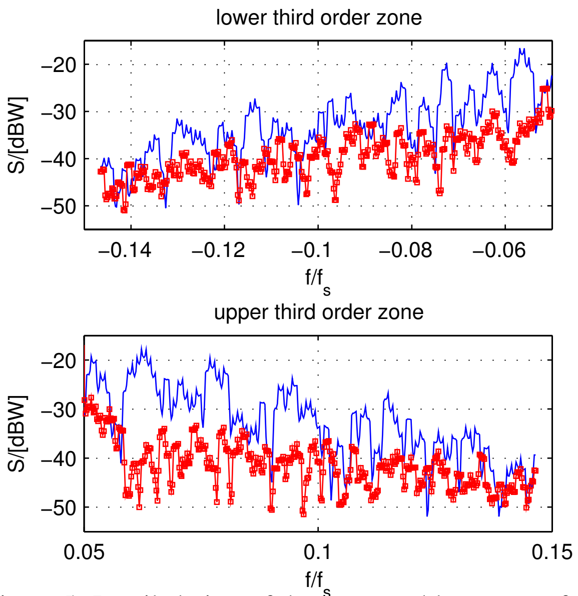

Figure 5: Detailed view of the upper and lower out-of-band third order harmonic zone for case 1, cf. Tab. 2. The square markers denote the measured output signal with pre-distortion.

The measured output power, as well as the reduction of the power in the out-of-band third order harmonic zone, g_3, and the reduction of the overall distortion, g_{MSE}, compared with backing-off, can be found in Tab. 2. In case 1, which corresponds to the least output power and therefore to the case where the power amplifier is not too much in saturation, pre-distortion yields a significant gain with respect to simple back-off. The spectral broadening due to the third order intermodulation could be reduced significantly, as well as the total distortion, both compared with an equivalent back-off. If the power amplifier is driven by a higher input power, the

case	$P_{out}/[dBm]$	$OBO/[dB]$	$g_{MSE}/[dB]$	$g_3/[dB]$
1	40,9	1,4	9,7	10,7
2	41,6	2,3	11,1	6,7
3	43,3	2,1	3,3	4,1

Table 2: Reduction of power in the third order out-of-band spectral zone by using pre-distortion vs. back-off (g_3) and reduction of overall distortion (g_{MSE}).

reduction in nonlinear distortion compared with the simple back-off becomes smaller, but is still present. The equivalent backing-off is larger in these cases, corresponding to a larger gain-decrease when pre-distortion is used.

5. CONCLUSIONS

A fast and simple iterative algorithm for pre-equalising a nonlinear dynamic system has been presented. The algorithm was applied for pre-distortion of a nonlinear power amplifier. For this, a black-box model of a microwave power amplifier-chain was created. It was shown that a relatively simple Wiener model is sufficient for the power amplifiers investigated. The Wiener model was compared to a Volterra model which yielded only slightly better modelling results but required far more parameters.

Digital pre-distortion, based on the presented method, was then compared to simple back-off, i.e., a reduction of the input power, resulting in a more linear behaviour of the power amplifier. Measurement results showed that pre-distortion can yield a significant reduction in nonlinear distortion with respect to a back-off. Large gains are achieved if the power amplifier is not driven too much into saturation. For higher saturation levels the linearization results in a significant gain-reduction and the gain of pre-distortion, compared to an equivalent back-off, decreases.

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