Wayfinding for Public Transportation Users as Navigation in a Product of Graphs

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Abstract: Navigation systems help car drivers and pedestrians to find their way in unknown environments; they are probably the most widely used GIS application. GIScience investigates the theoretical foundations for geoinformation. This article describes a series of recent investigations focusing on finding the shortest path in a network represented as graph..
To aid pedestrians in wayfinding and using public transportation systems effectively, they need information not only for their spatial decisions (e.g., also to buy a ticket). These business aspects can be represented as a second graph they navigate. The article shows a novel solution to merge two state-transition graphs using category theory. The resulting formula can be used to program simulation systems or wayfinding programs.

Zusammenfassung: Navigationssysteme helfen Autofahrern und Fußgängern ihren Weg zu finden. Sie sind wohl die am weitesten verbreiteten GIS Anwendungen; die Geoinformationswissenschaft legt die theoretischen Grundlagen dazu. Es werden hier drei neuere Untersuchungen, die alle die Suche nach dem kürzesten Weg in einem Graph als zentrale Operation enthalten, vorgestellt.
Fußgänger, die öffentliche Verkehrsmittel verwenden wollen, brauchen nicht nur Anweisungen für ihre Bewegungen im Raum, sondern auch Hinweise auf Benützungsregeln der Verkehrsmittel (Ticket kaufen, abstempeln etc.). Diese Regeln können ebenfalls als Zustands-Übergangsgraph dargestellt werden. Es wird hier eine neuartige mathematische Formulierung für die Verbindung der zwei Zustands-Übergangsgraphen angegeben, die auf Kategorientheorie beruht und zur Programmierung geeignet ist.

1 Introduction

Geodesy makes important contributions to the construction of geoinformation systems (GIS). It is therefore not surprising that some of the pioneering geoinformation researchers were geodesists. Today, universities where a geodesy curriculum is offered have often a chair in geoinformation. The research group for geoinformation in the Institute for Geoinformation and Cartography of the TU Vienna stands in this tradition, with a background in geodesy and surveying and a strong research orientation in geoinformation.

From the technological beginnings of GIS in Canada by Tomlinson in the early 1960s and LIS in Europe (Eichhorn 1979) emerged a need to understand and further develop the foundational theory. Geoinformation Science (GISc) developed, with now several scientific journals and regular conferences, supplementing the large number of applications, national and vendor oriented conferences and magazines. GISc is an interdisciplinary enterprise, bringing together researchers from geography, geodesy, cartography, computer science, mathematics, experimental psychology, linguistics, cognitive science, etc. My contributions over the past 25 years were mostly in putting mathematical theory to use in GISc. Egenhofer investigated point-set topology to characterize the relations between spatial regions (Egenhofer 1989; Egenhofer et al. 1990; Egenhofer et al. 1991), which opened a field that has now many research publications and his results eventually were integrated in an ISO standard (SQL Multi-Media). Bittner generalized raster based indexing methods, as used in location indices for city maps (indications like“B7”, i.e., second row, 7th column), to irregular subdivisions of space (Bittner 1999). This was one of the precursors for the application of rough set theory (Zdzisław 1991) to GIS and is now an active field of research (Bittner et al. 2003).

In this article I address another core theme of GIS, namely, navigation in a discrete network—for example a street network. This can be seen as an application of the geodesic, though not as usual in continuous space, but in discrete space.

The next section introduces navigation as a major GIS application and section 3 abstracts the street-network to a graph. The following section discusses why operations on a graph are seldom expressible as closed, analytical formula. Section 5 reports two investigations using agent simulation to assess the quality of street-network data and signage in a building. The following section generalizes navigation as graph traversal further, to apply it to connected actions in general, preparing for the application to navigation aids for pedestrians using public transportation systems discussed in the following section. These systems must combine navigation in a street-network with the fulfillment of business rules. Section 8 then gives a general, category theory based, solution for this and similar problems. The concluding section suggests future work.

2 Navigation Is a Major Application of GIS

Computerized car navigation systems are probably the most popular application of geographic information systems. They go back to one of the first thorough analyses of an algorithm by computer pioneer Edsger Dijkstra (1959): the task to find the shortest path in a (street) network. This one or the faster A* algorithm (Hart et al. 1968) are at the core of the programs running in millions of car navigation systems and produce navigation advice for drivers all over the world.

Car navigation needs, besides the algorithm and the hardware to run it on, a schematic representation of the street-network, usually provided by surveyors and cartographers. Car navigation systems became feasible with the construction of computerized street-network graphs in the 1970s and 80s by the U.S. Bureau of the Census (Corbett 1979; Witiuk 1988) and later by many European jurisdictions (see SORSA conferences). The viability of this industry depends on the availability of the data collected and maintained and the quality of the data, in particular, the encoding. This led to a research strand investigating the representation of space (Mark et al. 1991) and how people cognize space (Twaroch 2007), but also to research in the application of ontology to geographic space (Smith et al. 1998; Kuhn 2005).

3 An Abstract View of a Street-Network

The famous problem to find a path leading once over all bridges in Königsberg (Kalinagrad) led Euler to abstract graph theory. For present purposes, a graph is a structure, consisting of nodes and edges (many other terms are in use), related by incidences and adjacency.

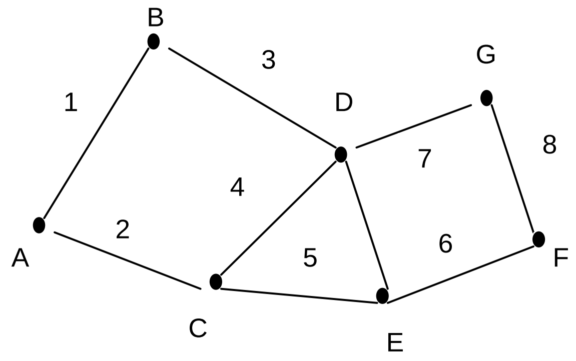

Figure 1: Example street-network graph

Node A in Figure 1 is incident with edge 1. The edges 1 and 2 are adjacent because they have node A in common; similarly, the nodes A and B are adjacent, because they have edge 1 in common.

4 Operations on Graphs

In continuous space the shortest path from A to B is called the geodesic (line). In a discrete space like a graph, this problem has in general no analytical solution. Most operations on a graph require some sort of traversal of the graph and at each node or edge traversed an operation is performed,. These operations are usually simple: count, find minimum or maximum. Efficient algorithms to determine the shortest path avoid traversing the complete graph and are the more effective the less blind alleys the algorithm chases and the more directly it is targeting the goal; this is the reason why A* is more efficient than Dijkstra's method—if applicable.

For some regular graphs (e.g., complete graphs) analytical solution for some questions are known. Recently random graphs (Barabási 2003) attracted attention, because they allow a statistical analysis. In general, however, solutions for applied graph problems cannot be given by analytical methods and require some sort of traversal, for which we use the conceptual framework of multi-agent simulation. In a computational agent simulation a programmed agent traverses a simulated network (graph) and performs at each node and edge some functions that determine the next action. The behavior of a person navigating in a street-network maps naturally to such a computational agent.

5 Agent Simulation for Navigation Problems

Initially, in our research on navigation problems we approached two quite different questions:

1. to assess the affects of data quality of street-network data on navigation (Krek 2002).
2. to identify places in a network, where road signs are missing (Raubal 2001).

The first reflects a geodesists concern for the quality of data, whereas the second is exploring novel applications for the geoinformation industry.

5.1 Assess effects of quality of street-networks on navigation

The quality of the data describing the street-network influences the quality of the navigation instructions produced. Before we can address the question to find a relation between data quality and navigation information quality, we have to define data and navigation quality. Quality of street-network data can be—in a first approximation—characterized by

1. the precision of the node positions (coordinate values),
2. the completeness of the connections between the nodes (omissions and commissions).

The quality of the navigation instructions is measured by the length of the path resulting when an agent follows the instructions till the goal is reached. Optimal instructions lead to the shortest path. Incorrect instructions lead to situations where the planned path cannot be executed, for example, because a one-way street cannot be traversed in the intended direction, this results in a longer than optimal path.

Krek constructed a simulation of an agent following instructions to navigate a street-network and she varied the amount of one-way streets not included (omissions) as the dependent variable representing data quality. The result for a part of the street-network of Vienna was:

- even quite large (meter) position inaccuracies have very little effect on the quality of navigation instructions
- omissions and commissions of one-way streets up to 25% have nearly no effect on the navigation quality.

5.2 Completeness of signage

Street signs and signs in buildings are crucial in all situations where the visitors do not know and do not intend to learn the environment. Raubal selected an airport situation, where a sufficient number of signs is crucial for passengers to find their departure gate in time. The simulated agents navigate in the graph representing the halls and corridors of the airport and follow only the signs they find at each bifurcation where they must make a decision. Three kinds of errors encountered are:

1. there are no relevant signs at a bifurcation point,
2. the signs do not lead to the desired goal, and
3. following the signs results in an infinite loop.

The simulation permits to test new buildings and the planned signage to avoid confusion and frustration of visitors.

6 Generalization: State Transition Diagram

Navigation in a street-network consists of a series of actions that lead to states; when a bifurcation point is reached, a decision about the next action is taken, following some rule incorporated in the agent. This schema of *action—state—decision—action* is applicable to other problems, where a sequence of steps is necessary to achieve a goal. The correspondence is well known and is widely applied to determine, e.g., the critical path in large projects (CPM). In using a public transportation system, a passenger follows the graph in Figure 2. Realistic business rules may contain bifurcations, when users have options, e.g., buying different types of tickets.

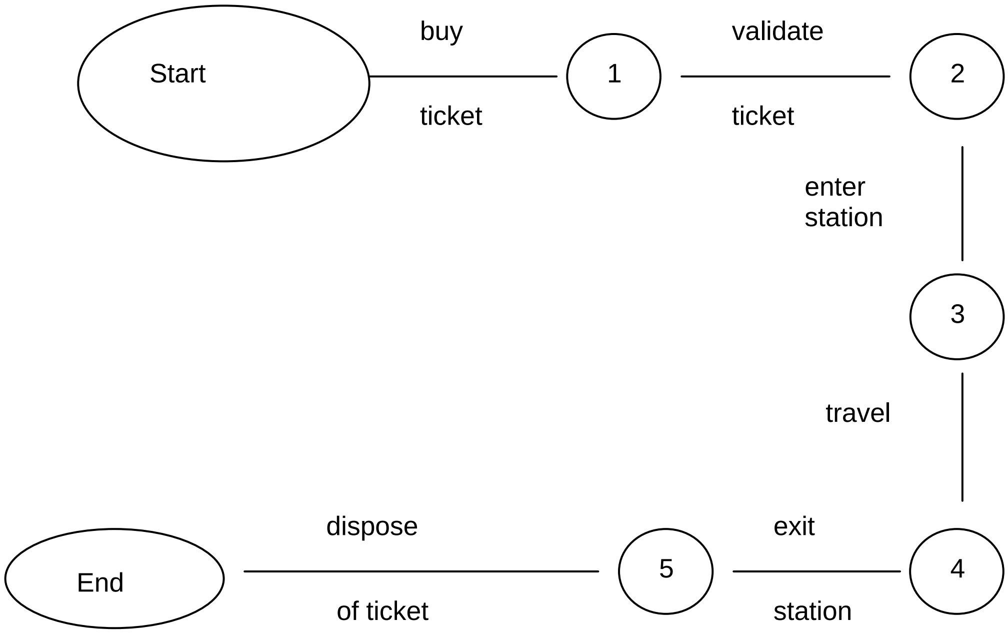

Figure 2: Graph, which describes actions necessary for a legal ride on Vienna's metro

7 Navigations Systems for Using Public Transportation

The future of large cities is threatened by the overflowing individual vehicle traffic; methods to make public transportation more attractive are urgently needed. We investigated how portable navigation aids (PDA, mobile phone or similar) could be used to assist users. Using a public transportation system requires surprisingly large amounts of information: one needs to know where lines are running and where stops are, but one needs also information about ticketing and where to buy tickets—tobacconist's shop, for example, are not an obvious place!—and what other actions are required to legally ride a bus or tram (e.g., validation of ticket). Our scientific goal was to understand how to combine spatial navigation with the "navigation" of the state-transition diagram of the public transportation rules (Figure 2 is an extremely simple example); this latter case will be called "business navigation" (Pontikakis 2006). An agent simulation was constructed and demonstrated clearly the typical problems; e.g., how do agents recognize the stop where they have to exit a vehicle?

8 Generalization

The result obtained was only partially satisfactory, because it was based on a single state transition diagram, which integrated spatial and "business navigation". In this section I will show how category theory gives a general solution that allows the integration of two (or more) state transition diagrams formally. Given two navigation problems with state spaces *W* (wayfinding) and *B* (business), and actions *V* and *T*. The state-transition diagrams are then described by two (partial) functions

$$w: W \times V \rightarrow W$$

$b: B \times T \rightarrow B.$

A combined state-transition diagram with states *S* and actions *P* is described as function

$f: S \times P \rightarrow S.$

How to construct *f* from *w* and *b*? The function *f*, representing the combined state-transition diagram describes the total behavior of the passenger; this is given separately as a set of navigation rules *w*—which are essentially a graph encoding of the city map with streets and public transportation lines—and the "business rules" *b* about tickets, how they are obtained, validated, etc.

A fully general solution can be achieved using category theory (Mac Lane 1998), a generalization of universal algebra (Whitehead 1898; Asperti et al. 1991). In this framework, we can write

$S = W \times B$
$P = V + T$

and then

$f: S \times P \rightarrow S$

becomes

$f: (W \times B) \times (V + T) \rightarrow (W \times B).$

The set of states of the combined wayfinding problem is the product (x) of the states *W* and *B* from each problem; each state of the combined problem consists of a pair of states of the individual problem. The set of actions is the union (+) of the actions *V* and *T* of the separated problem; a passenger can perform at each decision point a wayfinding or a business action. We construct two trivial operations (isomorphisms) *h*, *k* such that

$h: (W \times B) \times V \rightarrow (W \times V) \times B$
$k: (W \times B) \times T \rightarrow W \times (B \times T).$

From the given functions *w* and *b* we construct functions *w'* and *b'*

$w': (W \times V) \times B \rightarrow W \times B$
$w' = w \times id \cdot h$
$b': (W \times B) \times T \rightarrow W \times B$
$b' = id \times b \cdot k$

id: A → *A* denotes in category theory the function that does nothing. The function *l* is an isomorphism

$l: (W \times B) \times (V + T) \rightarrow (W \times B) \times V) + (W \times B) \times T)$

because we operate here in the distributive category of sets (Cockett et al. 1992). Combined this gives for *f*

$f = [w', b'] \cdot l = [(w \times id) \cdot h, (id \times b) \cdot k] \cdot l$

This solution translates directly to an executable computer program in a modern programming language that includes second order operations (like [,], ·, *id*) and categorical concepts (for example Haskell (Peyton Jones et al. 1999), which produced the expected result and demonstrates the correction of the solution.

Two or more navigation problems in real space and "business logic" space can be directly combined with this formula and we produce a state transition function that can be used for shortest path and similar algorithms. The formula needs to be extended to include possible interactions between space navigation and business rules, e.g., passengers must not enter the station unless they are in possession of a validated ticket.

The compact formula together with a description of the interaction may make it possible to analyze the interaction between the two navigation tasks—a probably fruitful field for future research.

9 Conclusions

The development of geographic information systems from a niche technology benefiting from geodesy, surveying, and cartographic theories to an important industry with its own body of theories has led to geographic information science. Assisting humans with navigation and wayfinding tasks is one of the most important applications of GIS—and most likely the one used by more people than any other. The article showed how graph theory and agent simulation are used to analyze the experiences expected for agents with a certain behavior. We have used it to assess the influence of street-network data quality on the quality of navigation instructions and to identify points in a building where existing signage is not sufficient. Such simulations are crucial to identify short comings in signage, which can lead to disasters and loss of life (Hajibabai et al. 2006).

The goal to construct portable navigation aid devices for pedestrians using public transportation systems requires the integration of navigation in space with "navigation in business logic". The novel contribution of this paper is a categorical formula to construct a combined state-transition function from individual ones. The compact representation of the combined formula suggests future research in the interaction between the two wayfinding tasks. Is it possible to identify situations where the individual problems can be solved, but the combined problem—which is the one relevant for the user—forces passengers to wasteful behavior, e.g., forth- and backward movements, or leads even to impossible catch-22 situations (e.g., you need a ticket to enter the station, but tickets are only sold inside)?

The discussion of three practical problems related to the "geodesic" in a discrete graph space and the solution found in a corresponding part of modern algebra could remind geodesists of the advancement achieved by C.F. Gauss in geodesy by extending the mathematics of continuous (real) spaces.

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