General Training of Spatial Abilities by Geometry Education in Augmented Reality

Hannes Kaufmann[1], Karin Steinbügl[1], Andreas Dünser[2], Judith Glück[2]

[1] Institute of Software Technology and Interactive Systems, Vienna University of Technology, Favoritenstr. 9-11/188-2, A-1040 Vienna, Austria; kaufmann@ims.tuwien.ac.at
[2] Institute of Psychology, University of Vienna, Liebiggasse 5, A-1010 Vienna, Austria.

Abstract

Geometry education has proven as one powerful means of improving spatial abilities, an important component of human intelligence. In the first part of this paper we summarize our development of a system that uses collaborative augmented reality as a medium for teaching, and uses 3D dynamic geometry to facilitate mathematics and geometry education. Our immersive collaborative educational application, specifically developed for geometry education, serves as the basis of a comprehensive evaluation study regarding its efficacy in training spatial abilities. The main contribution is the description of evaluation design including the test instruments, learning tasks and practical experiences with using our system for actual training of high school students. Results of a pre-study with spatial ability tests in high schools are presented. They point to interesting gender-specific differences of strategies when solving spatial ability tests, which have not been reported in literature before.

Keywords

Spatial ability, training, geometry education, augmented reality.

Introduction

Spatial abilities present an important component of human intelligence. Many studies have shown that spatial abilities can be improved by well-designed trainings [32]. Geometry education has proven as one powerful means of improving these skills [10]; recently, a number of training studies have shown the usefulness of virtual reality (VR) in training spatial ability [9, 29]. However, little to no work has been done towards systematic development of VR applications for practical education purposes in this field.

- No VR/AR application for actual use in high school or higher education has ever been developed with the main purpose of improving spatial skills.
- Hardly any evaluations can be found in literature which give hints to the actual learning transfer from a VR/AR learning environment to the real world.

The authors initiated a national research project which addresses these two key issues by developing a prototype AR system for geometry education. As a pilot study for this proposal, a three dimensional geometric construction tool called Construct3D [16-18] has been developed that serves as a platform for our work. A comprehensive evaluation study will be conducted in order to study the general and differential effects of the training on several components of spatial ability.

In this paper, we briefly describe our experiences in developing and using Construct3D in high school geometry education. To provide a natural face-to-face setting for teachers and students, this system uses an immersive setup, more specifically a collaborative augmented reality (AR) setup, based on see-through head-mounted displays (HMD). The main advantage of using AR is that students actually see three dimensional objects which they until now had to calculate and construct with traditional - mostly pen and paper - methods (Figure 1). The face-to-face setting allows for traditional pedagogic communication.

Figure 1: Students are working in 3D space with Construct3D.

It is important to note that while *geometry education* software shares many aspects with conventional computer aided *design* software at a first glance, its aims and goals are fundamentally different. Geometry education software is not intended for generating polished results, but puts an emphasis on the construction process itself. While relatively simple geometric primitives and operations will suffice for the intended audience of age 10 to 20 (especially 10 to 14), the user interface must be both intuitive and instructive in terms of the provided visualizations and tools. Commercial CAD software offers an overwhelming variety of complex features and often has a steep learning curve. In contrast, geometry educators are interested in simple construction tools that expose the underlying spatial process in a comprehensive way. The interface might be restricted to their students needs. In accordance to that our aim with Construct3D was not to create a professional 3D modeling package but a simple and intuitive 3D construction (not animation) tool in an immersive environment for educational purposes.

The main part of our work concentrates on our evaluation study - the test instruments, participants and the study design. We intend to address the following research questions in our evaluation study:

1. Effect of the training on performance in tasks central to the training;
2. Transfer of the training effect to more distant spatial tasks;
3. Effect of the training on strategy use in central and distant spatial tasks;
4. Aptitude-treatment interactions: Dependence of individual training effects on pretest spatial ability, verbal ability, and reasoning ability;
5. Gender differences in training effects.

Related Work

For the development of any educational VR/AR application, technological, domain specific, pedagogical and psychological aspects are of importance. Accordingly, literature from different and diverse research areas relates to our work: VR/AR collaborative distributed systems, desktop 3D modeling, immersive modeling, educational 2D/3D applications, dynamic 2D geometry, parametric CAD, pedagogic theories such as constructivism or activity theory and psychological literature from the field of spatial ability research. We will briefly mention work related to training and psychological aspects of this paper. For a comprehensive overview of related work we refer to [16].

Educational VR Applications

Since the early 1990th researchers have been working on virtual reality applications for purely educational use ([1, 5, 22] and many others). In the area of mathematics education the most recent and most advanced project is CyberMath [34]. CyberMath is an avatar-based shared virtual environment aimed at improving mathematics education. It is suitable for exploring and teaching mathematics in situations where both the teacher and the students are co-present and physically separated. A very good summary of educational and training applications is given by Mantovani [25].

Training Spatial Abilities in VR

Training of spatial abilities in VR is of strong interest in application areas such as surgery [35], navigation and way finding [8], for rehabilitation of patients with psychiatric illnesses such as traumatic stress disorder, schizophrenia, and Alzheimer's disease [28, 29] or for pre-flight space training.

A recent article by Durlach et al. [9] gives a very good overview of work that has already been done in the area of enhancing spatial skills within virtual environments but mainly identifies the indispensable need for comprehensive future research in this area.

Spatial Cognition

Psychological literature on spatial cognition ranges from studies on the way people process simple spatial objects to studies about how people orient themselves in real environments. In the following, we give a brief introduction into spatial cognition with references to related work.

Several authors [6, 13, 23, 33] suggested how spatial ability can be structured into sub domains. Most of the proposed structures focus on relationships and similarities among spatial tests and were developed through analysis of test inter-correlations. The results of structural analyses of spatial ability are highly convergent. In addition to some simple, basic performance aspects, at least two factors are consistently reported: “Spatial Relations” (as labeled by Lohman [23]), that is, speeded mental rotation, and “Visualization”, which includes all complex, multi-step spatial tasks. Tasks involving three-dimensional mental rotation are somewhat intermediate and have been grouped into each of these two factors. Tasks requiring participants to imagine different perspectives either form a third factor or are grouped into Spatial Relations.

A different approach to thinking about spatial cognition is by analyzing the processes people actually use to solve spatial tasks. Basically, all spatial tasks can be solved in different ways, and the more complex a task is, the more different strategies can be used to solve it (overview in [24]). People differ in the strategies they use, and people shift their strategies within a task if necessary. A basic distinction is between holistic and analytic strategies [11]. Holistic strategies involve representing and manipulating spatial information “in a spatial way”, that is, mentally using information about spatial relations between elements in the mental representation. A person using holistic strategies imagines, for example, how a stimulus figure is rotated, or how a two-dimensional shape can be folded into a three-dimensional object. Analytic strategies reduce spatial information to an essentially non-spatial, serial format. For example, a route can be represented as a list of landmarks, and the spatial relations among the patterns on a cube can be represented as a list of relations among pairs of patterns. Thus, the complexity of spatial information is reduced, and the focus is on parts of the object or the environment rather than on the object as a whole. Compared to holistic strategies, analytic strategies usually take more time, but less mental effort, as the information represented is less complex. Note that analytic and holistic strategies should not be viewed as mutually exclusive categories.

Gender differences in strategy use have frequently been found in studies of environmental orientation and navigation. Men more often acquire map-like knowledge about their environment, use euclidean information, and are aware of relative directions, whereas women more often rely on landmarks and represent routes as lists of landmarks. These gender differences have been found in self-reported strategy use in new environments [21] and in use of landmarks and euclidean information in learning routes from maps and giving directions ([4] and others). With respect to spatial tests, gender differences in strategy use have not been studied much, but at least one [11] found that men more often used holistic strategies than women, and women more often used analytic and mixed strategies than men in two different spatial tests.

Construct3D

Construct3D is based on the Studierstube AR system [30] and uses augmented reality to provide a natural setting for face-to-face collaboration of teachers and students.

Hardware Setup

In our standard Studierstube setup, we have 2 collaborating users wearing HMDs for a shared virtual space, and holding fully tracked interaction props in their hands. One dedicated host with 2 graphic ports renders stereoscopic views for both users. We are using this setup for demonstrations and evaluations. Other setups for educational use have been reported in [16].

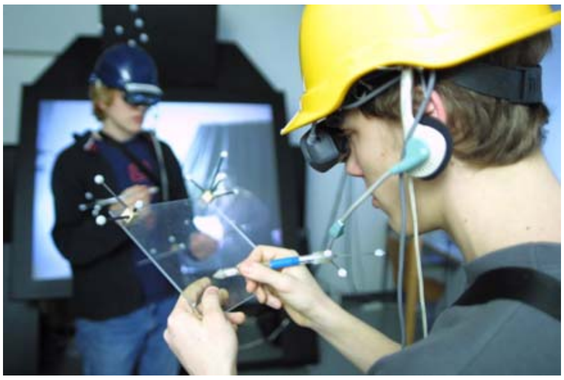

Figure 2: Two students collaborate in our standard lab setup, wearing an HMD, holding a wireless pen and panel. All devices are optically tracked.

Software Design

The current version of Construct3D offers functions for the construction of points, two-dimensional geometric primitives and three-dimensional geometric objects. It provides functionality for planar and spatial geometric operations on these objects, allows measurements, features structuring of elements into layers and offers basic system functions.

Construct3D promotes and supports exploratory behavior through dynamic geometry. A fundamental property of dynamic geometry software is that dynamic behavior of a construction can be explored by interactively moving individual defining elements such as corner points of a rigid body. It can be seen what parts of a construction change and which remain the same. The histories of constructions as well as dependencies between geometric objects are maintained. Experiencing what happens under movement allows better insight into a particular construction and geometry in general.

At its start Construct3D initializes a 3D window which has maximum size to cover the "whole" virtual space. The user interface is initialized, the menu system is mapped to a hand-held tracked panel called the personal interaction panel (PIP). The PIP allows the straightforward integration of conventional 2D interface elements like buttons, sliders, dials etc. as well as novel 3D interaction widgets. Haptic feedback from the physical props guides the user when interacting with the PIP, while the overlaid graphics allows the props to be used as multi-functional tools.

All construction steps are carried out via *direct manipulation* in 3D using a stylus tracked with six degrees of freedom. In order to generate a new point the user clicks with his pen exactly at the location in 3D space where the point should appear. Users can easily switch between point mode (for setting new points) and selection mode (for selecting 3D objects). We omit a detailed description of the user interface in this context.

All operations consistently support dynamic modifications of their input elements and reevaluate the resulting elements accordingly.

Learning Tasks

To illustrate the type of geometric examples and learning tasks that are best suited for teaching geometry with Construct3D we list actual educational tasks and examples that we used in previous evaluations. Students of age 16-18 who have geometry classes in high school worked collaboratively with a teacher and another student on these examples.

Surface of Revolution

Given an axis, students have to construct a surface of revolution by rotating a B-Spline curve (cubic, 5-6 control points) around the axis. The control points can dynamically be modified at any time resulting in a change of the surface of revolution.

In the next step, the tangential plane in an arbitrary point P of the surface must be found. Therefore pupils have to construct a meridian curve through P which they get by intersecting the surface with a plane through the axis. They also have to rotate the point around the axis to get its circle of latitude on the surface of revolution. The two tangents to the circle of latitude and to the meridian curve in P define the tangential plane. Figure 3 (left) shows the result of this construction.

Dynamic modifications of the start elements and all constructed parts allow to study and explore the construction. Students see immediately and learn that the surface normal in any point of the surface intersects the surface's axis, which is an important property of surfaces of revolution.

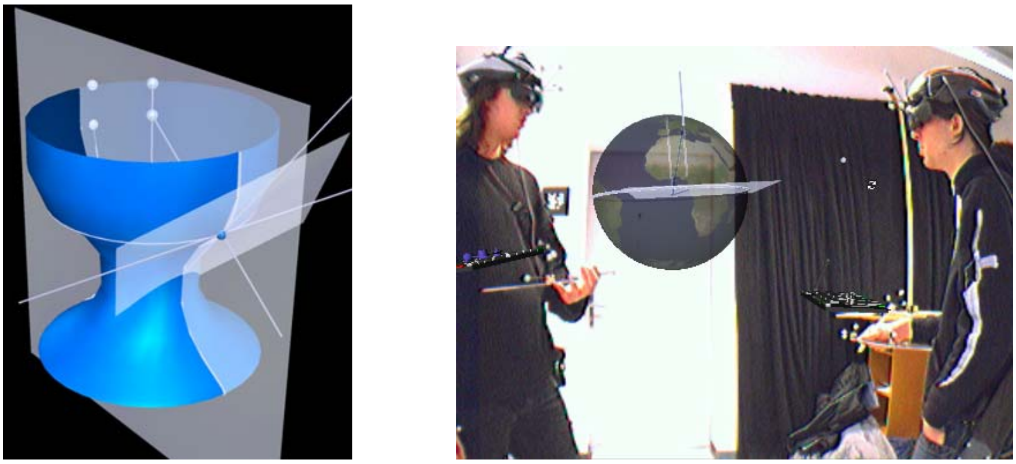

Figure 3: Left: Surface of revolution with the tangential plane in a surface point. Right: Two students are collaborating to solve the problem. Vienna is marked on the globe and a geostationary satellite (in the equatorial plane) is given too.

Adjusting a Satellite Dish

In the second problem a satellite dish, positioned in Vienna, has to be adjusted to point to the TV-SAT2 satellite, which is given. Students have to translate this real life problem into a geometric problem to be able to identify two angles which are needed to adjust the satellite dish. Web links are presented with additional information about geostationary satellites; given images help to understand and translate the problem. The given virtual scene in Construct3D (Figure 3 right) shows a model of earth (with texture) to help pupils find the correct places on earth and to immerse them further into the problem.

Deflection Sheave

A rope is being redirected from one given position to a final given position. Two deflection sheaves, which are drawn as circles redirect the rope. They have to be constructed as well as the piece of connecting rope between them. Only start and end position of the rope are given as well as the mid point of the first deflection sheave. Deflection sheaves can be found at skiing lifts, elevators and in many other machines. Students got an explanatory draft to lead them towards a correct solution.

Usability Evaluations

System Evaluation

Based on feedback from many trials with real high school students, we continuously improved Construct3D over a period of 3 years. In early 2004, we conducted a larger study based on interviews and the standardized ISONORM 9241/10 usability questionnaire [26] after systematic exposure of 15 students (9 male, 6 female) to Construct3D. A number of training exercises that fit the students' 11th and 12th grade curriculum were designed by us and then worked through by students from two high schools with the aid of their teachers. All students attended to geometry classes (descriptive geometry) since the beginning of grade 11. Each of them participated in 5 training sessions lasting 6 hours. Our main objective was to at least informally assess the usability and potential of our system and method for real high school work.

Construct3D Usability Results

At the end of all training sessions students had to answer an ISONORM usability questionnaire. Two questions regarding self-descriptiveness of the application had to be removed since they were related to desktop applications only. Afterwards students answered general questions regarding user acceptance, user behaviour, technical requirements and organisational aspects.

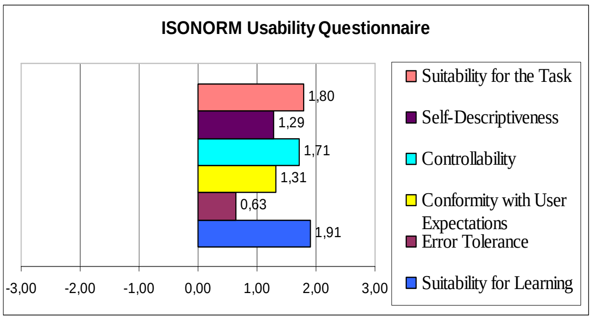

Table 1 summarizes the results of the ISONORM usability questionnaire in the 6 categories “suitability for the task”, “self-descriptiveness”, “controllability”, “conformity with user expectations”, “error tolerance” and “suitability for learning”.

A closer look at the data reveals that the categories “suitability for learning” and “suitability for task” received the highest grading which is very important in this context. In our opinion the highest priorities for an educational application that complies with pedagogic theories such as constructivism are that it (1) is easy to use and requires little time to learn, (2) encourages learners to try new functions and (3) can be used consistently and is designed in a way that things you learned once are memorized well. These are exactly the items that students rated very high. Nearly all students reported that they can imagine using the current version of Construct3D in high school or university education.

The categories “self-descriptiveness” and “conformity with user expectations” got lower grades than the rest. The self-descriptiveness of Construct3D is currently improved by adding better labeling and tool tips to widgets on our panel in order to explain all menu items. We are also restructuring the menu system to give a better overview of the functionality.

Some of the students reported negative side effects after working in the virtual environment. One female student reported headache and eyestrain after 20 minutes of work in the virtual environment but did not stop working and wanted to use Construct3D again (in total she worked for 3 hours with the system). In retrospect we know that our one hour lessons were simply too long for continuous work with an HMD. Since negative side effects are a general problem when working with HMDs and influence the user’s subjective experience of a VR/AR environment considerably they are relevant to all VR/AR applications that use HMDs. We identified some possible reasons of cybersickness that may be relevant to our virtual environment such as accommodation problems, low frame rate, lag or bad fitting helmets. In the meantime we already reduced some of these negative side effects (using light weight bicycle helmets and increasing frame rate).

We must stress that we designed Construct3D to be a valuable *addition* to traditional or computer aided geometry education and *not a substitute* of it. A very reasonable time for working and teaching with Construct3D during a class is about 20-30 minutes. According to our observations, this would guarantee that less than 5% of all students feel any negative side effects such as eyestrain or even headache. Of course it would be desirable to find the exact reasons of cybersickness and ways to avoid it – as pointed out by other researchers [20].

Evaluation Design on Training Spatial Abilities

The presented system serves as a base of further research and allows to do more in depth evaluations of various aspects regarding VR/AR learning that have not been studied before.

Currently we focus on the largest evaluation with Construct3D yet, involving over 250 students. It concentrates on 5 main questions about training and improving spatial abilities as mentioned in the introduction.

Test Groups

We will conduct a pre-/posttest experiment with four different training groups and one untrained control group. At pre-test, all participants will be presented with a battery of spatial tests (including strategy assessments) and tests of verbal and reasoning ability. Then, each participant will be randomly assigned to one of five groups:

a) Untrained control group: Participates in the pre- and post-test but does not receive any training;
b) Traditional school group: Participates in the pre- and post-test, but is taught geometry in their normal school classes between pre- and post-test with traditional paper & pencil methods;
c) Modern instruction/school group: Participates in the pre- and post-test, but is taught geometry in their normal school classes between pre- and post-test on the computer using CAD software;
d) Individual tutoring group: works with a tutor using standard teaching methods without Construct3D;
e) Construct3D group: works with Construct3D.

The reason we use an individual tutoring group is that this condition is probably “harder to beat” than standard class-wise Descriptive Geometry (DG) instruction. Tutors are likely to use hand drawing and sketching as teaching methods, thus, they will provide a “hands-on” experience that has proven to be particularly effective in other training studies. However, we expect the Construct3D tutorial to lead to broader transfer of training gains than the tutoring. As women more often than men use analytic strategies on spatial tests, the Construct3D training, which focuses on visualization, might have different effects on female than on male participants. It is certainly interesting to see how different types of training affect such differences.

Between pre- and post-tests five training sessions will be conducted. Participants will be students in their 11th and 12th year of schooling, attending secondary schools in Vienna. Austria has several different secondary-education curricula, some of which have a focus on the natural sciences, including Descriptive Geometry (DG). DG classes start in the 11th form.

Instruments

The following instruments will be presented to participants at pre- and post-test. A battery of spatial tests as listed below will be used:

- Mental Rotations Test (MRT; [36]), German version by [27]: speeded mental rotation.
- Purdue Spatial Visualization Test: Rotations (PSVT:R; [12]): shortform: mental rotation combined with visualization.
- Mental Cutting Test (MCT; [7]): shortform: visualization.
- Differential Aptitude Test: Space Relations (DAT:SR; [2]): shortform: visualization.
- Spatial Orientation Test (SOT, [19]); mental rotation – large scale
- Judgement of Line Orientation Test (JLO; [3]); visuospatial perception and processing.

Reasoning and Verbal Ability Tests:

- “Letter Sequence”: subtest from the German intelligence test battery “Wilde Intelligenz Test” (WIT) by [14]: shortform
- “Verbal analogies”: subtest from the German intelligence test battery “Berliner Intelligenzstruktur Test” (BIS) [15]
- "Vocabulary” ("Wortschatztest" WST) [31]: shortform

Strategy Questionnaire: Assessment of different aspects of holistic vs. analytic strategies, the most important ones are “Visualization / Imagination” vs. “Non-visual Reasoning / Thinking” and “Focus on the object as a whole” vs. “Focus on details”.

These tests have also been used at two preliminary studies that we did in Austrian high schools. In the first study 42 students (39 males, 3 females; aged 17-19 years, 11^{th} grade) participated, in the second bigger one there were 79 students (44 females, 33 males, 2 unknown; aged 16-19, 11^{th} and 12^{th} grade). Due to time restrictions in schools, we developed two short versions of each test except the MRT which was used in its full length. We split the tests (PSVT:R, MCT, DAT:SR) into halves by simply assigning all odd items to test version A and all even ones to test version B. In order to eliminate obvious inequalities, we thereafter exchanged some items or assigned some items to both test versions. The final test forms consisted of 15 (MCT and PSVT:R) respectively 25 (DAT:SR) items. The time limits set for the tests were calculated based on the regular test versions and the number of problems selected for the short versions and were therefore 12.5 minutes for the PSVT:R, 12 minutes for the MCT and 12.5 minutes for the DAT:SR. Our short test versions had reliability coefficients (Cronbach's alphas) ranging form 0.66 to 0.90.

Preliminary Study

The purpose of the preliminary study was to examine some frequently employed spatial tests that contain different kinds of tasks and are supposed to affect different aspects of spatial ability in regard to performance, strategy use and the relation between these two variables. More precisely, we aimed to find answers to the following questions:

a) How do performances in different spatial ability tests correlate with each other and what is the nature of the relationship to cognitive tests measuring analytical abilities (reasoning and verbal abilities)?
b) Are there *intra*individual differences in strategy use across tests? Do different spatial tests evoke different strategies?
c) Are there *inter*individual differences in the use of strategies? Do participants show a tendency to use holistic or analytic strategies irrespective of the nature of the task?
d) How are performance and strategy use associated with each other?

Results

The first study (39 males, 3 females; aged 17-19 years, 11^{th} grade) aimed to examine associations and intra- and inter-individual differences between several spatial ability tests regarding performance and strategy use and to make connections between performance and strategy use. The spatial ability tests used were the MRT, the PSVT:R, the DAT:SR and the MCT together with reasoning (verbal and non-verbal) and verbal ability (vocabulary) tests.

In an explorative pre-study subjects were asked to solve problems of these tests by thinking aloud. The findings of this pre-study suggest that a major part of these problems (especially PSVT:R and DAT:SR problems) can be successfully solved not only in a spatial (holistic) way but also by using analytic non-spatial strategies. The way the MCT was solved indicates that (geometry) knowledge may also be an important factor.

Applying a factor analysis on the test scores 3 factors could be identified: spatial ability (with loadings of the 4 spatial ability tests as well as a lower loading of non-verbal reasoning tests), reasoning ability (with loadings of both reasoning tests), and vocabulary. There were positive significant correlations between the DAT:SR and the PSVT:R on one hand and the non-verbal reasoning test on the other hand. This finding goes together with the one from the pre-study indicating that DAT:SR and PSVT:R scores are not a pure measure of "real" spatial (holistic / visualization) ability but confounded by reasoning abilities.

Analyses of the strategy use questions reveal that, on the one hand, there are differences in strategy use on an inter-individual level: Significant positive correlations over the tests indicate that subjects tend to certain strategies that are rather independent from the kind of spatial task. On the other hand, these personal strategy tendencies are not completely unaffected by the nature of the spatial test: The use of holistic strategies is significantly higher for the MRT as for the other three spatial tests.

When drawing the conjunction between performance and strategy use, we found that there were no correlation between performance and strategy regarding the DAT:SR, the PSVT:R, and the MCT, i.e. subjects using mainly holistic strategies could *not* achieve higher scores than subjects using mainly analytic scores. However, there was a significant correlation between strategy use and performance in the MRT, indicating that “holistic” subjects scored slightly better than “analytic” subjects.

Considering these results, it seems that the MRT stands out from the other three spatial tests we used in this study: It is processed more holistic than the other tests and the use of holistic strategies has more (positive) influence on the performance compared to the DAT:SR, the PSVT:R and the MCT.

Altogether, our findings give some evidence that spatial ability tests – in particular the PSVT:R and the DAT:SR - can be successfully solved not only with “real” spatial (holistic) strategies but also with analytical strategies – at least as long the time limit set is ample enough as these analytical strategies are generally more time-consuming than holistic processing. Results from an explorative pre-study also suggest, that knowledge plays an important role in the solving process of the MCT. Though our results are based on a rather small sample it seems to be justifiable to draw the tentative conclusion that the scores of spatial ability tests do not necessarily reflect “real” spatial abilities in the sense of holistic processing and visualization and are also affected by other factors like reasoning abilities and knowledge. These considerations imply that these tests probably do not differentiate between subjects with high and low spatial abilities as intended in certain cases. This should be kept in view especially when spatial ability tests are used for purposes like selection or career choice procedures.

In the second study with 79 students (44 females, 33 males, 2 unknown; aged 16-19, 11^{th} and 12^{th} grade) we added the JLO, which is usually used for clinical neurological assessment, to the test battery. This test examines the ability to estimate angular relationships between line segments and serves as an instrument to measure skills for recognition and processing of simple visual stimuli. We can only present preliminary results of this study yet but it points to very interesting gender specific differences in strategies used for solving spatial tests.

If we look at the test scores separated by gender we see that in case of male students almost all spatial ability scores are significantly correlated but performance in a logical reasoning test (WIT Letter Sequence subtest) is not correlated to any of them. In the female students, almost all spatial ability tests are significantly correlated both to each other and to the logical-reasoning test.

Separate factor analyses for male (Table 2) and female (Table 3) students identified two factors in both cases, however, factor loadings are different for men and women.

	Component	
	1	2
PSVT:R score	**,661**	,196
MCT score	**,735**	-,141
MRT score	**,632**	,231
DAT score	**,836**	,083
WIT score	,187	**,888**
JLO score	**,826**	-,056

Table 2: Component matrix for male participants

	Component	
	1	2
PSVT:R score	**,689**	,363
MCT score	**,761**	,269
MRT score	,094	**,890**
DAT score	**,707**	,481
WIT score	**,736**	,392
JLO score	,211	**,682**

Table 3: Component matrix for female participants

For men (Table 2) PSVTR, MCT, MRT, DAT, and JLO load on factor 1, which is clearly a “spatial” factor, whereas only logical reasoning loads on factor 2. For women (Table 3), PSVTR, MCT, and DAT, which are complex spatial tasks, load on factor 1, together with logical reasoning. MRT and JLO, which focus on elementary spatial functions, load on factor 2.

When logical reasoning is partialled out of the correlations among the spatial tests, the correlations in the male sample do not change much, whereas in the females, almost all correlations among spatial tests disappear. The reverse pattern shows when JLO performance is partialled out: In the male sample, all correlations among spatial tests disappear, while in the female sample, only the correlations of the MRT to the other spatial tests disappear and all correlations to logical reasoning get stronger.

These preliminary results suggest that in female students, high spatial performance is related to good reasoning skills and logical thinking, whereas for male students, speed and accuracy of basic spatial processes are the best predictor of spatial test performance.

Future Work

Within the next few months our evaluation regarding spatial intelligence, as described in previous sections, will be conducted. In the future we plan to study general improvements of spatial ability in contrast to task specific improvements in detail. We want to seek for an answer to the question if direct manipulation as used in Construct3D and explorative interaction is sufficient to improve spatial understanding and last but not least if training of spatial intelligence in a virtual environment is more “efficient” than in a real environment.

Further, we plan to utilize Construct3D as a tool for evaluating various aspects in virtual environments in our future research. For example a comprehensive pedagogic evaluation, evaluating e.g. teaching styles or transfer of learning in educational VR/AR applications would be interesting.

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