THE POTENTIAL OF AUGMENTED REALITY IN DYNAMIC GEOMETRY EDUCATION

Hannes KAUFMANN
Vienna University of Technology, Austria

ABSTRACT: This paper summarizes work done within the previous 6 years to integrate and implement Virtual and Augmented Reality (VR/AR) technologies in geometry education. A VR/AR application supporting dynamic 3D geometry is presented, intended for real use in high school and university geometry education. Related areas such as dynamic three-dimensional geometry, usability and user interface design, spatial abilities, pedagogy and low-cost VR systems influenced our work and the development of the educational geometry application Construct3D.
After describing the design of Construct3D, the strengths of this learning environment for geometry education are investigated. Dynamic three-dimensional geometric content is depicted that fully benefits from the advantages provided by the application.
In order to adapt software and hardware to users' needs, user interfaces were redesigned and in depth research was done on usability design. Very positive and useful feedback from teachers and students, who are excited by the possibilities, was collected in three evaluations with more than 100 students in over 500 teaching lessons. Results from these evaluations show that Construct3D is easy to use, requires little time to learn, encourages learners to explore geometry and can be used in a consistent way.
Various hardware setups have been studied that are suitable for educational purposes. An immersive setup that uses head mounted displays is most favored by teachers and students. It allows users to actually "walk around" geometric objects which are fixed in space.
Regarding spatial ability research, a currently running project for training spatial abilities with more than 300 participants is outlined. A review of shortcomings of existing spatial ability tests concludes with ideas to conduct future testings directly in VR/AR. Finally the recent development of a low cost optical tracking system, that allows to build affordable, immersive VR systems is described.
Our work enables teaching of three-dimensional dynamic geometry in an interactive, immersive learning environment, therefore offers new possibilities to modern geometry education. Despite the findings mentioned in this paper, many more research questions have emerged during the development of Construct3D. According to each of the related areas we indicate interesting topics that might require future work.
By summarizing our work, which aims to establish Augmented Reality in geometry education, and by providing insight into the problems and future challenges in each of the related areas, we want to highlight and advert the full potential of Augmented Reality in geometry education.

Keywords: Geometry Education, Collaborative Augmented Reality, 3D Dynamic Geometry.

1. INTRODUCTION

Spatial abilities present an important component of human intelligence. Many studies have shown that spatial abilities can be improved by well-designed trainings [23]. Geometry education has proven as one powerful means of improving these skills in general [5, 15]; recently, a number of training studies have shown the usefulness of virtual reality (VR) in training spatial ability [3, 21]. However, little to no work has been done towards systematic development of VR applications for practical education purposes in this field. No VR/AR application for actual use in high school or higher education has ever been developed with the main purpose of improving spatial skills.

This was the motivation to create an Augmented Reality (AR) application for geometry education. A three dimensional geometry construction tool called Construct3D [9, 10, 12] has been developed that serves as a platform for our work.

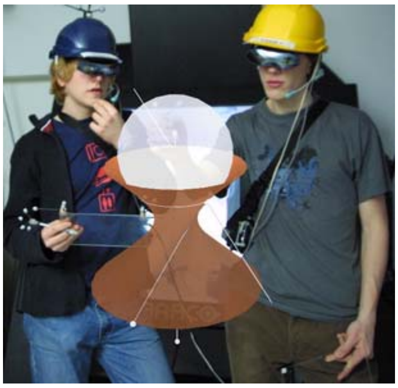

Figure 1: Students work with Construct3D in 3D space.

Augmented Reality (AR) [1] is mainly used (in contrast to Virtual Reality) to provide a natural setting for face-to-face collaboration of teachers and students. According to pedagogical theories [26], collaboration is a fundamental social process that supports learners' development of capabilities. Supporting natural collaboration in geometry education opens new possibilities to the educational process. The main advantage of using AR is that students actually see three dimensional objects which they until now had to calculate and construct with traditional - mostly pen and paper - methods (Figure 1). The face-to-face setting allows for traditional pedagogic communication.

The goal of creating an application for actual use in geometry education (in the long term) has many implications. It distinguishes Construct3D, being one of the longest actively developed educational VR/AR applications, from most other work in this area.

Construct3D is a combination of work done in various diverse research areas. Each of these areas will be mentioned after a basic description of Construct3D (section 2).

Creating innovative content for 3D dynamic geometry is a new challenge for educators where profound geometric knowledge is needed. Dynamic 3D geometry seems to be a new field of research and we will indicate some problems that were found during our work (section 3).

It is important to note that while *geometry education* software shares many aspects with conventional computer aided *design* software at a first glance, its aims and goals are fundamentally different. Geometry education software is not intended for generating polished results, but puts an emphasis on the construction process itself. While relatively simple geometric primitives and operations will suffice for the intended audience of age 10 to 20 (especially 10 to 14), the user interface must be both intuitive and instructive in terms of the provided visualizations and tools. Commercial CAD software offers an overwhelming variety of complex features and often has a steep learning curve. In contrast, geometry educators are interested in simple construction tools that expose the underlying spatial process in a comprehensive way. The interface might be restricted to their students needs. In accordance to that our aim with Construct3D was not to create a professional 3D modeling package but a simple and intuitive 3D construction tool in

an immersive environment for educational purposes (section 4).
In order to improve spatial abilities and to evaluate our work, psychological expertise in spatial intelligence is very useful (section 5).
Integrating a VR/AR application in education implies the use of a sound pedagogic theory (section 6).
VR/AR hardware is currently still very expensive. Therefore we are working on reducing the costs of the most expensive part, an accurate tracking system to acquire position and orientation data of the participants and their devices. Work on a low-cost infrared optical tracking system is summarized in section 7.

By presenting work done in these areas we give an insight into the potential that the use of Augmented Reality provides for geometry education.
We are fully aware that literature of diverse research areas relates to this work: VR/AR collaborative distributed systems, desktop 3D modeling, immersive modeling, educational 2D/3D applications, dynamic 2D geometry, parametric CAD, pedagogic theories such as constructivism or activity theory and psychological literature from the field of spatial ability research. A detailed literature review is omitted in the context of this paper though.

2. CONSTRUCT3D

2.1 Software Design

Construct3D is based on the Studierstube AR system [22]. It offers functions for the construction of points, two-dimensional geometric primitives and three-dimensional geometric objects. It provides functionality for planar and spatial geometric operations on these objects, allows measurements, features structuring of elements into layers and offers basic system functions.
Construct3D promotes and supports exploratory behavior through dynamic 3D geometry. A fundamental property of dynamic geometry software is that dynamic behavior of a construction can be explored by interactively moving individual defining elements such as corner points of a rigid body. It can be seen what parts of a construction change and which remain the same. The histories of constructions as well as dependencies between geometric objects are maintained. Experiencing what happens under movement allows better insight into a particular construction and geometry in general.
At its start Construct3D initializes a 3D window and the user interface. The menu system is mapped to a hand-held tracked panel called the personal interaction panel (PIP) [25].

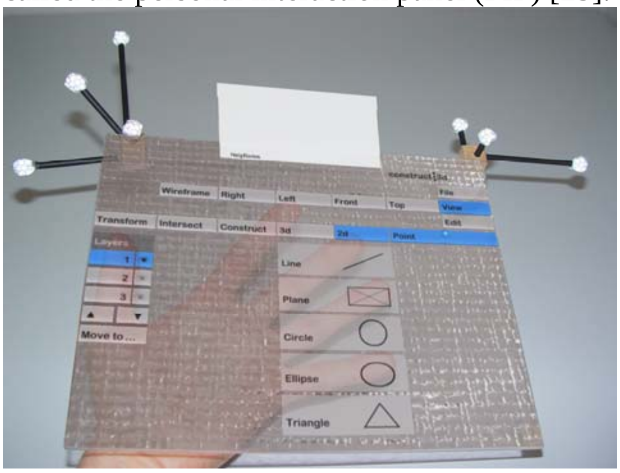

Figure 2: 2D Menu of Construct3D on the PIP. Spherical markers mounted at the upper corners of the plexiglass plate are used for optical tracking.

The PIP (Figure 2) allows the straightforward integration of conventional 2D interface elements like buttons, sliders, dials etc. as well as novel 3D interaction widgets. Passive haptic feedback from the physical props guides the user when interacting with the PIP, while the overlaid graphics allows the props to be used as multi-functional tools. Students can position written notes onto the tablet for instance that might help them during their work in the virtual environment.
All construction steps are carried out via direct manipulation in 3D using a stylus tracked with six degrees of freedom. In order to generate a

new point the user clicks with his pen exactly at the location in 3D space where the point should appear. Users can easily switch between point mode (for setting new points) and selection mode (for selecting 3D objects).

We support generation of and operation on these basic object types: Points (either freely positioned in space or fixed on curves and surfaces), lines, planes, circles, ellipses, cuboids, spheres, cylinders, cones, B-Spline curves with an unlimited number of control points and variable degree, interpolated B-Spline curves, NURBS surfaces up to 8x8 control points and variable degree, interpolated NURBS surfaces and surfaces of revolution (rotational sweep surfaces).

Regarding geometric operations we implemented Boolean operations (intersection, union, difference); intersections between all types of 2D and 3D objects resulting in intersection points and curves; planar slicing of objects; rotational sweep around an axis; surface normals in surface points; tangential planes in points of surfaces; tangents to all types of curves and in curve points; common tangent to two circles; plane normal to a line; line normal to a plane; plane of symmetry; measuring distances, angle bisector and mid point. Translations, rotations and mirroring of objects are supported as well.

All these operations consistently support dynamic modifications of their input elements and re-evaluate the resulting elements accordingly.

Necessary system operations such as selection and deselection of primitives, save, load, delete, undo, redo, export and import of VRML files are provided too. Details on the implementation, specifically the implementation of undo, redo and other features for multi-user environments are omitted in this context.

A professional geometry kernel ACIS [2] is used for robust geometric calculations. We utilize ACIS especially for calculating Boolean operations, all types of intersections, slicing, tangents and tangential planes, sweep surfaces as well as NURBS and B-Spline surfaces.

2.2 Hardware Setups

The standard immersive setup used for Construct3D supports two collaborating users wearing stereoscopic see-through head mounted displays (HMDs) (see Figure 1) providing a shared virtual space. The users interact with the system using pen and pad props (Figure 3).

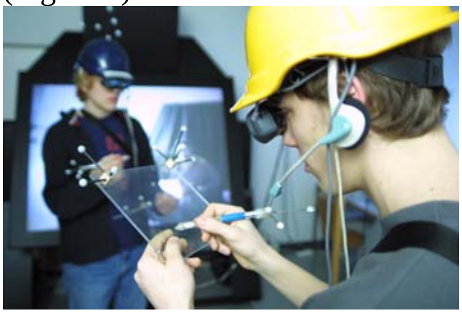

Figure 3: Collaborative HMD-Setup.

Both users see the same virtual objects as well as each others' pens and menu systems which provides a global shared space. In addition it allows one user to help the other one (i.e. with the menu system) if necessary. The same is valid in a distance learning scenario since input device data is shared amongst remotely located users. Because of see-through head mounted displays they perceive their real bodies, gestures and actions and those of people outside the virtual space, i.e. a teacher, as well which is especially important for co-located work. Position and orientation of head and hands are tracked using an ARTTrack1 4-camera optical tracking system. In a co-located setup one dedicated host with 2 graphic ports renders stereoscopic views for both users. In distributed setups rendering as well as computation of the geometric objects is done locally on each participant's PC (Figure 4).

For distance learning with VR/AR, reliable network distribution and replication of educational content is of prime importance. Recently we improved the robustness and scalability of our distributed software components to be able to serve larger groups of students. Construct3D can be configured in multiple ways by defining the number of users, its associated resources, specifying application retrieval method (by distribution or by file input) and tracking data obtaining strategy. Further on a central and persistent Construct3D service can be established as a background process. This allows joining and leaving a persistent Construct3D learning experiment at any time. In contrast to this, dynamically migrating Construct3D application hosts with directly associated users and rendering tasks is also easily possible without difficult configuration effort.

Figure 4: Students working in a distributed HMD setup in two different labs.

Other AR setups for educational use have been tested with Construct3D such as a basic desktop setup, semi-immersive, mobile and hybrid setups which are described in detail in [10]. The advantages of the collaborative immersive HMD setup to learning are mentioned in section 6.

3. DYNAMIC 3D GEOMETRY

In evaluations and discussions with teachers, three key strengths of Construct3D were identified:

(1) First and foremost the construction of three-dimensional dynamic geometry is a major asset. 3D dynamic geometry cannot be realized with pencil on paper or existing CAD programs. Nearly haptic interaction with geometric objects supports explorative learning. (2) Students can actively walk around an object which builds up a spatial relationship between the learner's body and object.

(3) Teachers especially emphasized the strength to visualize abstract geometric problems. Therefore the ideal content for this AR geometry learning environment exploits dynamic features, encourages modifications and is a visualization of (abstract) geometric problems.

Students and teachers were thrilled by simple dynamic interaction such as moving tangential planes on a cylinder or on a surface of revolution. These are things they have never seen or done before.

3.1 Content Design

Content design must reflect these findings. It is obvious that doing simple, introductory geometric tasks such as cutting cubic blocks out of a larger cube (creating a so called "Tschupik-cube") can be done faster and easier by hand drawing or simple PC CAD programs, saving a lot of costs of expensive VR hardware. Therefore a challenge was to design interesting content for a 3D dynamic geometry application that justifies its use.

In 3 evaluations with over 150 students we identified a number of very suitable examples that exploit the dynamic features of Construct3D.

We briefly want to mention three examples that are well suited for dynamic 3D geometry.

The example in Figure 5 discusses the milling of a surface of revolution. The surface (in red), generated by rotating a B-Spline curve around an axis, is given. The milling head is a sphere which must be constructed by students appropriately. The mid point of the sphere is located on the surface normal. The radius of the milling tool as well as all given elements (axis, B-Spline curve, touching point,…) can be dynamically modified at any time. Therefore differently shaped surfaces of revolution can be explored and cases where the milling head cuts into the surface (as shown in Figure 5). By moving the touching point on the surface of revolution the spherical milling head will move too. Due to exploration and teachers' guidance students can learn about properties of surfaces of revolution, about curvature, with the aid of a tangential plane about specific surface points (elliptic, parabolic, hyperbolic), about important criteria when choosing milling tools and much more.

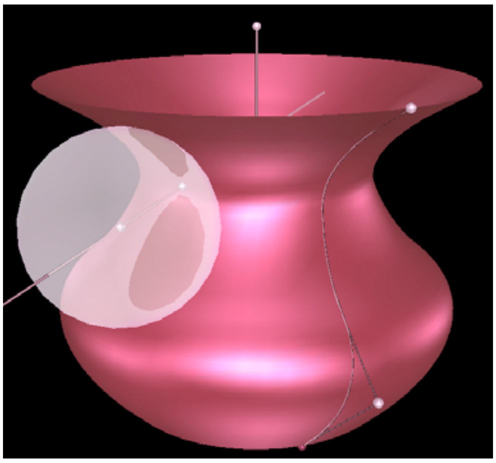

Figure 5: Milling of a surface of revolution. The milling head is a sphere.

Totally new examples and applications are possible when integrating dynamic 2D geometry in dynamic 3D space. Since planar constructions can be embedded in 3D space, spatial interpretations of planar constructions are an obvious application area. In some cases problems with conic sections [27] in high school education can be solved very elegantly and smartly by using a spatial interpretation of the problem.

In our example three points are given which lie in between two parallel lines. This planar problem requires finding an ellipse which touches both lines and goes through all 3 points. At the bottom of Figure 6 the planar configuration is shown. It is the top view of the spatial construction.

This problem can be solved in 2D by methods of projective geometry, taught in university courses. With spatial interpretations this problem can be solved by high school students. The solution to this problem lies in the following spatial interpretation of the planar situation. We assume that both parallel lines form the contour of a cylinder. The 3 given points are projected onto this cylinder. The normal line to the given plane through each point intersects the cylinder in 2 different points. Figure 6 basically speaks for itself.

After converting the planar problem into a spatial one, we need to find an ellipse (a planar intersection of the cylinder), that passes through the 3 given points. Since there are 2 solutions for each point, 8 possible planes, therefore 8 possible solutions are valid in general.

Projecting an ellipse back into the plane (via a normal projection) gives a solution to the planar problem.

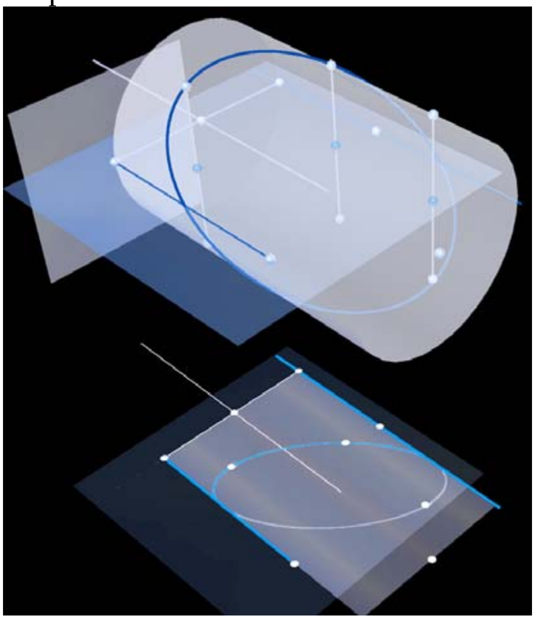

Figure 6: An ellipse through 3 points touching 2 parallel lines. Given elements can be dynamically changed to study interesting configurations or special cases.

Many excellent examples for high school geometry and university education regarding spatial interpretations can be found in [8].

The geometric proof of J. P. Dandelin (1822) shows that the intersection curve of a cone with a plane can only be circle, ellipse, hyperbola or parabola. These cases of conic sections can all be visualized with the same dynamic construction in Construct3D (Figure 8). In a geometry course such an interactive model can serve as the base for explaining the proof which might be difficult to understand without any model.

3.2 Ambiguities in 3D dynamic geometry

The main problems of all implementations of dynamic geometry are caused by ambiguities. Kortenkamp [14] isolated the problem and solved it for the dynamic 2D software Cinderella [20]. To name a simple example, let us assume a line intersects a circle in 2 real points. If the line is moved, no real intersection might occur. Moving the line back in its original position creates two intersection points again. The problem is to keep track of all possible solutions, where they have been before and where they move, to keep continuity. This and equivalent problems are resolved by Kortenkamp in general by using "complex tracing". Therefore all calculations are carried out in complex space with tools of complex analysis. These are needed to guarantee not only continuity, but analytic behavior.

Kortenkamp solved the problem for two dimensional constructions only. He mentioned that in the 3D case we can still apply the theory of complex tracing, although it is a little bit more involved. It would be a very challenging project to implement complex tracing for 3D geometry. After finding and solving related mathematical problems, the exploding complexity of geometric operations in 3-space would require to rewrite existing algorithms. No existing 3D kernel (such as ACIS [2]) supports complex tracing.

In Construct3D we implemented dynamic behavior in a standard way as done by most authors of 2D dynamic geometry software. Ambiguities are not handled correctly but internally an "intelligent" way is used to remember solutions in order to keep continuity to a certain extent. It is usually possible to keep continuity as long as points stay in Euclidean space. As soon as one of multiple intersection points for instance becomes a point at infinity we loose track and loose continuity. During development and testing of Construct3D a number of continuity problems were found that can only be solved by a general approach.

One example that imposes a hard test on implemented continuity algorithms is illustrated in Figure 7. A spline surface is intersected by a cubic spline curve (interpolating a set of given points). To keep track of intersection points, it is very helpful to know the maximum number of solutions. This helps to determine if intersection points that appear during dynamic changes already existed before or represent totally new solutions. Assuming that the surface is a NURBS surface, which is piecewise rational, it is theoretically not trivial to determine the exact number of possible solutions for that intersection problem. Both elements are only piecewise rational.

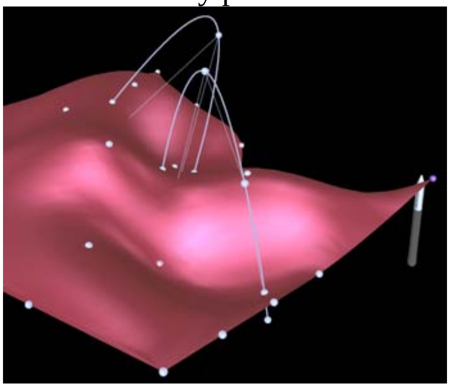

Figure 7: Intersection of a spline surface (cubic interpolation scheme) with an interpolating cubic spline curve (white).

We consider a robust implementation of dynamic 3D geometry to be of high importance. It is not only required for teaching correct geometric concepts but also for application areas such as parametric CAD and spatial kinematics.

4. USABILITY DESIGN

In this section we describe improvements in the user interface and visual design. Three usability evaluations have been conducted with the goals to identify usability issues and to investigate Construct3D’s strengths and weaknesses for learning and teaching geometry. In June 2000 a first informal pilot study has been conducted [12]. In the second evaluation 14 high school students participated in 5 training sessions lasting 6 hours in total. Results have been published in [9]. A third evaluation with 50 students and over 300 teaching lessons in total was conducted in 2005. In this context we briefly summarize the major usability improvements that we consider important for a VR/AR geometry learning environment (details are in [11]).

The purpose of visual design of geometric objects constructed by the user, is to support the user's understanding of the construction. Unlike desktop visualization of the same content, using stereoscopic see-through HMDs requires to deal with limited contrast, resolution and viewing angle. Moreover, it is important to present scenes of high depth complexity in a clear way to aid the user in “reading” the construction. Among the techniques employed in Construct3D to support these goals are the use of transparency, color coding to allow distinguishing between multiple users’ contributions, separation into layers to support semantic structuring of a construction, and automatic previewing of new objects. These techniques make scene handling and graphical rendering quite expensive despite the simple look of the application, but we feel that the usability improvements we observed after introducing these features are worth the additional computational load.

In cooperation with a graphics designer the interface, using a pen and tablet, was completely redesigned. A new wireless pen was designed and manufactured to improve user interaction. User requirements were defined as a result of pre-tests. The menu system was completely re-implemented to create a professional yet intuitive interface. In post-evaluations [9] it was confirmed that the new user interface is very easy to use, easy to remember and requires little time to learn. Construct3D supports distributed work (students working on geometric constructions in remote places/labs over the internet) and a specific color design allows the teacher to distinguish which student did which part of the construction. 4 different states of objects can clearly be distinguished within a color scheme: selected and deselected objects, objects in the currently active layer or in inactive layers.

Figure 8 depicts a dual-user setup. Each user has an own menu system (working within a pre-defined color scheme). In this case the whole construction was done by one user (with the blue menu system).

Students can work in 3D layers to structure their construction. Transparencies, as used in technical drawings, allow to see "inside" objects (illustrated in Figure 8 with the proof of Dandelin). Transparencies were carefully designed to allow seeing objects which are occluded by other objects (through 3 different layers of occluded objects).

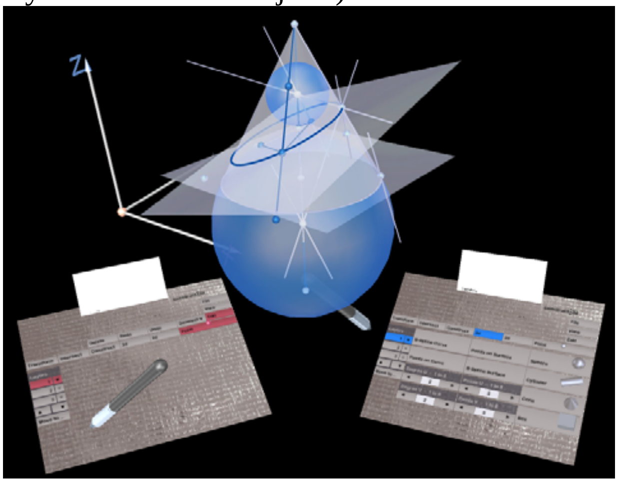

Figure 8: Two users collaborate on a construction in Construct3D. To distinguish users’ contribution each user is working within an own color scheme (note the differently colored menus).

4.1 Designing Complex User Interfaces

Construct3D currently provides sufficient functionality for high school geometry education. The menu system consists of more than 70 widgets (buttons and sliders) in various sub-menus to access all features that are provided by the application. It must be noted that because of tracking inaccuracies all widgets must be at least of a size of 1x1 cm. If they are smaller, it is very difficult (mostly because of small tracking jitter and jittering hands) to accurately hit buttons.

In contrast to educational applications, CAD software usually offers hundreds or thousands of features. Integrating and manipulating CAD data in VR environments is currently a topic of research. However we think the bigger problem and one of the main challenges for the future is: How to design an interface of a VR/AR application that allows to control a complex CAD application such as AutoCAD, CATIA or the like. Hardly any work has been done towards designing very extensive user interfaces in VR/AR yet.

5. TRAINING SPATIAL ABILITIES

5.1 Large-Scale Training Study

In cooperation with experts in spatial ability research from the Institute of Psychology at University Vienna, a national project with the title "Educating Spatial Ability with Augmented Reality" (P16803-N12) was initiated. The major research question is: Can we improve spatial abilities by geometry education with Construct3D?

Six spatial ability pre- and post-tests were conducted in 20 high schools with over 300 students. 50 of those students were trained with Construct3D in 6 teaching lessons, 50 used the educational PC software CAD3D [24], 50 had traditional geometry education at school and the rest was an untrained control group.

Our evaluation study addressed the following research questions:

1. Effect of the training on performance in tasks central to the training;
2. Transfer of the training effect to more distant spatial tasks;
3. Effect of the training on strategy use in central and distant spatial tasks;
4. Aptitude-treatment interactions: Dependence of individual training effects on pretest spatial ability, verbal ability, and reasoning ability;
5. Gender differences in training effects.

We are currently at the end phase of the project and gathered a lot of data. There are interesting results which will be published later this year. Going into more detail would go beyond the scope of this paper. The evaluation, test setup and first results of the pre-study were published in [13].

5.2 Spatial Ability Tests

There are several shortcomings of traditional paper-pencil spatial ability test and some authors have argued that these may be one reason for the relatively low predictive power of spatial ability tests when it comes to real-life spatial tasks [7, 19]. First of all, although most existing spatial ability tests – especially the more complex ones – aim at assessing three-dimensional spatial abilities, virtually all of them use two-dimensional presentations of the stimulus material. Thus, solving these test items requires participants to mentally transform a two-dimensional picture into a three-dimensional figure, to perform some mental manipulations on the figure, and to re-transform the result into a two-dimensional picture. One could argue that this 2D-3D transformation adds a difficulty facet that is not directly related to what is supposed to be measured.

A second shortcoming is that the large majority of existing tests require participants to select the correct solution to each item from a (small) number of response alternatives. Thus, participants can solve tests by excluding the alternatives one by one, often by concentrating on single features, as opposed to mentally constructing the correct solution [4]. There are several possibilities for avoiding this problem. One is to have participants actively construct their solutions [16], which is facilitated by the use of computerized systems. Another is to present the task in a stepwise fashion, so that participants need to keep track of a sequence of manipulations. In such cases, participants need to mentally manipulate at least parts of the stimulus in order to be able to identify the correct solution.

These two points of critique are true not only for paper-pencil, but also for the majority of computerized tests. Most computerized spatial ability tests that are currently available are simply computerized versions of existing paper-pencil tests [19]. One general benefit of computerized tests is that additional performance measures, e.g. response latencies and information on solution strategies, can be collected [6]. As we found in our national project, self-report measures of strategy use can be problematic because they require a high amount of introspection [7].

5.3 Future Work on Spatial Ability Testing

New technologies such as augmented reality allow the development of a new generation of spatial tasks that are three-dimensional in presentation and response format. With a VR/AR tool such as Construct3D it will be possible to actively construct solutions in three-dimensional space. Participants can move around the objects they are working on and can virtually "touch" and manipulate them. Compared to the use of real three-dimensional objects, these systems allow for the free creation of an unlimited number of objects, and manipulations can be easily undone and redone.

Therefore we plan to design a spatial ability test in AR and continue our work in this very interesting field.

6. PEDAGOGY

6.1 Constructivism

Constructivist theory provides a valid and reliable basis for a theory of learning in virtual environments [18, 28]. The core commitment of a constructivist position is that knowledge is not transmitted directly from one knower to another but is actively built up by the learner. As constructivism underlines, learning takes place when students can build conceptual models that are both consistent with what they already understand and with the new content. In order to ensure successful adaptation of old knowledge to new experience, flexible learning direction should be provided [17]. One possibility is to integrate known types of information and educational supports other than the 3D representation (such as audio and text annotations, images etc.). In an educational geometry application this can be familiar 2D views such as top view, front view and side views which we integrated in Construct3D.

VR environments can be tailored to individual learning and performance styles. Therefore we implemented support of different learning modes in virtual environments from teacher-supported to autodidactic learning (described in [9, 10]).

6.2 VR/AR Hardware Setups for Classrooms

For productive use in the classroom, a number of circumstances must be accommodated: Support for a variety of social settings including students working alone and together, a teacher working with a student or teaching a whole class, a student or the whole class taking an exam, etc. Collaboration in these situations is largely determined by roles, and the teacher should be able to retain control over the activities. Moreover, it is not realistic to expect that schools can afford extensive installations of expensive equipment such as used in our lab, and therefore the software must run on a variety of immersive and non-immersive hardware platforms including heterogeneous and hybrid setups. Other AR setups for educational use have been tested with Construct3D such as a basic desktop setup, semi-immersive, mobile and hybrid setups which are described in detail in [10]. Most of these setups can be utilized for face-to-face as well as remote collaboration (e.g. a teacher can remotely advise a student at a homework problem).

Our immersive setup that uses head mounted displays is most favored by teachers and students. The big advantage of this setup is that it allows users to actively "walk around" geometric objects which are fixed in space. Excited students sometimes lie down on the floor to view objects from below (Figure 9) or step on a chair to look down from above.

Figure 9: Actively constructing students in the immersive HMD setup. Instead of a top view a bottom view is chosen by one student.

This is a unique feature of an HMD setup which cannot be provided by monitor or projection screen based hardware configurations. It actively involves students and therefore complies with constructivist learning theories. Geometric objects are not abstract anymore but in spatial relation to the learner's own body, they can be manipulated directly and are nearly tangible. We think these are key features to learning and to improving spatial abilities with Construct3D.

In order to use Virtual or Augmented Reality applications in realistic, educational classroom settings, a large group of students must be able to participate either actively or passively in the activities taught in VR/AR. A distributed setup where the geometric content is distributed to all student PCs is one alternative to realize a desktop-based collaborative setup for all students in a class. We plan to evaluate such a setup in the near future.

Another possibility is to use several low-cost immersive VR setups (which may be connected via distribution) to allow multiple students to work with HMDs. In the next section our approach towards a low cost VR setup is described.

6.3 Learning in VR/AR

One of the main questions of learning in VR/AR is: Do students really learn or understand "better" or "more" in a VR/AR environment. To this date no study exists in literature that clearly shows the superiority of learning in VR/AR compared to education with other media. In geometry education such a comparative study would mean comparing traditional paper&pencil geometry education to education with CAD programs on desktop PCs and finally comparing it to geometry education in VR. In our opinion it is not possible to determine a "best" learning medium because of multiple reasons. The most obvious reason is that there is no single learning medium which is best suited for learning geometry in general, independent of the actual content. A combination of multiple media seems to be ideal, which is also a result of studies about learning with new media. For example various studies show that drafting by hand is very important to improve students' spatial abilities. Dynamic 2D geometry can (only) be taught by using PC software. Analogically we believe that there are good reasons to integrate VR/AR technologies in modern geometry education in addition.

7. LOW-COST AR/VR SYSTEMS

Current VR/AR hardware is expensive mainly because of no existing mass market for VR/AR solutions. It is unrealistic that an average high school can afford an immersive setup, which is most favored by teachers and students for use with Construct3D.

Tracking position and orientation of multiple users and all their devices accurately is the most expensive part of our system. This is the main reason why the author initiated a project to build a low cost optical tracking system.

Its development is nearly finished and we are currently beta-testing. Details about the hardware and implementation will be published at a later date. The future goal is to sell and spread the technology at low costs to the masses. With this advancement we are able to build a fully immersive dual-user VR system including two low-cost HMDs, a PC and an optical 4-camera tracking system for a total price of approximately 7.000 - 8.000 EUR. The price is still high but such a system becomes at least affordable for some university institutions.

8. CONCLUSIONS

Construct3D is one of the longest actively developed educational VR/AR applications. Its development, originally motivated by the idea to improve students' spatial abilities and overall understanding of geometry, is influenced by many diverse research areas. This paper summarizes our development in the areas of dynamic 3D geometry, usability design, spatial abilities, pedagogy in VR/AR environments and of low-cost VR systems.

After developing robust dynamic 3D geometry software in Augmented Reality, the strengths of this learning environment have been investigated. Content has been created that fits into the curriculum and fully benefits from all of the advantages provided by the application. In order to adapt software and hardware to users' needs user interfaces were redesigned and in depth research was done on usability design. Extremely positive feedback from teachers and students who are excited by the possibilities, encouraged the development throughout the years. Results from three evaluations show that Construct3D is easy to use, requires little time to learn, encourages learners to explore geometry and can be used in a consistent way.

Regarding spatial ability research, a currently running project is outlined with ideas to conduct future spatial ability tests directly in VR/AR.

Various hardware setups have been studied that are suitable for educational purposes. An immersive setup that uses head mounted displays is most favored by teachers and students. It allows users to actually "walk around" geometric objects which are fixed in space.

Finally the recent development of a low cost optical tracking system, that allows to build affordable immersive VR systems is described.

By summarizing our work, which aims to establish Augmented Reality in geometry education, and by providing insight into the problems and future challenges in each of the related areas, we highlight and advert the full potential of Augmented Reality in geometry education.

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ABOUT THE AUTHOR

Dr. Hannes Kaufmann, is an Assistant Professor at the Interactive Media Systems Group, Institute of Software Technology and Interactive Systems at Vienna University of Technology, Austria. His research interests are in Virtual and Augmented Reality, applications of these technologies especially in education, psychological topics in AR (e.g. spatial ability research), low-cost tracking technologies and AR/VR & CAD integration. His email address is kaufmann@ims.tuwien.ac.at; website: http://www.ims.tuwien.ac.at/staff_detail.php?ims_id=kaufmann