The classical analysis of turbulent boundary layers in the limit of large Reynolds number \( Re \) is characterised by an asymptotically small velocity defect with respect to the external irrotational flow. As an extension of the classical theory and as a first step towards a more general approach which also accounts for separating flows, it has been shown, [1], that the defect may become moderately large and essentially independent of \( Re \) but still remain small compared to the external streamwise velocity component for non-zero pressure gradient boundary layers. That wake-type flow turns out to be characterised by large values of the Rotta–Clauser parameter, which serves as an appropriate measure for the defect and hence as a second perturbation parameter besides \( Re \). Most important, it is demonstrated that also this case can be treated by rigorous asymptotic analysis on basis of the Reynolds-averaged Navier–Stokes equations, independent of the choice of a specific Reynolds stress closure. The asymptotic investigation predicts the remarkable result that in the special case of quasi-equilibrium flow the transition from classical small-defect to a pronounced wake flow is accompanied by double-valued solutions, a phenomenon seen to agree well with early experimental observations. In addition, a numerical treatment of the nonlinear boundary layer equations, supplemented with an asymptotically correct shear stress closure which provides the well-known logarithmic velocity portion close to the surface, confirms those theoretically predicted results for finite values of \( Re \). The solutions exhibit good agreement with experimental data taken from the literature and convergence to the asymptotic limit obtained analytically, [2], [3]. As a minor but nevertheless interesting point, certainly relevant for engineering applications, it is found that the influence of the wall curvature, which must be included in the analysis in the case of an asymptotically small velocity defect from a rational point of view, on the irrotational bulk flow associated with double-valued near-equilibrium boundary layers allows for self-similar potential flow solutions. They describe flows along curved diffuser ducts. Such flow configurations are seen to withstand the maximum pressure rise possible in order to maintain equilibrium, [3].

The results summarised so far indicate that turbulent boundary layers having a moderately large velocity defect represent the ‘missing link’ between the classical flow with a defect of \( O(1/\ln Re) \) and boundary layers where the defect is a quantity of \( O(1) \). The investigation of the latter, [4], suggests the need of a second perturbation parameter, denoted by \( \alpha \), besides \( Re \) which measures the slenderness of the boundary layer. The asymptotic analysis of that general type of turbulent boundary layers has been completed as far as the limit \( \alpha = 0, Re^{-1} = 0 \) is concerned. This primary limit gives rise to weakly singular solutions of the resulting shear layer approximation of the wake flow if the prescribed adverse pressure gradient is controlled suitably. The local behaviour of these solutions is studied analytically. They indicate marginally separated flows for finite values of both \( \alpha \) and \( Re \). However, a correct treatment of the separation process must take into account the local interaction of the boundary layer and the external potential flow. This is a topic of the subsequent research.