Mixed convection flow past a Horizontal Plate

Ljubomir Savić and Herbert Steinrück

Institute of Fluid Mechanics and Heat Transfer, Vienna University of Technology, Vienna, Austria

1 Introduction

The flow past a horizontal heated plate which is aligned under a small angle of attack ϕ to the oncoming parallel flow with velocity U_∞ in the limit of large Reynolds Re and large Grashof $\mathrm{Gr} = g\beta \Delta T L^3/\nu^2$ number will be investigated (see figure 1). As usual β and ν denote the isobaric expansion coefficient and the kinematic viscosity, respectively. The difference between the plate temperature and the temperature of the oncoming fluid is ΔT and L is the length of the plate. A measure for the influence of the buoyancy onto the boundary layer flow along a horizontal plate is the buoyancy parameter $K = \mathrm{Gr}\,\mathrm{Re}^{-5/2}$ as defined in [4]).

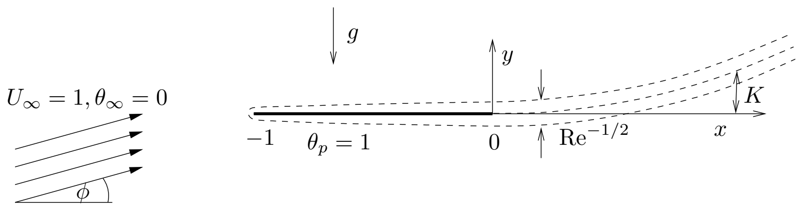

Fig. 1. Mixed convection flow past a horizontal plate

The starting point of the analysis are the Navier Stokes equations for an incompressible fluid using Boussinesq's approximation to take buoyancy forces into account and the energy equation. Additionally to the above mentioned dimensionless parameters the Prandtl number Pr, which is assumed to be of order one, and the angle of attack ϕ enter the problem.

We define the reduced buoyancy parameter $\kappa = K\,\mathrm{Re}^{1/4}$ and the reduced inclination parameter $\lambda = \phi K \sqrt{\mathrm{Re}}$ and perform an asymptotic analysis for re $\to \infty$ under the condition that κ, λ are of order one. The choice of the similarity parameters λ and κ is dictated by the analysis of the wake interacting with the potential flow and the analysis of the trailing edge region as well. In the present paper we discuss the far field and the trailing edge behavior of the flow as well.

2 The wake

Due to the fact that the fluid in the wake has a different temperature a hydrostatic pressure difference across the wake builds up. In order to determine the outer (potential) flow field a vorticity distribution compensating the hydrostatic pressure difference is placed at the center line of the wake. Since the wake is inclined there is a non vanishing component of the buoyancy force in the tangential direction of the wake. Thus the potential flow and the wake have to be determined simultaneously. Results for the inclination of the wake are shown in figure 2 for $\lambda = 1$ and various values for κ. In case of $\kappa = 0$ the potential flow field is not influenced by buoyancy at all. Thus the inclination of the wake increases monotonically to its limiting value 1. Increasing κ a local minimum of the inclination form approximately one plate length after the trailing edge. For $\kappa > 0.9$ the wake even bends downwards and the fluid in the wake decelerates. For $\kappa > 0.914$ no numerical solution has been obtained. It is expected that in the wake a small back-flow region forms (see [5]). This will be investigated in the future.

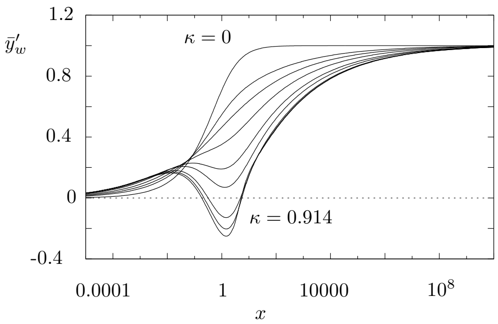

Fig. 2. Scaled inclination of wake for different buoyancy parameters κ and $\lambda = 1$

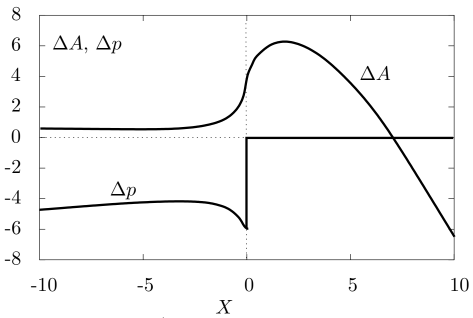

Fig. 3. Antisymmetric part of the pressure Δp and the negative displacement thickness near the trailing edge

3 The trailing edge

To discuss the flow field asymptotically near the trailing edge the method of interacting boundary layers is applied [1,2]. The displacement thickness influences the pressure in the boundary a usually by the displacement of the potential flow and additionally by hydrostatic pressure perturbation. However, the interaction problem can be decoupled into a symmetric part, and an antisymmetric part. In fig. 3. the antisymmetric part of the interaction pressure Δp and oft negative displacement thickness ΔA are shown, respectively. Surprisingly the interaction pressure is discontinuous at the trailing edge, and thus a sub-layer is needed where this singularity is removed. Thus on the scales of the interaction problem there is a flow around the trailing edge, but on the scales of the potential flow the Kutta condition is satisfied.

References

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