Mixed Convection Flow Past a Horizontal Plate: The Global Flow

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The mixed convection flow past a horizontal plate which is aligned under a small angle of attack to a uniform free stream will be considered in the limit of large Reynolds number and small Richardson number.

1 Introduction

The effect of weak buoyancy on the laminar flow past a horizontal plate which is aligned under a small angle of attack ϕ to the oncoming free stream will be investigated in the limit of large Reynolds numbers Re (see figure 1). Since the gravity force is almost perpendicular to the main flow direction buoyancy influences the flow field only indirectly by a (non-uniform) hydrostatic pressure distribution.

The influence of buoyancy on the potential flow can be characterized by the buoyancy parameter $K = \mathrm{Gr}/\mathrm{Re}^{5/2}$ introduced in [1] which is equivalent to a Richardson number [4] where $\mathrm{Gr} = \mathrm{g}\beta\Delta\mathrm{TL}^3/\nu^2$, $\mathrm{Re} = \mathrm{u}_\infty\mathrm{L}/\nu$, $\mathrm{Nu} = \dot{\mathrm{Q}}/\mathrm{k}\Delta\mathrm{T}$, $\mathrm{Pr} = \rho\mathrm{c_p}\nu/\mathrm{k}$ are the Grashof, Reynolds, Nusselt and Prandtl number and β, c_p, ν, k, ρ are the isothermal expansion coefficient, the isobaric heat capacity, kinematic viscosity, thermal conductivity and density of the fluid. The plate of length L is assumed to be isothermal with plate temperature $T_p = T_\infty + \Delta T$. The temperature of the ambient fluid is T_∞ and u_∞ is the velocity of the oncoming parallel flow.

In a recent paper Schneider [2] showed that for the flow past a finite plate the outer (potential) flow field is markedly influenced by buoyancy. In order to simplify the problem he neglected the viscous boundary layer and wake, by setting the Prandtl Pr number to zero. Considering a large Peclet number Pe temperature and density perturbations are limited to a thin thermal boundary layer and wake, respectively. An essential assumption to determine the perturbation of the outer flow field is the validity of the Kutta condition. Thus a vortex distribution on the wake and the plate has been introduced to compensate the hydrostatic pressure differences at the trailing edge and across the wake.

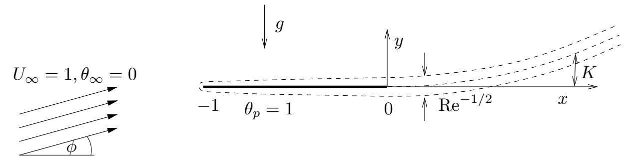

Fig. 1 Mixed convection flow past a horizontal plate

The goal of the paper is to describe the global field. Due to the inclination of the wake the flow in the wake is accelerated or decelerated by the tangential (to the wake) component of the hydrostatic pressure gradient. This effect has been neglected by Schneider 2005 limiting his analysis to Richardson numbers $\mathrm{K} \ll \mathrm{Re}^{-1/4}$. For technical reasons Schneider 2005 considered the flow problem in a channel with a width of the order $O(\mathrm{K}^{-\mathrm{n}})$.

In section 2 we introduce the governing equations for the potential flow and wake. They are formulated in local coordinates around the centerline of the wake. The position of the centerline has to be determined by the first order correction of the potential flow. Thus the wake and the potential flow field have to be determined simultaneously. The potential flow correction consists of two contributions: one due to the angle of attack ϕ and a second one due to buoyancy differences across the wake.

Accordingly two dimensionless coupling parameters for the the angle of attack ϕ the buoyancy parameter K and the Reynolds number are introduced: The dimensionless parameter $\lambda = \phi K\sqrt{\mathrm{Re}}$ describes the effect of buoyancy in the far wake. Similarity solutions for the velocity and temperature profile in the far wake exist only for positive values of λ. The reduced buoyancy parameter $\kappa = K\mathrm{Re}^{1/4}$ is a measure for the hydro-static pressure difference across the wake. The ratio $K/\phi = \kappa^2/\lambda$ measures the influence of the hydrostatic pressure differences onto the potential flow.

In section 3 results regarding the form of the wake, the velocities in the wake and the resulting lift force on the plate are presented and discussed.

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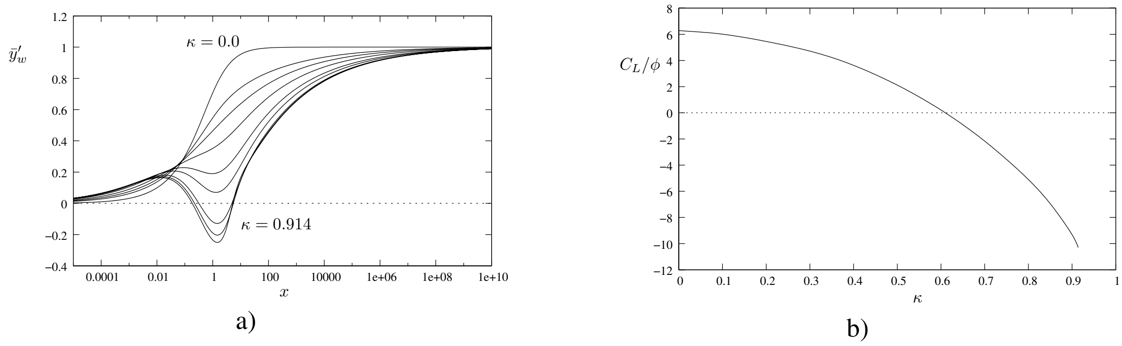

Fig. 2 a)Inclination of wake, $\mathrm{Pr} = 0.71$, $\lambda = 1$, κ =0, 0.5, 0.6, 0.7 ,0.8, 0.85, 0.9, 0.91, 0.914, b)Lift coefficient, $\mathrm{Pr} = 0.71$, $\lambda = 1$

2 Governing equations

To obtain the global flow picture we have to determine the wake and the potential flow simultaneously. Due to the temperature perturbation in the wake there is a hydrostatic pressure difference across the wake which has to be compensated by the potential flow. Formally this can be done by a vortex distribution $\gamma_w(x)$ along the wake. On the other hand the potential flow induces an inclination of the wake which in turn affects the velocity and temperature distribution in the wake.

The potential flow is a superposition of the flow around a horizontal plate under angle of attack ϕ and a flow induced by a vortex distribution γ along the plate and wake. Thus we can express the scaled inclination $\bar{y}' = \mathrm{Re}^{1/4} y'_w$ of the wake in terms of the vortex distribution γ_w along the wake.

$$\bar{y}'_w = \frac{\lambda}{\kappa}\sqrt{\frac{x}{x+1}} + \kappa\frac{1}{2\pi}\sqrt{\frac{x}{x+1}}\int_0^\infty \frac{\gamma_w(\xi)}{x-\xi}\sqrt{\frac{\xi+1}{\xi}}\,\mathrm{d}\xi, \quad x > 0, \tag{1}$$

To describe the wake we introduce the scaled stream function F and temperature profile D

$$\psi = (x+1)^{3/5}F(x,\eta), \quad \bar{\theta} = \frac{1}{(x+1)^{3/5}}D(x,\eta), \quad \text{with} \quad \eta = \sqrt{\mathrm{Re}}\frac{y - \mathrm{Re}^{-1/4}}{(x+1)^{2/5}}, \tag{2}$$

where ψ is a stream function. Note that the horizontal velocity $\bar{u}_w = (x+1)^{1/5}F'$ will grow unbounded for $x \to \infty$ if F' tends to a non-vanishing limit. Thus we expect (due to the scaling) a velocity overshoot in the wake.

We obtain the transformed wake equations:

$$F''' + \frac{3}{5}F''F - \frac{1}{5}(F')^2 + \kappa\bar{y}'_w D = (x+1)(F'F'_x - F''F_x), \tag{3}$$

$$\frac{1}{\mathrm{Pr}}D'' + \frac{3}{5}(FD)' = (x+1)(F'D_x - D'F_x), \tag{4}$$

subject to the boundary conditions $F(x,0) = F''(x,0) = D'(0)$, $F'(x,\infty) = \frac{1}{(x+1)^{1/5}}$, $D(0,\infty) = 0$, and at the "initial conditions" at the trailing edge $x = 0$, $F(0,\eta) = F_B(\eta)$,, $D(0,\eta) = D_B(\eta)$. The vortex distribution in the wake is given by $\gamma_w(x) = \int D(\hat{\eta})\,\mathrm{d}\hat{\eta}/(x+1)^{1/5}$.

3 Results

Equations(1)-(5) are solved simultaneously for $\lambda = 1$ and several values of κ. The case $\kappa = 0$ corresponds to the flow around an inclined plate where buoyancy effects are limited to the wake. However, for $\kappa > 0$ the outer flow is influenced by the wake. In figure 2a the inclination of the wake is shown. About one plate length after the plate the wake bends downward. For $\kappa > 0.914$ the solution procedure failed and we believe this heralds a region of reverse flow direction. Most interestingly buoyancy influences the lift coefficient and if $\kappa > 0.65$ the lift coefficient becomes negative. For a detailed discussion we refer to [4].

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