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Mixed Convection Flow Past a Horizontal Plate: The Trailing Edge

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The mixed convection flow past a horizontal plate which is aligned under a small angle of attack to a uniform free stream will be considered in the limit of large Reynolds number and small Richardson number close to the trailing edge.

1 Introduction

The mixed convection flow past a horizontal plate in the limit of large Reynolds number Re and small buoyancy parameter $K = Gr/\mathrm{Re}^{5/2}$ near the trailing edge is considered using triple deck analysis. However, by considering the wake it turns out that K has to be chosen of order $\mathrm{Re}^{-1/4}$. The analysis can be simplified by decomposing all quantities, e. g. the horizontal velocity component u into the mean value $\bar{u}(x,y) = (u(x,y) + u(x,-y))/2$ and the difference $\Delta u(x,y) = (u(x,y) - u(x,-y))/K$ of their values on the upper and lower side of the plate.

2 The triple deck problem

For the mean values $\bar{u}$, $\bar{v}$ the unperturbed trailing edge problem first analyzed by Stewartson[1] and Messiter [2] is obtained. For the differences Δu. Δv we obtain the linearized triple deck problem, where the independent varibels x and y are the lower deck variables scaled with $\mathrm{Re}^{-3/8}$ and $\mathrm{Re}^{-5/8}$, respectively. Then the equation for the flow in the lower deck read as

$$\bar{u}\frac{\partial \Delta u}{\partial x} + \Delta u \frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \Delta u}{\partial y} + \Delta v \frac{\partial \bar{u}}{\partial y} = -\frac{\partial \Delta p}{\partial x} + \frac{\partial^2 \Delta u}{\partial y^2}, \tag{1}$$

and the continuity equation. The boundary conditions are

$$\begin{aligned} \Delta u(x,0) = \Delta v(x,0) = 0, &\qquad x < 0 \quad \text{plate}, \\ \Delta u(x,0) = \Delta p(x,0) = 0, &\qquad x > 0 \quad \text{wake}. \end{aligned} \tag{2}$$

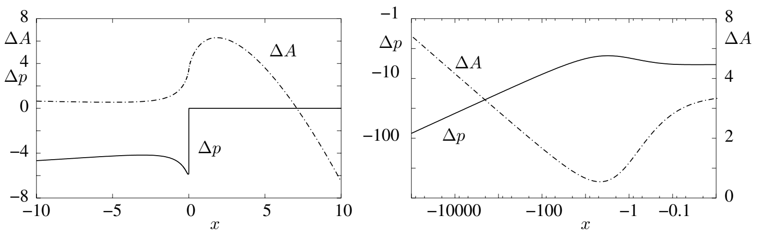

Fig. 1 Negative displacement thickness ΔA and interaction pressure $\Delta p^{(3,5)}$ a). Asymptotic behavior of ΔA, and $\Delta p^{(3,5)}$ for $x^{(3)} \to -\infty$ b).

For the interaction law we obtain

$$\Delta A'(x) + \sqrt{3}a_s x^{1/3}\, h(x) = -\left[\frac{1}{\pi}\int_{-\infty}^{0} \frac{\Delta p(\xi) + 2a_s|\xi|^{1/3}}{x-\xi}\,\mathrm{d}\xi - \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\bar{A}(\xi) - a_s h(\xi)|\xi|^{1/3}}{x-\xi}\,\mathrm{d}\xi\right]. \tag{3}$$

where ΔA is the difference of the displacement thicknesses and Δp is difference of the pressures on the upper and lower side of the plate. The asymptotic behavior o the displacement thickness $\bar{A}$ of the trailing edge poblem for the mean values is given by $\bar{A}(x) \sim a_s x^{1/3}$ with $a_S = 0.892$ [3].

Taking the logarithmic behavior of Δu for $y \to \infty$ into account we have $\Delta A(x) = \lim_{y\to\infty} y \ln y u_y(x,y) + u(x,y)$. The logarithmic behavior of Δu becomes obvious when considering the asymptotic behavior for $x \to -\infty$. For that limit

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a similarity solution $\Delta\tilde{u}(\zeta)$ in terms of the variable $\zeta = y/(x^2+1)^{1/6}$ can be found which behaves asymptotically like $\Delta\tilde{u}(\zeta) \sim c_1 \ln\zeta + c_2 = c_1 \ln y + c_2 - (c_1 \ln x)/3$ for $\zeta \to \infty$. Defining $\Delta A(x)$ as the constant (y independent) part of Δu for $y \to \infty$ we deduce that ΔA behaves logarithmically for $x \to -\infty$. A numerical solution for Δp and ΔA is shown in figure 1. In figure 1b the logarithmic behavior of ΔA is verified.

The numerical solution shows that Δp has a jump discontinuity at $x = 0$. Since $\Delta p + \bar{A}$ and $\Delta A'$ can be considered as the real and imaginary part of an analytic function of the complex variable $z = x + iy$ we conclude that $\Delta A'(x) \sim \ln|x|$ for $x \to 0$. The numerical solution in figure 1a) shows the vertical tangent of ΔA at $x = 0$.

Integrating the lower deck equation across the discontinuity we obtain an ordinary differential equation for the jump $[\Delta u]$ of the horizontal velocity component at $x = 0$. It has the solution

$$[\Delta u](y) = \Delta p(0-)\left(\frac{1}{\bar{u}(0,y)} + \int_{\infty}^{y} \frac{\bar{u}'(0,y)}{\bar{u}^2(0,\xi)} \mathrm{d}\xi\right) \sim -\Delta p(0-)\frac{\bar{u}''(0)}{\bar{u}'(0)^2} \ln y, \quad y \to 0 \tag{4}$$

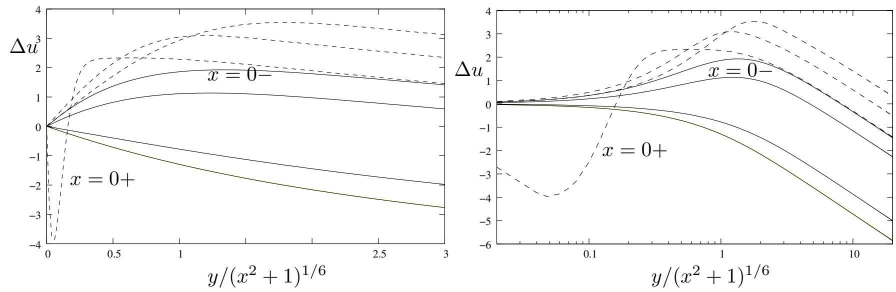

Fig. 2 Velocity profiles Δu at different locations x, $x < 0$: soild lines, $x > 0$: dashed lines. Δu is plotted versus a linear and a logarithmic scale of $\zeta = y/(x^2+1)^{1/6}$

In figure 2 velocity profiles are shown at different locations x as a function of the similarity variable ζ. The logarithmic behavior for $y \to \infty$ can be observed. Moreover the velocity profiles at $x = 0-$ and $x = \Delta x$ are shown. As predicted we observe the logarithmic behavior for $y \to 0$. Since the no-slip boundary condition at $y = 0$ has to be satisfied a viscous sub-layer forms.

3 Additional sub-layers

To resolve the pressure discontinuity at $x = 0$ additional sub-layers have to be introduced. Here we mention only the inviscid sub-layer with dimensions of $\mathrm{Re}^{-5/8}$ in x and y-direction. We define the local coordinate $\hat{x} = \mathrm{Re}^{-2/8}\mathrm{x} = \mathrm{Re}^{-5/8}\mathrm{X}$ where X is the original horizontal coordinate scaled with the plate length. For the interaction pressure we obtain the elliptic equation

$$\bar{u}_y(0,y)\Delta\hat{p} = \bar{u}(0,y)\left(\Delta\hat{u}_{xx} + \Delta\hat{u}_{yy}\right), \tag{5}$$

subjected to the boundary conditions

$$\Delta\hat{p}(x,0) = \begin{cases} \Delta p(0-) & x < 0 \\ 0 & x > 0 \end{cases}, \quad \Delta\hat{p}(r \to \infty, \varphi) \sim \Delta p(0-)\frac{2\varphi - \sin(2\varphi)}{4\pi}. \tag{6}$$

The unique solution of this ellipitc equation is smooth with the exception of the origin. To resolve this singularity further sub-layers reproducing locally the full Navier-Stokes equations have to be introduced.

4 Conclusions

Considering the trailing edge problem for mixed convection flow past a horizontal plate a new interaction mechanism has been analyzed. The solution exhibits new features such as a pressure discontinuity at the trailing edge. Thus on triple deck scales there will be a flow around the trailing edge although on the scales of the potential flow the Kutta condition is satisfied [4]

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