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TOPOLOGICAL BIRKHOFF

MANUEL BODIRSKY AND MICHAEL PINSKER

ABSTRACT. One of the most fundamental mathematical contributions of Garrett Birkhoff is the HSP theorem, which implies that a finite algebra **B** satisfies all equations that hold in a finite algebra **A** of the same signature if and only if **B** is a homomorphic image of a subalgebra of a finite power of **A**. On the other hand, if **A** is infinite, then in general one needs to take an *infinite* power in order to obtain a representation of **B** in terms of **A**, even if **B** is finite.

We show that by considering the natural topology on the functions of **A** and **B** in addition to the equations that hold between them, one can do with finite powers even for many interesting infinite algebras **A**. More precisely, we prove that if **A** and **B** are at most countable algebras which are oligomorphic, then the mapping which sends each term function over **A** to the corresponding term function over **B** preserves equations and is *Cauchy-continuous* if and only if **B** is a homomorphic image of a subalgebra of a *finite* power of **A**.

Our result has the following consequences in model theory and in theoretical computer science: two ω-categorical structures are primitive positive bi-interpretable if and only if their topological polymorphism clones are isomorphic. In particular, the complexity of the constraint satisfaction problem of an ω-categorical structure only depends on its topological polymorphism clone.

1. INTRODUCTION

The algebraic result we present has a motivating application in model theory, which in turn has implications for the study of the computational complexity of constraint satisfaction problems in theoretical computer science. We start our introduction with this model-theoretic perspective on our result, and describe the central algebraic theorem of this article later in the introduction, in Section 1.2.