Plastic Deformation of Axially Moving Continuum in Mixed Eulerian-Lagrangian Formulation

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Abstract

We present a new approach to model the motion of a metal sheet during a rolling mill process. In particular, we focus on the planar motion of the sheet as seen from the bird's eye view in the area between two consecutive roll stands. The outflow of the sheet from the preceding roll stand, as well as the infeed of the sheet into the subsequent roll stand are subject to given entry and exit velocity profiles, which are inhomogeneous and varying in time. The method takes advantage of a mixed Eulerian-Lagrangian formulation, meaning that the governing equations are based on spatial coordinates and material points at the same time. The resulting Finite Element mesh is spatially fixed with respect to the direction of the rolling process, whereas material is transported across the geometry of the mesh. The overall deformation is described by a multiplicative scheme, allowing for an exact geometrical representation and the study of specifically non-linear effects. Particular emphasis is placed on modeling plastic material behavior, and the transport of the inelastic strain field required thereby.

Keywords: material generation, spatial description, finite element method, geometric nonlinearity, elastoplasticity, axially moving structures

1. Introduction

We investigate the numerical computation of the planar deformation of an axially moving strip with prescribed velocity distribution at spatially fixed interfaces. Such problems naturally occur in material forming processes as, e.g., the extrusion, rolling or coiling of thin strips of material. Analytical results on vibrations of axially moving one-dimensional structures are summarized in the review paper Ref. [3]. A Finite Element approach of this problem based on a Lagrangian formulation, however, would ultimately lead to meshes with materially fixed elements. Spatially fixed interface conditions would have to be imposed away from elements' edges, and serious numerical troubles would arise, cf. the discussion of *variational crimes* in, e.g., Ref. [2].

In contrast to that, the presented approach utilizes a mixed Eulerian-Lagrangian formulation, leading to meshes which are spatially fixed in transport direction, whereas they move with the material in transverse direction. A consistent mathematical description of imposing the interface conditions at spatially fixed positions is thereby possible. We further focus on a consistent formulation of finite strain plasticity in the framework of the presented kinematic description, and we discuss the question of how to transport the plastic strain field in an optimal manner. The latter has been investigated lately by the authors of Ref. [5] for the case of one dimensional structures.

2. Kinematic Description

Let us consider a strip of continuous material, which is moving in the (x, y)-plane, see Fig. 1. The gross motion takes place in x-direction, such that at each time $t \in [0, T]$ a certain set of the strip's material particles, called the *active domain* $\Omega(t)$, is located between the interfaces Γ_{entry} at $x = 0$ and Γ_{exit} at $x = L$.

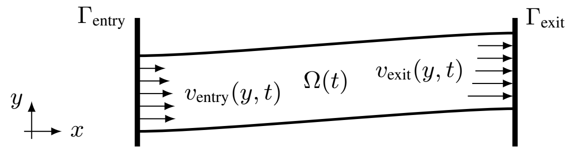

Figure 1: Material strip $\Omega(t)$ moving in between two spatially fixed interfaces Γ_{entry} and Γ_{exit}.

Material particles are considered to enter the active domain in form of strip segments with variable length
$l(y, t) = v_{\text{entry}}(y, t)\ \mathrm{d}t$
as outlined in the top left scene of Fig. 2. In the infinitesimal framework we may instead equivalently consider the application of an intrinsic deformation gradient $\mathbf{F}_*$ applied to strip segments of constant length $l_* = v_*\ \mathrm{d}t$, with v_* denoting the strip's nominal velocity. Thereby – and under consideration of additional plastic deformation $\mathbf{F}_p$ – the active domain of the strip forms its reference configuration (see bottom left scene of Fig. 2) with $\mathring{\mathbf{r}}$ denoting the position of each material particle of the strip's active domain. Elastic deformation $\mathbf{F}_e$ finally leads to the actual configuration of the active domain (top right scene of Fig. 2), where

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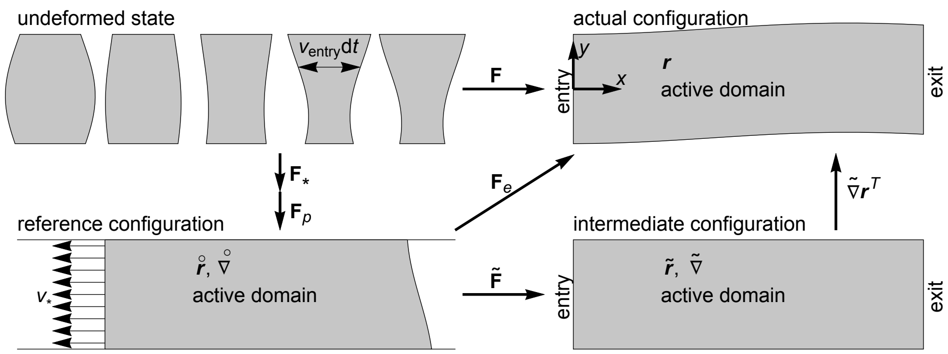

Figure 2: Deformation gradient **F** and its multiplicative decomposition.

material particles now have position

$$\mathbf{r} = \mathring{\mathbf{r}} + u_x\,\mathbf{i} + u_y\,\mathbf{j}\,, \tag{1}$$

with $\mathbf{i}$ and $\mathbf{j}$ denoting unit vectors in x- and y-direction, respectively. The overall deformation gradient is given by

$$\mathbf{F} = \mathbf{F}_e\,\mathbf{F}_p\,\mathbf{F}_*\,. \tag{2}$$

In a classical Lagrangian formulation material particles are identified by their position in reference configuration $\mathring{\mathbf{r}}$. In other words, the displacement solution $\mathbf{u} = (u_x, u_y)$ is obtained by integration of the total strain energy on the active domain in reference configuration as a function in $\mathring{\mathbf{r}}$. In contrast to that, we prevent the finite element mesh from traveling across the interface lines in axial direction by using a mixed Eulerian-Lagrangian formulation. Material particles are identified by their position vector

$$\tilde{\mathbf{r}} = \mathring{\mathbf{r}} + u_x(\tilde{\mathbf{r}})\,\mathbf{i} = \mathbf{r} - u_y(\tilde{\mathbf{r}})\,\mathbf{j} \tag{3}$$

in an intermediate configuration (bottom right scene of Fig. 2). Integration of the total strain energy is performed on the rectangular shape of the active domain in intermediate configuration, meaning, all field variables are considered to be functions of $\tilde{\mathbf{r}}$.

3. Material model

Having accepted the multiplicative decomposition of the total deformation gradient in Eq. (2), we then turn to the application of a finite strain plasticity law. In this context we assume a rate dependent Mises solid with isotropic hardening. The presented mathematical model follows the monograph Ref. [1], the numerical solver is closely related to a a method suggested by the authors of Ref. [4]. Numerical results, as those shown in Fig. 3, give rise to the question of proper strategies concerning the transport of the plastic and intrinsic deformation gradients $\mathbf{F}_p$ and $\mathbf{F}_*$.

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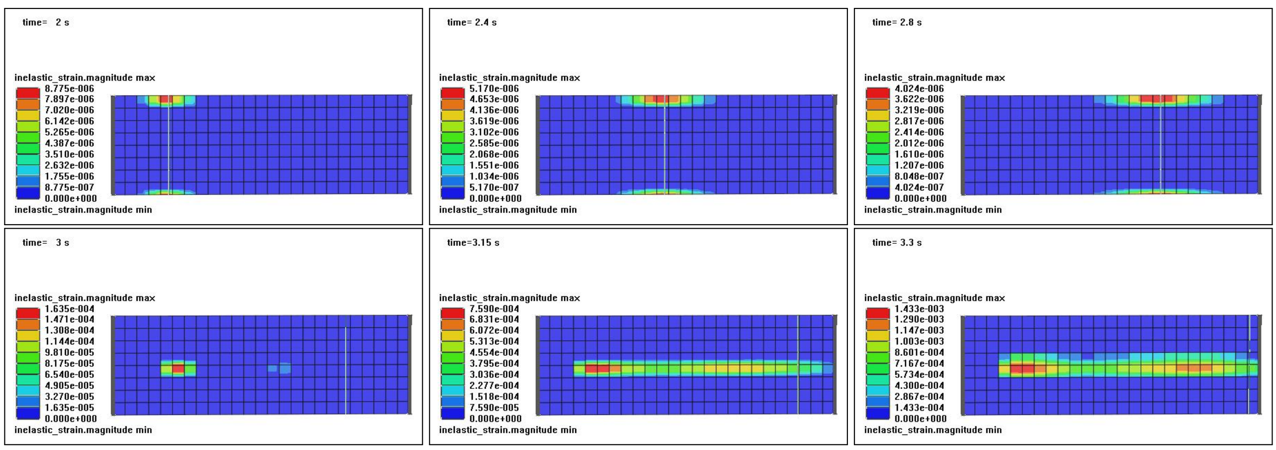

Figure 3: Simulation results reporting on transport of localized old plastic zones (upper plots), as well as sudden and global development of new plastic zones (lower plots).