

SPRINGER BRIEFS IN MATHEMATICS

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Entropy Methods for Diffusive Partial Differential Equations

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Preface

The concept of entropy in mathematics. Entropy methods for partial differential equations. Some entropy methods over the last decades. These methods for solutions to diffusive equations. The large-time asymptotics, the existence of inequalities, the existence of discrete and geometric entropy.

The purpose of this book is to provide a survey of entropy methods which can be found in the literature. It is not stated in the widest general sense that the functional entropy is a new concept. My hope is that in this book, many PhD students and may be some other researchers will find it useful.

The book consists of two parts. The first part is devoted to entropy in physics and to the entropy methods for Fokker-Planck equations. Entropy methods for Fokker-Planck equations are a huge topic, investigated by many authors over the last decades. In particular, the entropy methods of Markowich, Toscani, and Villani are given, based on the work of Dolbeault. Many aspects of the entropy methods are discussed, such as the placement convexity, Ricci curvature, and the entropy-entropy dissipation principles. More details are given in the monograph by Villani. Chapter 3 is concerned with the entropy methods for the heat equation, as treated by Matthes, Bukal,

methods in an efficient, higher order equations.

Entropy methods are here proved the global existence of solutions. These techniques were developed by Di Francesco, Pietschmann, and others. These methods are rather technical since they rely on weak convergence arguments which are treated in the appendix.

The book is motivated by the aim of providing a comprehensive treatment of the numerical level. Since various approaches taken from the literature are discussed (recently investigated by Mielke; Fathi and Maas; and solutions of Fokker–Planck equa-

schools in Vienna (Austria) in 2012, Kacov (Czech Republic) in 2015. Some material in this book is based on notes of Matthes and Evans, and the work of Burger, Gajewski, Mielke, and others. The corresponding sections. In some cases, the book is exhaustive. I do not claim

the cooperation with my colleagues (in alphabetical order) J.A. Carrillo (Barcelona), P. Degond (London), G. Di Francesco (Vienna), N. Zamponi (Vienna), M. Burger (Darmstadt), M. Groisman (Berlin), J. Fuhrmann (Darmstadt), and others. Last but not least, I would like to thank my wife for her fruitful discussions. Last but not least, I would like to thank me to write this brief

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Ansgar Jüngel

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Chapter Intro

Abstract ...
in Sects. 1.
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Keywords
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1.1 Entr

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