Coverage Distribution of Heterogeneous Cellular Networks Under Unsaturated Load

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***Abstract*—Heterogeneous cellular networks have received considerable attention in the literature owing to increased data demands of cellular network users. Recently, Poisson Point processes (PPPs) are vastly used for modeling and analyzing cellular networks because of their accuracy and tractability. However, results in this area suffer from a big drawback. They considered a saturated load model in their analysis or used a base station (BS) activity factor independent of the network condition. In this work, we evaluate the performance of cellular networks under a more accurate definition of load. In cellular networks, a BS is active when it has at least one user associated and is inactive when it has no associated user. The load definition used in this paper is a user-based load model and denotes whether at least one user is associated with the BS or not. This paper provides a closed form relation for the probability of inactive BSs of each tier. Based on this value, the coverage probability of the network has been derived. Finally, the performance of analytical expressions has been validated with simulations.**

***Index Terms*—Heterogeneous Cellular Networks; Poisson Point Process; Load-aware User Association; Coverage distribution.**

I. Introduction

Recently, the development of new devices such as smartphones, and tablets along with data hungry applications (i.e, games, maps etc.), enforced cellular network operators to answer this demand by introducing new technologies. Adding low power base stations (BSs), which are generally called small cell BSs, to the network are the most promising way for increasing the data rate of cellular networks, and the resulting networks are called heterogeneous cellular networks (HCNs). Although, HCNs aim to increase the capacity of cellular networks, the performance of HCNs is questionable if we assign the users to BSs based on traditional user assignment techniques [1].

In traditional cellular networks, a user is generally assigned to a BSs with respect to the received power and the BS that provides the highest SINR at such user is considered as serving BS. Since, the lower tier BSs have lower transmit power, most of the users receive the highest power from macro cell BSs, and choose the nearest one as serving BS. So, to overcome the problem of imbalanced load distribution between the tiers, an efficient load balancing technique is required [2]. Biasing users to the lower tier BSs, by multiplying a bias value to their received power from lower tier, is one of the most popular ways of load balancing [3–5].

Because of, the stochastic distribution nature of lower tier BSs, and the tractability of Poisson point processes (PPPs), modeling and analyzing networks using PPPs has received considerable attention in HCNs literature [6]. In [7], the authors considered the problem of performance evaluation of cellular networks in a single tier network. They showed that even in a single tier network, PPPs could provide results in performance evaluation of networks at least as tight as a grid model. A similar study for validating performance of PPPs for modeling HCNs has been done in [8].

In [9], the authors provide a general framework for performance evaluation of K-tier cellular networks. They assumed each user chooses the strongest BS in terms of signal to interference plus noise (SINR), as serving BS and based on this assumption they derived the coverage probability and average data rate of the network. They proved that in open access networks, the coverage probability is independent of the number of tiers or the density of BSs where the target SINR of all tiers are identical. In [10], the authors present a framework to evaluate the performance of indoor users, in urban two-tier cellular networks. In their model of urban cellular networks, the buildings are situated in the network area according to a PPP, and are equipped with small cell BSs with some given probability. The users are associated to macro BSs when no small cell BS is situated inside the building and the users choose the small cell BS when the building is occupied with small cell BS. Based on this system model, they derive the coverage probability of a typical indoor user in two-tier cellular network.

The performance evaluation of HCNs under flexible user association and load balancing has been considered in [11]. They present the coverage probability of each tier, where the users choose the BS with maximum long-term biased received power as serving BS. Based on the results of [11], the authors in [3, 12] tried to find the optimal value for the bias factor under different system settings.

Even with vast efforts in modeling of HCNs, the assumption of a saturated load model is still a major drawback in most of the literature. To date, most of the works assumed each BS has always data to transmit. Among them, [13] is one of the work that tried to relax this assumption by considering the load of each BS as an independent activity factor. However, their definition of load suffers from the independency of the activity factor from the actual network conditions. The independent activity factor p used in that paper does not reflect the actual user association of network, and could be far from real network conditions.

Main contributions: The main contributions of this paper can be summarized as follows:

1) This paper provides a closed form definition for the probability distribution function (PDF) of the number of users

associated to a BS in tier i, in addition to the probability of inactive BS.

2) By relating the BS load definition to the number of users assigned to it, a more accurate definition of load is presented. Since finding the exact relation of the BS load and number of users associated to it is not easy to compute, we use a simplified model of load and a more accurate model will be addressed in future work. We assume that, a BS is inactive when no user is associated to it, and is active when it has at least one user. In this paper, first we find the probability of inactive BS as function of the tier association probability, BS density, and user density. Then, based on this definition of load the user coverage probability when the user is associated to tier i is derived.

3) In [13], the authors only computed the total coverage of networks where the user chooses the BS with highest SINR as serving BS. Finding the coverage probability of each tier, and coverage probability under biased user association is another interesting contributions of this paper.

The rest of the paper is organized as follows. The system model and assumptions are presented in Section II as well as the tier association policy employed in this paper. Section III, provides an exact definition of load and introduces the inactive probability of BSs. Section IV presents coverage probability of a typical user in the network and numerical results are presented in Section V before the paper is concluded in Section VI.

II. System Model

We consider the downlink of stochastically distributed K-tier heterogeneous cellular networks. The BSs of tier i are distributed according to some PPP ϕ_i with density λ_i in the Euclidean plane, and the users are distributed with another independent PPP ϕ_u with density λ_u. The path loss exponent is set to α for all tiers and all of the BSs of tier i use identical transmit power P_i. Small scale fading between the user, and the BS, m, in tier i, denoted by $h_{i,m}$, follow an exponential distribution with unit mean. Suppose that, $y_{i,m}$ is the distance between the BS m in tier i, and a typical user under consideration u. The received power at this user is modeled as $P^u = P_i h_{i,m} y_{i,m}^{-\alpha}$. Referring to the definition of received power, (1) provides the SINR at node u, when the user is associated with a BS in tier i.

$$\mathrm{SINR}(i,u) = \frac{P_i h_{i,m} r_{i,m}^{-\alpha}}{\sum_{k=1}^{K} \sum_{x \in \phi_k, x \neq m} P_k h_{k,x} y_{k,x}^{-\alpha} + \sigma^2}. \quad (1)$$

In (1), we assumed that the distance between the user and its serving BS in tier i, called m, is $r_{i,m}$, which is a random variable, and σ^2 is the noise power. The user will be covered by BSs of tier i if the received SINR of the user is at least equal to an SINR threshold τ_i.

A. User Association and Distance Distribution

We adopt the long-term biased received power as association metric throughout this paper. Each user chooses the BS with maximum long-term biased received power as serving BS

$$u \in \mathcal{U}_i \quad if \quad B_i P_i r_{i,u}^{-\alpha} > \max_{k \in \mathcal{K}, k \neq i} P_k B_k r_{k,u}^{-\alpha} \quad (2)$$

where the B_i is the user association bias value, that is identical for all BSs of tier i, and $\mathcal{U}_i$ is the users set of tier i. The coverage area of tier i is expanded, if the larger bias value is used by the BSs of this tier. Under this user association policy, (3) denotes the association probability of a user to the BSs of tier i [11]

$$A_i = \left(1 + \frac{\sum_{j=1, j \neq i}^{K} \lambda_j (P_j B_j)^{\frac{2}{\alpha}}}{\lambda_i (P_i B_i)^{\frac{2}{\alpha}}}\right)^{-1}. \quad (3)$$

Using the results of association probability, (4) computes the distance distribution between a user and its serving BS given that the user is associated with tier i [11].

$$f_{R_i}(r) = \frac{2\pi\lambda_i r}{A_i} \exp\left(-\pi \sum_{k=1}^{K} \lambda_k \left(\frac{P_k B_k}{P_i B_i}\right)^{\frac{2}{\alpha}} r^2\right). \quad (4)$$

III. Definition of Load

It is clear that the load of different BSs in the network is correlated. Increasing the load in one BS will increase the interference at the users of the other BSs which leads to decreasing the rate of the cell and increasing the backlogged load in that cell. Involving the load in performance evaluation of HCNs is intricate due to this correlation. We try to use a relaxed definition of load by relating the load of the BS to the number of users associated with it.

There is a direct relation between the load of the BS, and the number of users associated with it. Increasing the users of a BS, will increase the amount of load of that BS. To present the user-based load definition, let us first introduce the probability distribution function (PDF) of the number of users associated to a BS in tier i.

Theorem 1. *The probability distribution function of number of users associated to a BS in tier i is equal to:*

$$f_{N_i}(n) = \frac{(A_i\lambda_u)^n}{n!} \frac{3.5^{3.5}}{\Gamma(3.5)} \left(\frac{\lambda_i}{A_i}\right)^{3.5} \frac{\Gamma(n+3.5)}{\left(\frac{3.5\lambda_i}{A_i} + A_i\lambda_u\right)^{n+3.5}}. \quad (5)$$

Proof. The PDF of a Voronoi cell area under association probability A_i , presented in [14], is equal to:

$$f_{C_i}(c) = \frac{3.5^{3.5}}{\Gamma(3.5)} \frac{\lambda_i}{A_i} \left(\frac{\lambda_i}{A_i} c\right)^{2.5} \exp\left(-\frac{3.5\lambda_i}{A_i} c\right). \quad (6)$$

The user distribution in the association area of the BSs of tier i, is a thinned PPP with density $A_i\lambda_u$. Using (6), the PDF of the number of users associated to the tier i is equal to:

$$f_{N_i}(n) = \int_0^\infty \frac{(A_i\lambda_u c)^n e^{-A_i\lambda_u c}}{n!} \frac{3.5^{3.5}}{\Gamma(3.5)} \frac{\lambda_i}{A_i} \left(\frac{\lambda_i}{A_i} c\right)^{2.5} \exp\left(-\frac{3.5\lambda_i}{A_i} c\right) dc$$

$$f_{N_i}(n) = \frac{(A_i\lambda_u)^n}{n!} \frac{3.5^{3.5}}{\Gamma(3.5)} \left(\frac{\lambda_i}{A_i}\right)^{3.5} \int_0^\infty c^{n+2.5} \exp\left(-\frac{3.5\lambda_i}{A_i} c - A_i\lambda_u c\right) dc,$$

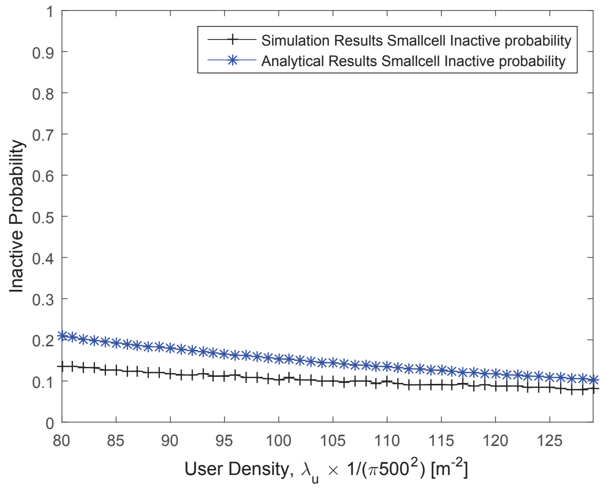

Fig. 1: Small cell inactive probability under different user density.

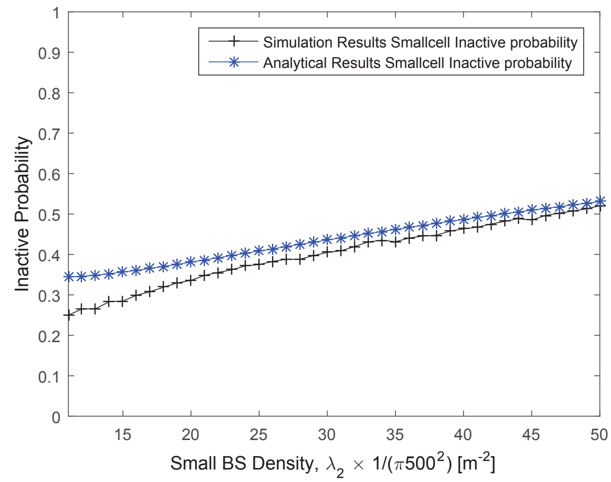

Fig. 2: Small cell inactive probability under different small cell density.

as $\left(\frac{3.5\lambda_i}{A_i} + A_i\lambda_u\right) > 0$, we reach (6). □

Clearly, when the BS has no user associated, it has no load and it is inactive. So, in this paper we use an on\off load model. The BS is inactive when no user is associated with it and is active when there is at least one user associated. So, the network load is defined as:

$$L(i) = \begin{cases} 0 & ,\mathcal{U}_i = \varnothing \\ 1 & ,\text{otherwise} \end{cases}. \tag{7}$$

Using the results of Theorem 1, the probability that a BS in tier i has no associated user(inactive probability) is equal to:

$$p_0(i) = \frac{(3.5\lambda_i)^{3.5}}{(3.5\lambda_i + (A_i)^2\lambda_u)^{3.5}}. \tag{8}$$

We modify the BSs distribution of tier i, to a thinned PPP of density $(1-p_0(i))\lambda_i$, that reflects the density of BSs that have at least one associated user.

IV. Coverage Probability

This section presents the coverage probability of each tier under the defined load model, and flexible user association. Without loss of generality, and using the Slivnyak theorem [15], we assume that a typical user under consideration is located in the origin. The coverage probability is equal to the probability that the received SINR at the typical user is higher than the SINR threshold.

Theorem 2. *The coverage probability of a typical user in cellular network when the user is associated to a BS in tier i is equal to:*

$$\mathcal{C}(i,\tau_i) = \int_0^\infty \exp\left(\frac{-\tau_i r^\alpha}{P_i}\sigma^2\right)\prod_{k=1}^{K}\exp\Bigg\{-2\pi\lambda_k\frac{\tau_i P_k}{P_i(\alpha-2)}\left(\frac{P_kB_k}{P_iB_i}\right)^{\frac{2}{\alpha}-1} r^2\left(1-\frac{(3.5\lambda_k)^{3.5}}{(3.5\lambda_k+(A_k)^2\lambda_u)^{3.5}}\right)\frac{2\pi\lambda_i r}{A_i}\, {}_2F_1\left(1,1-\frac{2}{\alpha},2-\frac{2}{\alpha},-\tau_i\frac{B_i}{B_k}\right)\Bigg\}\exp\left(-\pi\sum_{k=1}^{K}\lambda_k\left(\frac{P_kB_k}{P_iB_i}\right)^{\frac{2}{\alpha}} r^2\right). \tag{9}$$

Proof. The coverage probability of a network is computed using (10):

$$\mathcal{C}(i,\tau_i) = \int_0^\infty \mathbb{P}\left(\frac{P_i h_{i,x} r^{-\alpha}}{\sum_{k=1}^{K}\sum_{z\in\hat{\Phi}_k\backslash x} P_k h_{kz} {y_{kz}}^{-\alpha} + \sigma^2} > \tau_i\right) f_{R_i}(r)dr \tag{10}$$

by conditioning on r we have:

$$\mathcal{C}(i,\tau_i) = \mathbb{P}\left(h_{i,x} > \frac{\tau_i r^\alpha}{P_i}\left(\sum_{k=1}^{K}\sum_{z\in\hat{\Phi}_k\backslash x} P_k h_{kz} {y_{kz}}^{-\alpha} + \sigma^2\right)\right),$$

since the fading in a network has an exponential distribution with unit mean, we have:

$$\mathcal{C}(i,\tau_i) = \exp\left(\frac{-\tau_i r^\alpha}{P_i}\left(\sum_{k=1}^{K}\sum_{z\in\hat{\Phi}_k\backslash x} P_k h_{kz} {y_{kz}}^{-\alpha} + \sigma^2\right)\right)$$

$$= \exp\left(\frac{-\tau_i r^\alpha}{P_i}\sigma^2\right)\prod_{k=1}^{K}\exp\left(\frac{-\tau_i r^\alpha}{P_i}\left(\sum_{z\in\hat{\Phi}_k\backslash x} P_k h_{kz} {y_{kz}}^{-\alpha}\right)\right)$$

$$\overset{a}{=} \exp\left(\frac{-\tau_i r^\alpha}{P_i}\sigma^2\right)\prod_{k=1}^{K}\mathbb{E}_{\phi_k}\left[\prod_{z\in\hat{\Phi}_k\backslash x}\frac{1}{1+\frac{-\tau_i r^\alpha P_k}{P_i}{y_{kz}}^{-\alpha}}\right]$$

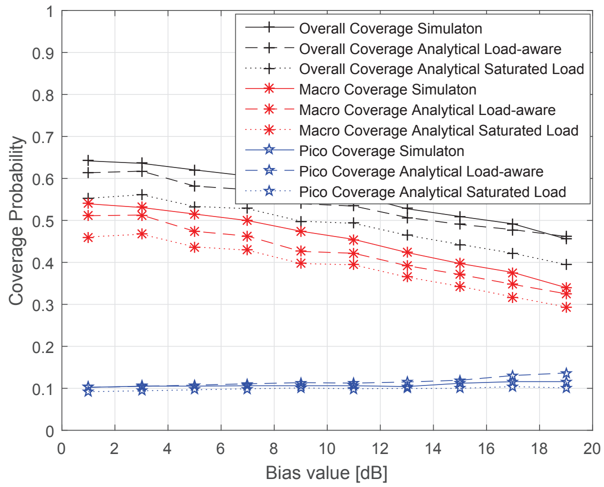

Fig. 3: Effects of bias value on coverage probability of network.

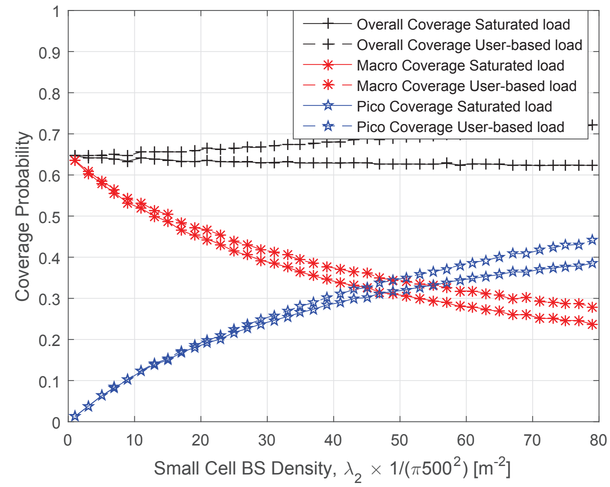

Fig. 4: Effects of small cell density on coverage probability of network.

$$\stackrel{b}{=} \exp\left(\frac{-\tau_i r^\alpha}{P_i}\sigma^2\right)\prod_{k=1}^{K}\exp\Bigg(-2\pi\lambda_k\left(1-p_0(i)\right) \int_{d_k}^{\infty} 1-\frac{1}{1+\frac{-\tau_i r^\alpha P_k}{P_i}y^{-\alpha}}dy\Bigg),$$

where (a) is obtained using the Laplace transform of exponential distributions and (b) is obtained using the probability generating functional (PGFL) of the PPP [15]. Considering the user assignment policy, the distance between the user and its nearest interferer of tier k is equal to $d_k = (\frac{P_k B_k}{P_i B_i})^{(\frac{1}{\alpha})} r$,

$$\mathcal{C}(i,\tau_i) = \exp\left(\frac{-\tau_i r^\alpha}{P_i}\sigma^2\right)\prod_{k=1}^{K}\exp\Bigg(-2\pi\lambda_k\left(1-p_0(i)\right) \frac{\tau_i P_k}{P_i(\alpha-2)}\left(\frac{P_k B_k}{P_i B_i}\right)^{\frac{2}{\alpha}-1} r^2 {}_2F_1\left(1,1-\frac{2}{\alpha},2-\frac{2}{\alpha},-\tau_i\frac{B_i}{B_k}\right)\Bigg), \tag{11}$$

by putting (11) and (4) in (10), we can obtain (9). □

V. Numerical Results

This section, provides some simulations to evaluate the performance of HCNs under a user-based load model. We simulate a two-tier network in a $4km \times 4km$ square area. All the results could be obtained for a general K-tier HCN, but the assumption of a two-tier cellular network is made solely for simplicity. For all numerical results, the macro BSs and small cell BSs transmit power is set to 46 dBm and 26 dBm, respectively.

Before comparing the coverage probability of the network, we validate our expression for inactive probability presented in (8). Fig. 1 and Fig. 2, compare the simulation and the analytical results for the inactive probability of the small cell BSs, in a two tier cellular network. The results show that, our analytical approximation of inactive probability closely matches the corresponding simulated results, which validates our load definition in Section III.

Fig. 1 shows the effect of different user densities on the inactive probability of the small cell BSs. In this simulation, the bias value of both tiers is set to 1, the macro BS distribution density is $\lambda_1 = \frac{1}{\pi 500^2}$, the small cell BS distribution density is $\lambda_2 = 10\lambda_1$, and the user density changes from $80\lambda_1$ to $130\lambda_1$. It shows that, by increasing the user density, the inactive probability of small cell BSs will decrease.

In Fig. 2, as the small cell density increases, the number of users associated with each small cell BS will decrease. So, we observe an increase in inactive probability of small cell BSs. In this simulation, the user density is set to $\lambda_u = 50\lambda_1$. Fig. 3, depicts the coverage probability of the network obtained by the analytical evaluation and the simulation results, under the different small cell bias values. We compared the overall coverage, macro BSs coverage, and small cell BSs coverage of load-aware analytical expression, against the results of fully saturated model of load presented in [11]. The proposed framework based on the user-based load model, provides a tighter lower bound approximation of coverage probability compared to the saturated load model. By increasing the bias value of the small cell BSs, the coverage probability of the network will decrease. This is completely in line with the findings of [11], and is a direct result of assigning the users to the small cell BSs with lower received SINR. To overcome the problem of decreasing the coverage probability, which is the result of increasing the bias value, an efficient interference management technique is required.

Fig. 4 shows the effect of changing the small cell density on overall coverage of the network in the saturated and user-based load model. In this simulation we use the bias 1 for both tiers to eliminate the effect of biasing on coverage of the network. The user density is set to $100\lambda_1$ when the small cell BS density of the network has been changed from λ_1 to $80\lambda_1$. Analogue to the result of [9, 11], the overall coverage probability of the saturated load model remains fixed under a different small cell density. However, increasing the small cell BS density improves the overall coverage probability of the user-based load model. By increasing the small cell density the number of users associated with each small cell BS is decreased, so we observe an improvement in the coverage probability of the user-based load model, and also the larger difference between the results of the two load models.

VI. Conclusion

This paper develops a new analytical model for the evaluation of SINR distributions of HCNs under an unsaturated load model, and flexible user association. We derived the coverage probability of the network by incorporating a user-base notion of the load to the analysis. In our proposed model, the BSs of each tier are inactive, when no user is associated with them, otherwise they are fully loaded. Supported by numerical results, our model provides a more accurate framework for the performance evaluation of HCNs, when the fully loaded model is pessimistic in terms of coverage.

The load definition used in this paper is a simplified model. Developing a load-aware framework for performance evaluation of the network, and incorporating the load correlation into the model can be an extension of this paper. It would also be very interesting to perform the analytical evaluation of HCNs by considering the other types of point process that models repulsion or the minimum separation distance between BSs, such as determinantal and Matern processes. This paper implicitly assumed that the bandwidth is completely shared between the BSs. Another important extension is evaluating the performance of the network by considering the more advanced interference management techniques.

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