Calculation of delivery rate in fault-tolerant network-on-chips

Junshi Wang, Letian Huang, Guangjun Li and Axel Jantsch

Reliability for network-on-chips has been widely researched in last decades not only to support the rapid growing requirement of on-chip communication but also to address the challenge on reliability due to aggressive technology scaling. Simulation is the most common method to evaluate the capacity of reliability design but its cost is significant. Therefore, it is necessary to calculate the capacity for reliability of a design with light-weight mathematical models. A model for faults on links is described to calculate the delivery rate of networks based on a Markov chain fault model, fault-tolerant configuration and network traffic. The calculation results show accurate approximations of simulation results.

Introduction: Since the emergence of the deep sub-micro era, inter-core communication on-chip has become the critical design issue for system-on-chips. Network-on-chips (NoCs) have caught the attention of researchers and engineers as they can provide high bandwidth, flexibility and redundancy. Aggressive technology scaling also weakens the reliability of devices due to process variation, thermal issue, wear-out and so on. Techniques of fault-tolerant design are necessary to extend the lifetime of NoCs.

Many interesting works have been published in this field [1]. Usually, reliability design is evaluated using system level or circuit level simulation, which requires a lot of manpower and time. Reliability design has to be implemented in a simulator. During simulation, many configurations with different parameters about fault injection, traffic distribution, fault detection and tolerance methods have to be tested to cover as many cases as possible. Even with parallel computation, it still costs easily from 1 week to 1 month. Thus, calculation models would be very useful to estimate the capacity of fault-tolerant methods and guide the reliability design.

To compare the capacity of reliability designs, delivery rate is taken as a uniform measurement. First, the curves of delivery rate to faults show the differences between different reliability designs on mean-time-to-failure (MTTF) and life time. For transient faults, with same fault probability, the design with higher delivery rate achieves higher MTTF. For permanent faults, the design with higher fault proportion can work for a longer life than the designs with lower fault proportion with the same deliver rate (as shown in Fig. 1). Furthermore, delivery rate only depends on network parameters. Hence, delivery rate can show the capacity for reliability and is easy to calculate.

A simple mathematical model based on Markov chains is proposed to calculate the capacity of methods to tolerate faults on links. It exhibits three characterising features: (i) faults are described using Markov chains at the circuit layer to provide a uniform formula for different kinds of failures; (ii) fault probability of a packet is estimated based on a fault model and routing algorithm; (iii) the delivery rate of a network is calculated based on traffic distribution. Comparing simulation results, the proposed model can provide very close approximate results with much lower computation effort.

Basic assumptions: To simplify the fault model, we make some assumptions. (i) Probability steadiness: the fault probability and proportion of faulty wires do not change. (ii) Spatial independence: the fault states of wires are independent from each other. (iii) Temporal dependence: the fault states of wires are only dependent on the state in the previous cycle. (iv) Packet continuity: the packet is continued in transmission, without empty slots.

Model architecture and flow: As shown in Fig. 2, we propose model in four steps. First, the fault model of a group of wires is investigated based on a circuit layer fault model and fault-tolerant design. The wires within one group should belong to the same group of a fault-tolerance method. Then, by combining states of groups together, a fault model of a flit passing one link is determined in step 2. After that, the fault probability of a packet from a given source and destination is determined by the results from step 2 and the given routing algorithm. Finally, delivery rate of the network is calculated based on packet fault probability and traffic distribution.

Fig. 2 Flowchart of proposed model

Circuit layer fault model: Fault phenomena can be described at physical, circuit and network layers. A circuit layer fault model describes the fault in signals, like stack-at-0, stack-at-1 and bit-reversal. Compared with the physical and network layer circuit layer fault models, circuit layer fault models have the advantage to be simple and uniform.

According to assumptions of probability steadiness, spatial independence and temporal dependence, the state of each wire $e_i(t)$, in which $i$ denotes the wire and $t$ denotes the cycle, follows the first-order homogeneous Markov chain. The stochastic variable $e$ has two states: living ($L$) and faulty ($F$), described by two-tuple $FM_e = \{ P_{O_e}, P_{F_e} \}$, as shown in Fig. 3a. The probability to switch from living to faulty is determined by the occurrence probability $P_{O_e}$, and the probability to switch from faulty to living is determined by the recovery probability $P_{R_e}$. $P_{O_e}$ and $P_{R_e}$ may be different for different wires. The steady-state probability of $F$ and $L$ is $P_{O_e}(t) = F = P_{O_e}/(P_{O_e} + P_{R_e})$ and $P_{O_e}(t) = L = P_{R_e}/(P_{O_e} + P_{R_e})$, respectively.

Fig. 3 Group fault model

a) Markov chain of one wire $e_i$

b) Markov chain of group of wires $E$. Numbers in circles indicate value of states

c) Markov chain of group $G$

Group fault model: Assume a group $G$ with $N$ bit wires with fault models $FM_i(0 \leq i < N)$. The state of the group is $E(t) = \{ e_1(t), e_2(t), \ldots, e_N(t) \}$, as shown in Fig. 3b. Owing to the spatial independence assumption, the state probability and the state transition probability are the product of the state probability of state transition probability of each wire

$$P(E(t) = a) = \prod_{i=0}^{N-1} P(e_i(t) = a_i)$$

$$P(E(t+1) = b|E(t) = a) = \prod_{i=0}^{N-1} P(e_i(t+1) = b_i|e_i(t) = a_i)$$

where $a_i$ and $b_i$ are the states of $i$th wire in group state $a$ and $b$.

Given a method that tolerates $n$ faults, it is easy to identify the living and the failed states as illustrated in Fig. 3b. If the state of $E$ is a member of $S_l$, the group is living after fault-tolerant methods, otherwise it is in a failed state.
state. Therefore, the state graph of a group can be reduced to a two-state graph by merging states in $S_i$ and $S_r$ as shown in Fig. 3c. The steady-state probability and state transition probability can be calculated by

$$P(G(t) = L) = \sum_{a \in E} P(E(t) = a)$$  \hspace{1cm} (3)

$$P(G(t) + 1 = L | G(t) = L) = \sum_{i \in E} \sum_{j \in E} P(E(t) + 1 = b | E(t) = a) P(E(t) = a)$$  \hspace{1cm} (4)

**Flit-component fault model:** We define the state of a flit passing one link at time $t$ as $P(t)$. If $P(t)$ is living, all groups are correct at $t$. As a link contains $M$ different groups, the correct probability of a flit at $t$ is the product of the state probabilities of each group, as shown in the following equation:

$$P(F(t) = L) = \prod_{j=0}^{M-1} P(G_j(t) = L)$$  \hspace{1cm} (5)

If a flit can go through a link correctly at $t$, the probability of a flit being correct at $t+1$ is shown in the following equation:

$$P(F(t+1) = L | F(t) = L) = \prod_{j=0}^{M-1} P(G_j(t+1) = L | G_j(t) = L)$$  \hspace{1cm} (6)

**Packet fault probability:** A correct packet means all flits within a packet can pass all links on its path correctly. Owing to spatial independence, the probability of a packet being correct from router $s$ to $d$ is shown in the following equation:

$$P_{\text{PPS}} = \prod_{k=0}^{N-1} [P(F_k(t+1) = L | F_k(t) = L)]^{S-1} P(F_k(t) = L)$$ \hspace{1cm} (7)

in which, $F_k$ is the state of $k$th link on paths, $S$ is the size of packets. If the packets are confirmed by acknowledgement packets, the correct probability of acknowledgement packets should be considered as well. Hence, the correct probability of packets is the product of the correct probabilities of normal packets and acknowledgement packets.

**Network delivery rate:** The delivery rate is defined as the proportion of successfully accepted packets and shown in the following equation:

$$D = \sum_s \sum_d P_{\text{PPS}} \cdot P_{\text{ack}}$$ \hspace{1cm} (8)

in which, $s$ and $d$ are the source and destination ids, $P_{\text{PPS}}$ is the proportion of the packets from $s$ to $d$ among the entire traffic distribution, $P_{\text{ack}}$ is determined in (7).

**Experiment setup:** Our experiment environment is an 8 x 8 wormhole-switch network simulated by a modified POPPET simulator [3]. The network is simulated with a low traffic injection rate of 0.01 packets/cycle/router in a uniform random traffic profile. A packet is confirmed when its acknowledgement packet is accepted correctly. A normal packet contains five flits and an acknowledgement packet contains only one flit. The width of a flit is 128 bits.

Each wire shares the same fault model. Two cases of fault models are exercised, transient and permanent faults. For transient faults, $P_{\text{t}} = 0.9$, $P_{\text{f}}$ ranges from 0 to 0.005. For the permanent case, we simulate the steady state after the faults have accumulated to compress the simulation results. If $P_{\text{t}} = F$ ranges from 0 to 0.04 and $P_{\text{f}} = 0$. XY-routing is implied as routing algorithm. To cover as many cases as possible, for each data point, the result of 100 simulations is averaged.

Two fault-tolerant methods are considered: error correcting code (ECC) and spare wire (SW). Hamming (12,8) is implied at each port, which can tolerate one faulty bit and detect two faulty bits in 12 bits. The spare-wire architecture can replace two permanently faulty wires in 16 bits wires.

Three reliability designs are evaluated using simulation as well as our proposed model: (i) ECC with transient fault (TF + ECC); (ii) ECC with permanent fault (PF + ECC) and (iii) ECC and SW with permanent fault (PF + ECC + SW). The details of reliability designs and simulation results have been published in [4].

**Simulation results and discussion:** Fig. 4 illustrates the simulation and calculation results for these three cases. The proposed model accomplishes a close approximation to the simulation results. The third column in Table 1 shows that the maximum errors between calculation and simulation results with the same fault probability in the experiments are not larger than 0.03. The cause for these errors will be subject of future research.

![Image](419x657 to 470x727)

**Table 1:** Maximum error and time cost for different experiments

<table>
<thead>
<tr>
<th>Fig.</th>
<th>Experiment</th>
<th>Max. error</th>
<th>Sim. time (s)</th>
<th>Cal. time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4a</td>
<td>TF + ECC</td>
<td>0.0082</td>
<td>324.97</td>
<td>0.5019</td>
</tr>
<tr>
<td>4b</td>
<td>PF + ECC</td>
<td>0.0257</td>
<td>142.15</td>
<td>0.0103</td>
</tr>
<tr>
<td>4c</td>
<td>PF + ECC + SW</td>
<td>0.0218</td>
<td>148.83</td>
<td>0.0102</td>
</tr>
</tbody>
</table>

Moreover, the fourth and fifth columns in Table 1 list the time cost of simulation and calculation of one point in the curve. The time of one simulation is more than 600 times higher than that of one calculation. Also, to cover as many cases as possible, simulations have to be repeated several times, but repeating is not necessary for calculation because proposed model has already covered all patterns in the experiments. Hence, the gap of time between simulation and calculation is even larger.

**Conclusion:** Delivery rate, the proportion of successfully accepted packets, can measure the fault-tolerance capacity of reliability designs. To approximate the delivery rate of NoCs quickly, a light-weight model based on Markov chains is proposed. The network delivery rate can be directly calculated from the fault model of each wire, the fault-tolerance design, routing algorithm and traffic distribution.

Comparing the simulation and calculation results of three reliability designs, it is obvious that the proposed model can provide high accuracy with no more than 0.03 absolute error. At the same time, it can reduce the time significantly.

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