Semi-holography for heavy ion collisions

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in collaboration with
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direct QCD approach to heavy ion collisions

glasma picture

\( f \sim \frac{1}{\alpha_s}, \alpha_s(Q_s) \ll 1 \)

F. Gelis, E. Iancu, J. Jalilian-Marian, R. Venugopalan, 

kinetic theory

takes over around \( f \sim 1 \)

dual $\mathcal{N} = 4$ SYM approach to heavy ion collisions

shockwaves ↔ nuclei, black hole formation ↔ thermalization

\[ ds^2 = \frac{1}{z^2} \left[ dz^2 + \left( g_{\mu\nu}^{(0)} + z^2 g_{\mu\nu}^{(2)} + z^4 g_{\mu\nu}^{(4)} + z^4 \ln z \ h_{\mu\nu}^{(4)} + \mathcal{O}(z^6, z^6 \ln z) \right) dx^\mu dx^\nu \right] \]

\[ g_{\mu\nu}^{(0)} = \eta_{\mu\nu} \]

Choose $ds_{\text{initial}}^2$ such that

\[ \frac{1}{z^2} g_{\mu\nu} \ t^{\mu\nu}(t_i) \propto g_{\mu\nu}^{(4)}(t_i) \] mimicks a CGC
dual $\mathcal{N} = 4$ SYM approach to heavy ion collisions

shockwaves $\leftrightarrow$ nuclei, black hole formation $\leftrightarrow$ thermalization

\[ ds^2 = \frac{1}{z^2} \left[ dz^2 + \left( g_{\mu\nu}^{(0)} + z^2 g_{\mu\nu}^{(2)} + z^4 g_{\mu\nu}^{(4)} + z^4 \ln z \ h_{\mu\nu}^{(4)} + \mathcal{O}(z^6, z^6 \ln z) \right) dx^\mu dx^\nu \right] \]

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Choose $ds_{\text{initial}}^2$ such that
\[ t^{\mu\nu}(t_i) \propto g_{\mu\nu}^{(4)}(t_i) \] mimicks a CGC

compare both approaches to heavy ion collisions

to get both on the same page one employs a geometric quench

\[ ds^2 = \frac{1}{z^2} \left[ dz^2 + \left( g^{(0)}_{\mu\nu} + z^2 g^{(2)}_{\mu\nu} + z^4 g^{(4)}_{\mu\nu} + z^4 \ln z h^{(4)}_{\mu\nu} + \mathcal{O}(z^6, z^6 \ln z) \right) dx^\mu dx^\nu \right] \]

\[ g^{(0)}_{\mu\nu} dx^\mu dx^\nu = -dt^2 + dx^2 + dy^2 + dl^2 g(t) \]

\[ g(-\infty) \rightarrow 1, \quad g(+\infty) \rightarrow t^2 \]
compare both approaches to heavy ion collisions
to get both on the same page one employs a geometric quench

\[ ds^2 = \frac{1}{z^2} \left[ dz^2 + \left( g^{(0)}_{\mu\nu} + z^2 g^{(2)}_{\mu\nu} + z^4 g^{(4)}_{\mu\nu} + z^4 \ln z \ h^{(4)}_{\mu\nu} + \mathcal{O}(z^6, z^6 \ln z) \right) dx^\mu dx^\nu \right] \]

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\[ g(-\infty) \to 1, \quad g(+\infty) \to t^2 \]

But heavy ion collisions don’t happen in curved space.
But heavy ion collisions don’t happen in curved space. Right?
semi-holographic approach to heavy ion collisions

the gravitational dual encodes the RG flow


\[ ds^2 = \frac{1}{z^2} \left[ dz^2 + \left( g^{(0)}_{\mu\nu} + z^2 g^{(2)}_{\mu\nu} + z^4 g^{(4)}_{\mu\nu} + z^4 \ln z h^{(4)}_{\mu\nu} + \mathcal{O}(z^6, z^6 \ln z) \right) dx^\mu dx^\nu \right] \]

\[ g_{\mu\nu} = \eta_{\mu\nu} + \frac{2\pi^2}{N_c^2 Q_s^4} t_{\mu\nu}^{UV} + \ldots \]

For the course grained \( t_{\mu\nu} \) at the scale \( z = Q_s^{-1} \)

\[ \nabla^\mu t^{Q_s}_{\mu\nu} = 0 \]
semi-holographic approach to heavy ion collisions

semi-holographic proposal

E. Iancu, A. Mukhopadhyay, JHEP 1506 (2015) 003
A. Mukhopadhyay, FP, A. Rebhan, S.A. Stricker JHEP 1605 (2016) 141

solve gravity dual with boundary conditions (sources) at $z = 0$ deformed by
gauge invariant operators (glasma)

$$
\phi^{(b)} = \frac{\beta}{4N_c Q_s^4} \text{Tr} \left( F_{\sigma\tau} F^{\sigma\tau} \right) \quad \chi^{(b)} = \frac{\alpha}{4N_c Q_s^4} \text{Tr} \left( F_{\sigma\tau} \tilde{F}^{\sigma\tau} \right)
$$

$$
g^{(b)}_{\mu\nu} = \eta_{\mu\nu} + \frac{\gamma}{Q_s^4} \left[ \frac{1}{N_c} \text{Tr} \left( F_{\mu\alpha} F_{\nu}^{\alpha} - \frac{1}{4} \eta_{\mu\nu} F_{\sigma\tau} F^{\sigma\tau} \right) \right]
$$

solve the UV theory deformed by marginal operators obtained from the gravity
dual

$$
S = -\frac{1}{4N_c} \int \text{Tr} \left( F_{\sigma\tau} F^{\sigma\tau} \right) + W_{\text{CFT}} \left[ g^{(b)}_{\mu\nu}, \phi^{(b)}, \chi^{(b)} \right]
$$
semi-holographic approach to HICs

equations of motion and conserved energy momentum tensor

\[ D_\mu F^{\mu\nu} = \frac{\gamma}{Q_s^4} D_\mu \left( \hat{T}^{\mu\alpha} F_{\alpha\nu} - \hat{T}^{\nu\alpha} F_{\alpha\mu} - \frac{1}{2} \eta_{\alpha\beta} F^{\mu\nu} \right) \]
\[ + \frac{\beta}{Q_s^4} D_\mu \left( \hat{\mathcal{H}} F^{\mu\nu} \right) + \frac{\alpha}{Q_s^4} \left( \partial_\mu \hat{A} \right) \tilde{F}^{\mu\nu} \]

\[ t^{\mu\nu} = T^{\mu\nu} + \hat{T}^{\mu\nu} \]
\[ - \frac{\gamma}{Q_s^4 N_c} \hat{T}^{\alpha\beta} \left[ \text{Tr}(F^{\mu\alpha} F_{\beta\nu}) - \frac{1}{2} \eta_{\alpha\beta} \text{Tr}(F^{\mu\rho} F_{\rho\nu}) + \frac{1}{4} \delta^{\mu}_{(\alpha} \delta^{\nu)}_{\beta} \text{Tr}(F^2) \right] \]
\[ - \frac{\beta}{Q_s^4 N_c} \hat{\mathcal{H}} \text{Tr}(F^{\mu\alpha} F_{\nu\alpha}) - \frac{\alpha}{Q_s^4 N_c} \eta^{\mu\nu} \hat{\mathcal{A}} \text{Tr} \left( F_{\sigma\tau} \tilde{F}^{\sigma\tau} \right). \]

\[ \partial_\mu T^{\mu\nu} = 0 \]

on shell and by the gravitational Ward identities.
Let's get familiar with the model
Simple toy example

Setup

homogeneity, isotropy, $\alpha = \beta = 0$, $N_c = 2$, $A_0^a = 0$, $A_i^a = \delta_i^a f(t)$

$\Rightarrow g^{(b)}_{\mu\nu} = \eta_{\mu\nu} + \frac{\gamma}{Q_s^4} t_{\mu\nu}$ is conformally flat, $p(t) = 1/3 \varepsilon(t) = 1/2[f'(t)^2 + f(t)^4]$

$\Rightarrow$ the bulk metric is diffeomorphic to AdS-Schwarzschild with mass $c$

$$f''(t) + \frac{1 - \frac{1}{2} \frac{\gamma}{Q_s^4} (\hat{E} + \hat{P})}{1 + \frac{1}{2} \frac{\gamma}{Q_s^4} (\hat{E} + \hat{P})} f(t)^3 + \frac{1}{2} \frac{\gamma}{Q_s^4} \frac{(\hat{E} + \hat{P})'}{1 + \frac{1}{2} \frac{\gamma}{Q_s^4} (\hat{E} + \hat{P})} f'(t) = 0$$

$$\hat{E} + \hat{P} = \frac{N_c^2}{2\pi^2} \frac{c}{\sqrt{1 - 3\gamma p(t)[1 + \gamma p(t)]^{3/2}}}$$

$$+ \frac{N_c^2}{2\pi^2} \frac{\gamma^3 p'(t)^2}{64[1 - 3\gamma p(t)]^{5/2}[1 + \gamma p(t)]^{7/2}}$$

$$\left(2[1 + \gamma p(t)][3\gamma p(t) - 1]p''(t) - \gamma[1 + 6\gamma p(t)]p'(t)^2\right)$$
Simple toy example

Iterative algorithm

Initial values: $f(0) = (2p_0)^{1/4}$, $f'(0) = 0$

$$f(t) = (2p_0)^{1/4} cd(\omega | - 1) \quad \omega = (2p_0)^{1/4}$$
Simple toy example

Iterative algorithm

Initial values: $f(0) = (2p_0)^{1/4}$, $f'(0) = 0$

\[
f(t) = (2p_0)^{1/4} cd(\omega|1)
\]

\[
\hat{E} + \hat{P} = \frac{N_c^2}{2\pi^2} \frac{c}{\sqrt{1 - 3\tilde{\gamma}p_0[1 + \tilde{\gamma}p_0]^{3/2}}}
\]
Simple toy example

Iterative algorithm

Initial values: $f(0) = (2p_0)^{1/4}$, $f'(0) = 0$

\[
f(t) = (2p_0)^{1/4} \cdot c d(\omega | - 1) \quad \omega = (2p_0)^{1/4} \left( \frac{1 - \frac{1}{2} \frac{\gamma}{Q_s^4} (\hat{E}_0 + \hat{P}_0)}{1 + \frac{1}{2} \frac{\gamma}{Q_s^4} (\hat{E} + \hat{P})} \right)^{1/2}
\]

\[
\hat{E} + \hat{P} = \frac{N_c^2}{2\pi^2} \frac{c}{\sqrt{1 - 3\bar{\gamma}p_0[1 + \bar{\gamma}p_0]^{3/2}}}
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Simple toy example

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Initial values: \( f(0) = (2p_0)^{1/4} \),
\( f'(0) = 0 \)

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f(t) = (2p_0)^{1/4} c d(\omega |-1) \quad \omega = (2p_0)^{1/4} \left( 1 - \frac{1}{2} \frac{\gamma}{Q_s^4} (\hat{E}_0 + \hat{P}_0) \right) \frac{1}{2} \left( 1 + \frac{1}{2} \frac{\gamma}{Q_s^4} (\hat{E} + \hat{P}) \right)^{1/2}
\]

\( \Downarrow \)

\( \hat{E} + \hat{P} = \) lengthy
Iterative algorithm

Initial values: \( f(0) = (2p_0)^{1/4}, \)
\( f'(0) = 0 \)

\[ f(t) = \text{numerical} \]

\[ \hat{E} + \hat{P} = \text{lengthy} \]
Simple toy example

Iterative algorithm

Initial values: \( f(0) = (2p_0)^{1/4} \),
\( f'(0) = 0 \)

\[
f(t) = \text{numerical}
\]

\[\downarrow\]

\( \hat{E} + \hat{P} = \text{numerical} \)
Simple toy example

Iterative algorithm

Initial values: \( f(0) = (2p_0)^{1/4}, \)
\( f'(0) = 0 \)

\[
f(t) = \text{numerical}
\]

\[
\hat{E} + \hat{P} = \text{numerical}
\]
Simple toy example

Results

A. Mukhopadhyay, FP, A. Rebhan, S.A. Stricker *JHEP* 1605 (2016) 141
Simple toy example

Outlook

study isotropization/thermalization by including anisotropy or the dilaton

C. Ecker, A. Mukhopadhyay, FP, A. Rebhan, S.A. Stricker

for late times replace glasma by kinetic theory

A. Mukhopadhyay, FP, A. Rebhan, A. Soloviev