1. Introduction

- Heavy Ion Collisions and Quark-Gluon Plasma
  - Quark-gluon plasma (QGP), a deconfined state of quarks and gluons, is produced in high-energy heavy ion collisions at RHIC and LHC.
  - The QGP in these experiments behaves like a strongly coupled liquid, not like a weakly coupled gas.
  - The plasma thermals on a very short time scale of 10^{-22} sec, which is theoretically not well understood yet.
  - Due to strong coupling perturbative QCD is not suitable to study the quantum dynamics of this system.

- AdS/CFT Correspondence [1]
  - AdS/CFT maps strongly coupled symmetric Yang-Mills (SYM) theory in 4D to classical gravity on 5D anti-de Sitter (AdS5) space.
  - We use SYM theory as a toymodel for QCD.
  - Thermalizing in the strongly coupled 4D SYM theory is mapped to black hole formation in AdS5.

2. Local and Non-Local Observables from AdS/CFT

- Local Observables
  - The energy-momentum tensor can be extracted from the metric g_{\mu\nu}, near the boundary, (\partial^\mu \phi) (x) = \partial_\mu g(\phi) (x).

- Non-Local Observables
  - Two-point functions for gauge invariant operators \text{C}_\alpha of large conformal weight \alpha are given by the length of geodesics [2]: [\mathcal{O}(i, \mathbf{z}_1, \mathbf{z}_2) \approx e^{-2\pi g_{\text{AdS}}/\mathcal{L}}]
  - Entanglement entropy of a spatial region \mathcal{A} is given by the area of a minimal surface [\mathcal{L}]: [S_{\mathcal{A}} = \frac{1}{16\pi} \mathcal{A} - \frac{1}{2} \mathcal{A}^2]

3. Shock Wave Collisions in SYM theory

The Lorentz contracted “scalet” in SYM are modeled as two Gaussian energy distributions approaching each other at the speed of light. The time evolution of the energy-momentum tensor is extracted from a numerically relativity simulation of colliding gravitational shock waves in the 5D gravity theory [3].

- Energy-Momentum Tensor
  - \( \langle T_{\mu\nu}(x) \rangle = \frac{1}{\sqrt{-g}} \int d^4y \sqrt{-\gamma} \gamma_{\mu\nu} \langle \mathcal{O}(x, y) \rangle \)
  - Wide and narrow shocks show qualitatively different behavior [3].

- Wide Shocks: Full-Stopping
  - Wide shocks stop each other at the collision when the plasma is formed which then explodes hydrodynamically.
  - Energy and pressure stay positive.

- Narrow Shocks: Transparency
  - Narrow shocks pass each other almost “transparently” and the plasma is formed only after the collision.
  - Energy and pressure can be negative for a short time period after the collision.

4. Geodesics in AdS5

In the calculation of two-point functions we need the length of spacelike geodesics that are attached to the boundary at z=0 and extend into the 5D shock wave geometry.

- These geodesics can be found by numerically solving the geodesic equation (1) subject to boundary conditions (2) that fix the endpoints at the boundary at some spatial separation \( \mathcal{L} \).

\[
\begin{align*}
X^\mu \dot{X}_\mu &= -1, \\
X^0(x) &= \langle \mathcal{O}(x, z) \rangle, \\
X^\mu(x) &= \langle \mathcal{O}(x, z) \rangle_{\mathcal{L}} = \langle \mathcal{O}(x, z) \rangle (1, \alpha \gamma, \pm \alpha \gamma, 0)
\end{align*}
\]

5. Two-Point Functions

- Time Evolution Two-Point Functions
  - The system starts in some correlated state.
  - As the shocks approach each other without interaction they destroy these initial correlations.

- After the collision correlations are restored because of the interactions during the collisions new correlations are formed.

- For the narrow shocks these new correlations grow significantly beyond their initial value.

- The shock wave system follows a top-down thermalization pattern where short range correlations (small \( \mathcal{L} \)) reach the equilibrium first.

6. Entanglement Entropy

- Time Evolution of Entanglement Entropy
  - We start with zero entanglement by construction.
  - As the shocks enter the entangling region the entanglement entropy rapidly grows.
  - After the rapid initial growth follows a regime of linear growth which goes approximtely, until the shocks collide.

- Right after the collision the wide shocks show a smooth fall off where the narrow shocks have a pronounced minimum which is related to the ratio in the energy density and the longitudinal pressure.

- As the two-point function the entanglement entropy shows a top-down thermalization pattern.

7. Summary

- We use collisions of shock waves in SYM theory as toymodel for real (QCD) heavy ion collisions.

- Using the AdS/CFT correspondence the dynamics in these collisions can be extracted from numerical relativity simulations of colliding gravitational shock waves.

- Within AdS/CFT non-local observables such as two-point functions and entanglement entropy can be computed from geodesics and minimal surfaces in the gravity theory.

- From our numerical simulation we find that both, two-point functions and entanglement entropy, show qualitatively different behavior for narrow and wide shocks.

- A Mathematica code for shock wave collisions is available at Wilke van der Schee homepage: [www.sites.google.com/site/wilkwanderschee/phdthesis](http://www.sites.google.com/site/wilkwanderschee/phdthesis)

- A Mathematica code for the entanglement entropy and the two-point function can be downloaded from: [www.christianecker.com](http://www.christianecker.com)

8. References


9. Contact

- christian.ecker@tuwien.ac.at
- grumil@hep.tuwien.ac.at
- wille@mtu.edu
- philipp.stanzer@tuwien.ac.at
- stricker@hep.tuwien.ac.at

- Institut für Theoretische Physik, TU Wien, Wiedner Hauptstr. 8-10, A-1040 Vienna, Austria

- Center for Theoretical Physics, MIT, Cambridge, MA 02139, USA