Entanglement entropy in shock wave collisions

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Central question:

How does a strongly coupled quantum system which is initially far-from equilibrium evolve to its equilibrium state?
Quark-gluon plasma (QGP) is a deconfined phase of quarks and gluons produced in heavy ion collision (HIC) experiments at RHIC and LHC.
Why AdS/CFT?

The QGP produced in HIC's behaves like a strongly coupled liquid rather than a weakly coupled gas.
AdS/CFT correspondence

**AdS/CFT correspondence:** [Maldacena 97]

**Type IIB string theory** on $\text{AdS}_5 \times S^5$ is equivalent to $\mathcal{N}=4$ super symmetric $\text{SU}(N_c)$ Yang-Mills theory in 4D.

**Supergravity limit:**

**Strongly coupled large $N_c$ $\mathcal{N}=4$ SU($N_c$) SYM theory** is equivalent to **classical supergravity** on $\text{AdS}_5$

$$e^{-S_{\text{sugra}}[\phi]}\left|_{\lim_{z \to 0} \phi = \phi_0} = \left\langle \exp \left( \int d^d x \phi(x) \mathcal{O}(x) \right) \right\rangle_{CFT}.\]

**Strategy:**

- Use $\mathcal{N}=4$ SYM as **toymodel** for QCD in the strongly coupled regime.
- Build a **gravity model** dual to HICs, like colliding gravitational shock waves.
- Switch on the computer and solve the 5-dim. gravity problem **numerically**.
- Use the **holographic dictionary** to compute **observables in the 4 dim. field theory** form those gravity result.
Holographic thermalization

Thermalization = Black hole formation
Entanglement entropy

Divide the system into two parts A,B. The total Hilbert space factorizes:

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

The reduced density matrix of A is obtained by the trace over $\mathcal{H}_B$

$$\rho_A = \text{Tr}_B \rho$$

Entanglement entropy is defined as the von Neumann entropy of $\rho_A$:

$$S_A = -\text{Tr}_A \rho_A \log \rho_A$$
Holographic entanglement entropy

Within AdS/CFT entanglement entropy can be computed form the area of minimal (extremal) surfaces in the gravity theory.

\[ S_A = \frac{\text{Area}(\Sigma)}{4G_N} \]

[Ryu-Takayanagi 06, Hubeny-Rangamani-Takayanagi 07]
Holographic entanglement entropy

- In practice computing extremal co-dim. 2 hyper-surfaces is numerically involved.

\[ \delta A = \delta \left[ \int d^3 \sigma \sqrt{\det \left( \frac{\partial X^\mu}{\partial \sigma^a} \frac{\partial X^\nu}{\partial \sigma^b} g_{\mu \nu} \right)} \right] \]

- Can we somehow simplify our lives?

Yes we can!

[Example: minimal surface for a star shaped boundary region (red) in AdS4 computed with Surface Evolver]
Entanglement entropy from geodesics

Consider a **stripe region** of infinite **extend** in **homogeneous directions** of the geometry. The **entanglement entropy** is prop. to the **geodesics length** in an **auxiliary spacetime**.

\[
A = \int d^3 \sigma \sqrt{\det \left( \frac{\partial X^\mu}{\partial \sigma^a} \frac{\partial X^\nu}{\partial \sigma^b} g_{\mu\nu} \right)} = \int dx_3 \int dx_2 \int d\sigma \sqrt{\Omega^2 g_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma} \frac{\partial X^\nu}{\partial \sigma}}
\]

\[
S_A = \text{const.} \frac{\text{Length}(\Gamma)}{4G_N} \quad \tilde{g}_{\mu\nu} = \Omega(z, t, x_1)^2 g_{\mu\nu}
\]
Numerics: relax, don't shoot!

Geodesic equation as two point boundary value problem:

$$\ddot{X}^\mu + \Gamma^\mu_{\alpha\beta} \dot{X}^\alpha \dot{X}^\beta = -J \dot{X}^\mu,$$

BC's: $X(\pm 1)^\mu \equiv (V(\pm 1), Z(\pm 1), X(\pm 1)) = (t, 0, \pm l/2)$

- There are two **standard numerical methods** for solving two point boundary value problems:
  - **Shooting:** Very sensitive to initialization on asymptotic AdS spacetimes.
  - **Relaxation:** Converges very fast if good initial guess is provided.

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Holographic shock wave collisions

HIC is modeled by **two colliding sheets of energy with infinite extend in transverse direction** and **Gaussian profile in beam direction**. [Chesler-Yaffe 10]
Wide vs. narrow shocks

Two qualitatively different dynamical regimes

- **Wide** shocks (~RHIC): full stopping
  - $\mu t = -2.0$
  - $\mathcal{E}/\mu^4$

- **Narrow** shocks (~LHC): transparency
  - $\mu t = -1.6$
  - $\mathcal{E}/\mu^4$
Violation of the null energy condition

“Well behaved” classical theories satisfy the null energy condition (NEC)

\[ T_{\mu \nu} k^\mu k^\nu \geq 0, \quad k_\mu k^\mu = 0. \]

- In quantum theories the NEC can be violated. [Epstein 65]
- In narrow shock wave collisions the null energy condition (NEC) is violated in some region in the forward light cone shortly after the collision.
The quantum null energy condition is (preliminarily) fulfilled

Recently the quantum null energy condition (QNEC) was proposed [Bousso 15]

\[ \langle T_{\mu\nu} k^\mu k^\nu \rangle \geq \frac{1}{2\pi} S'' \].

- Our preliminary results suggest that the QNEC is fulfilled.
Extremal surfaces

- In the case of entanglement entropy the extremal surfaces do not reach beyond the AH.
- If there is something like an entanglement entropy horizon it is very close to the AH.
Two point functions for operators $O(t, x)$ of large conformal weight $\Delta$ can be computed from the length of geodesics. [Balasubramanian-Ross 00]

$$\langle O(t, x)O(t, x') \rangle \propto e^{-\Delta \text{Length}(\Gamma)}$$
Time evolution of entanglement entropy

Characteristic behavior:

- **Rapid initial growth** when the shocks enter the entangling region.
- **Linear growth** when the shocks start to overlap.
- **Post collisional regime** which is a featureless fall off for wide shocks and shows an additional shoulder for narrow shocks.
- **Late time regime**: polynomial fall off very close to $1/t$.
Summary

- AdS/CFT allows to study the real time dynamics of strongly coupled QFT's by solving the IVP of (classical) supergravity theories.

- The NEC can be violated in holographic shock wave collisions, but the QNEC seems to be fulfilled.

- Entanglement entropy may serve as an order parameter for the full stopping–transparency transition.

- Interestingly the 2-point functions contain information from behind the apparent horizon, where the entanglement entropy does not.

Work in progress

- Improve the quality of the QNEC simulation.
  [CE-Grumiller-Van der Schee-Stanzer]

- Going beyond supergravity: string corrections, semi-holography, …
  [CE-Mukhopadhyay-Preiss-Rebhan-Stricker]