Turbulent breakage of ductile aggregates

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Abstract

In this paper we study breakage rate statistics of small colloidal aggregates (modelled as sub-Kolmogorov massless particles) in non-homogeneous anisotropic turbulence. We focus specifically on ductile rupture, which is activated when the hydrodynamic fluid stress acting on the aggregate exceeds a critical value, \( \sigma > \sigma_{cr} \), and is brought to completion when the energy absorbed by the aggregate meets the critical breakage value. We show that ductile rupture is associated to significant reductions of the breakage rate with respect to instantaneous rupture, which produces breakage as soon as the condition \( \sigma > \sigma_{cr} \) is met. These discrepancies are due to the different energy values at play as well as to the statistical features of energy distribution in the anisotropic turbulence case examined.

Keywords: breakage rate, colloidal aggregates, turbulence, energy dissipation, ductile rupture

1. Introduction

Breakage rates of micro and nano aggregates in turbulent flow are of high relevance to a wide variety of applications \([1, 2]\). When aggregates are small with respect to the characteristic flow length scale and have density close to that of the fluid, breakage is caused by the hydrodynamic stresses exerted by the flow around the aggregate, which in turn induce internal stresses acting on the structure of the aggregate. If the response time of the aggregate to stress-induced deformations is small then breakage is instantaneous and the aggregate rupture can be referred to as brittle; otherwise breakage depends on the stress history and the aggregate rupture can be referred to as ductile. In either brittle or ductile rupture, the phenomenology of turbulent breakage is still not fully understood because the complexity of the flow field adds to the intricacy of the aggregate morphology in determining how the hydrodynamic forces redistribute and stresses accumulate over the structure of the aggregate. To provide a basic understanding of turbulent breakage, many investigations (see for instance \([3, 4]\)) have focused on the influence that the hydrodynamic stresses have on the rate at which breakage occurs, neglecting the details of the aggregate inner structure. However, in the size range of interest for the present study (aggregates smaller than the Kolmogorov length), analyses were carried out considering brittle aggregates and instantaneous breakage \([3, 4]\). This assumption would be fully justified in highly viscous fluids, such as dispersions in liquid polymers, where the stresses required to break the aggregate are very low \([5]\). But in low-viscosity systems the effective hydrodynamic stress required for breakage changes significantly depending on the size of the fluid and rupture can frequently be determined by the stress history \([6]\).

The first attempt to assess the effects of flow inhomogeneity and anisotropy was recently put forward in the collaborative study of \([3]\) where the breakage of small inertialless aggregates in different archetypal bounded flows (turbulent channel flow and developing boundary layer) was compared with those of homogeneous isotropic turbulence. This study showed that, regardless of the flow configuration, the breakage rate decreases when the critical stress required to break the aggregate increases. For small values of the critical stress ("weak" aggregates) the breakage rate develops a universal power-law scaling that appears to be independent of the flow configuration; whereas for high values of the critical stress ("strong" aggregates) large differences in the breakage rate arise among the different flows and no clear scaling is observed anymore. In the paper we examine a more realistic breakage process resulting from ductile rupture, focusing on turbulent channel flow. We assume that the breakage process is activated when the hydrodynamic stress acting on the aggregate, \( \sigma \), exceeds a critical value characteristic of a given type of aggregate: \( \sigma > \sigma_{cr} \) (activation condition). As long as this condition is met the process continues, mimicking the situation in which the aggregate is storing energy from the surrounding fluid. The process ends when the energy transferred from the fluid to the aggregate (deformation energy hereinafter): \( E = \int_0^\tau \epsilon(t)[\sigma > \sigma_{cr}] \, dt \), with \( \tau \) being the time spent by the aggregate in regions of the flow where \( \sigma > \sigma_{cr} \) and \( \epsilon \) being the dissipation rate of fluid kinetic energy, exceeds the critical breakage value, which is also characteristic of the type of aggregate under investigation: \( E > E_{cr} \) (breakage condition). Following \([3]\) we assume \( \sigma \sim \mu (\epsilon/\nu)^{1/2} \), where \( \mu \) is the dynamic (kinematic) viscosity.

2. Physical Problem and Numerical Methodology

The physical problem considered in this study is the dispersion of tracer aggregates in turbulent channel flow. The reference geometry consists of two infinite flat parallel plates separated by a distance \( 2h \). The origin of the coordinate system is located at the center of the channel with the \( x,y \) axes pointing in the streamwise, spanwise, and wall-normal directions, respectively. Periodic boundary conditions are imposed on the fluid velocity field in the homogeneous directions (streamwise, \( x \), and spanwise, \( y \)), no-slip boundary conditions are imposed at the walls. The size of the computational domain is \( L_x \times L_y \times L_z = 4\pi h \times 2\pi h \times 2h \). The shear Reynolds number is \( Re_s = u_* h/\nu = 150 \), where \( u_* = \sqrt{\tau_w/\rho} \) is the shear velocity based on the mean wall shear stress. All variables discussed in this paper are expressed in wall units, obtained using \( u_* \) and \( \nu \). The flow solver is based on a Fourier-Galerkin pseudospectral method that solves for the full Navier-Stokes equations and thus yields the spatial derivatives required to calculate \( \epsilon \) along the aggregate trajectory with spectral accuracy. Lagrangian tracking is based on the following equation of motion: \( \dot{x}_p = u_{ap} \), with \( x_p \) the aggregate position and \( u_{ap} \), the fluid velocity at \( x_p \).
This equation is solved in time using a fourth-order Runge-Kutta scheme, whereas sixth-order Lagrangian polynomials are used to obtain the fluid velocity and the fluid velocity derivatives at the instantaneous aggregate position. Further details on the numerical methodology can be found in [7]. Breakage was measured by releasing $2 \cdot 10^7$ aggregates in two distinct regions of the channel: the wall region, $\Omega W$, which comprises a fluid slab 10 wall unit thick where the viscous stress (representing the mean fluid shear) is maximum while the turbulent stress is close to zero; and the center-plane, $\Omega C$, where all wall stress contributions drop to zero and turbulence is closer to homogeneous and isotropic.

3. Results and Discussion

Figure 1 shows the rates of ductile breakage obtained for the two release locations $\Omega C$ and $\Omega W$, highlighting the effect of increasing the critical deformation energy for different values of the critical energy dissipation. Results refer to three different values of $E_{cr}$: $E_{cr} = 0.04, 0.4$ and $4.0$, representing a case of low, intermediate and high ductility for the present flow configuration. Focusing first on the aggregates released in the channel center, we find that the breakage rate of brittle aggregates (solid curve) generally decreases with increasing aggregate strength, in agreement with the intuitive idea that weak aggregates in wall-bounded flows are broken by turbulent fluctuations faster than strong aggregates [3]. For small $\epsilon_f$, the breakage rate is known to exhibit a power-law behavior of the type $f(\epsilon_f) \propto \epsilon_f^\chi$, where $\chi$ is a flow-dependent scaling exponent: [3] have demonstrated that the value of $\chi$ for aggregates released in the channel center is very similar to that of aggregates released outside a developing boundary layer but slightly larger than that of aggregates released in homogeneous flows. In figure 1(a) the power-law scaling of $f(\epsilon_f)$ for brittle aggregates is observed when $\epsilon_f < -3$ and the best fit is obtained for $\chi \approx 0.5$. When ductile aggregates are taken into account (dashed curves), breakage rates change dramatically, especially for weak aggregates with low $\epsilon_{cr}$ threshold. The values of $f(\epsilon_f, E_{cr})$ decrease significantly with respect to the case of instantaneous breakage, already at low thresholds for the critical deformation energy (e.g. $E_{cr} = 0.04$). In addition, no clear power-law scaling is observed anymore and the breakage rate profiles tend to flatten as the aggregate “ductility” increases. As could be expected, the effect of ductile rupture on $f(\epsilon_f, E_{cr})$ becomes less important for strong aggregates: These must be subject to extremely violent fluid stresses, typical of the intermittent nature of small-scale turbulence, to activate the breakage process and thus can store the level of energy required to break almost impulsively. As a result, there is just a little increase of the exit time with respect to strong brittle aggregates.

Breakage rates depend quantitatively on the specific location chosen to release the aggregates at time $t_0$. In the center of the channel strong aggregates, no matter if subject to brittle or ductile rupture, are mainly broken by the rare extreme excursions of dissipation from the mean, which are caused by intermittency. Most of such aggregates must therefore reach the high-dissipation, high-shear regions of the flow near the channel walls to undergo breakage. To examine the influence of the release location on breakage rates, in figure 1(b) we also show the behavior of $f(\epsilon_f, E_{cr})$ for aggregates released in the region $\Omega W$. For brittle aggregates (solid curve), we observe that the power-law scaling at small values of $\epsilon_f$ is followed by a flattening for intermediate values of the threshold. For the very large threshold values associated to the right end of the profile, a drop-off in the breakage rate is observed, representing the case of aggregates that are too strong to be broken by the mean shear alone: intense but rare turbulent fluctuations within the near wall region are required to overcome the cohesive force of these aggregates [3]. The inclusion of ductile rupture effects (dashed curves) produces again a clear decrease of the breakage rates, which vanishes for large values of $\epsilon_{cr}$. Compared to the results of figure 1(a), we observe that the decrease is now almost negligible for aggregates with low ductility and flattening of the profiles is only attained for very high threshold values of the deformation energy. We also note that error bars are generally smaller, indicating a lower variability of the statistics: this is due to the fact that aggregates are already placed in the high-shear regions of the flow where they preferentially break and hence sample a reduced portion of the domain compared to aggregates released in $\Omega C$. In spite of these quantitative differences, however, the reduction of $f(\epsilon_{cr}, E_{cr})$ associated with ductile rupture is evident independently of the initial aggregate injection location.

References