A stochastic model for Lagrangian particle tracking in large-eddy simulation velocity fields

A. Innocenti\(^1\), S. Chibbaro\(^2\), M.V. Salvetti\(^3\), C. Marchioli\(^4\), A. Soldati\(^5\)

\(^1\)DICL, University of Pisa, Pisa (Italy)
e-mail: alessioinnocenti@yahoo.it, mvsalvetti@ing.unipi.it
\(^2\)D'Alembert Institute, Pierre and Marie Curie University and CNRS, Paris (France)
e-mail: chibbaro@ida.upmc.fr
\(^3\)DIEG, University of Udine & Department of Fluid Mechanics, CISM, Udine (Italy)
e-mail: marchioli@uniud.it, soldati@uniud.it

Abstract

A Lagrangian stochastic model is proposed for tracking the inertial particles in fluid velocity fields obtained in large-eddy simulations; the model is formulated for the fluid velocity seen by the particles along their trajectory. The behaviour of the model is first investigated for tracer particles in a turbulent channel flow. It is checked that in this limit case similar statistics to these given by a fluid phase LES are obtained. Next, stochastic model is used for Lagrangian tracking of particles of different inertia. The results will be shown in the final presentation and will be compared to particle statistics and concentration obtained in DNS and in LES with no model for particle equations.

Keywords: inertial particles, Lagrangian tracking, large-eddy simulation, stochastic subgrid scale modeling

1. Introduction

The dispersion of small inertial particles in inhomogeneous turbulent flows is important in a number of industrial applications and environmental phenomena, such as, mixing, combustion, deposition, spray dynamics, pollutant dispersion or cloud dynamics. Direct Numerical Simulations (DNS) of turbulence coupled with Lagrangian Particle Tracking (LPT) demonstrated their capability to capture the mechanisms characterizing particle dynamics in turbulent flows. Due to the computational requirements of DNS, however, analysis of problems characterized by complex geometries and high Reynolds numbers demands alternative approaches; Large-Eddy Simulation (LES) is increasingly gaining popularity, especially for cases where the large flow scales control particle motion. LES is based on a filtering approach of the fluid phase governing equations; thus, only the filtered fluid velocity is available for particle tracking and particles are prevented from interacting with the small (unsolved) Sub-Grid Scales (SGS) of turbulence. This may strongly influence clustering of inertial particles and lead to significant underestimation of particle preferential concentration and deposition rates (see e.g. [1]). Hence, there is currently a general consensus about the need to model the effect of SGS turbulence on particle dynamics.

Different kinds of SGS models for particle motion equations were proposed in the literature, e.g. filtering inversion or approximate deconvolution [2, 3], fractal interpolation [2], stochastic modeling [4, 5] or mixed models [6]. Previous studies [7, 8, 9], focusing on the error purely due to filtering of the fluid velocity field seen by the particle along its trajectory, showed that this error is stochastic and may exhibit a non-Gaussian and intermittent nature.

The work is aimed at developing and appraising a new stochastic model for subgrid scales of large eddy simulation of turbulent polydispersed two-phase flows. The model is based on the formalism for the filtered density function (FDF) approach in LES simulations. Contrary to the FDF used for turbulent reactive single-phase flows, the present formalism is based on Lagrangian quantities and, in particular, on the Lagrangian filtered mass density function (LFMDF) as the central concept [10]. A first example of Langevin model constructed within the above formalism is proposed considering isotropic sub-grid fluctuations, but taking into account crossing-trajectory effects and paying attention to the consistency of the model with the fluid limit case.

2. Physical Problem, Numerical Methodology and Modeling

The physical problem considered in this study is the dispersion of inertial particles in turbulent channel flow. The reference geometry consists of two infinite flat parallel plates separated by a distance \(2h\). The origin of the coordinate system is located at the center of the channel with the \(x\), \(y\) and \(z\) axes pointing in the streamwise, spanwise, and wall-normal directions, respectively. Periodic boundary conditions are imposed on the fluid velocity field in the homogeneous directions (streamwise, \(x\), and spanwise, \(y\)), no-slip boundary conditions are imposed at the walls. The size of the computational domain is \(L_x \times L_y \times L_z = 4\pi h \times 2\pi h \times 2h\). The shear Reynolds number is \(Re_x = u_s h / \nu = 300\), where \(u_s = \sqrt{\tau_w / \rho}\) is the shear velocity based on the mean wall shear stress.

The flow solver is based on a Fourier-Chebyshev pseudospectral method to discretize the LES equations. The SGS models considered for the fluid phase are the classical and the dynamic Smagorinsky model.

The Lagrangian tracking is based on the following equation of motion:

\[
\begin{align*}
\frac{dx_p(t)}{dt} &= U_p(t) \\
\frac{dU_p(t)}{dt} &= \frac{1}{\tau_p} (U_s(t) - U_p(t))(1 + 0.15 Re_x^{0.685})
\end{align*}
\]

In these equations, \(U_s(t) = U(t, x_p(t))\) is the fluid velocity "seen", i.e. the fluid velocity sampled along the particle trajectory \(x_p(t)\), \(\tau_p\) is the particle relaxation times and \(Re_x\) the par-
particle Reynolds number. The fluid velocity seen is given by the following stochastic Lagrangian model:

\[
\begin{align*}
dU_{x,i}(t) &= -\frac{1}{\rho_i} \frac{\partial p}{\partial x_i} dt + \sum_{j=1}^{3} (U_{p,j} - \bar{U}_i) \frac{\partial \bar{U}_i}{\partial x_j} dt - \frac{1}{T_{L,i}} (U_{x,i} - \bar{U}_i) dt + \sqrt{\varepsilon_r (C_9 b_i + \frac{2}{3} (b_i - 1))} dW_i(t)
\end{align*}
\]

(2)

where the overbar denotes filtered quantities, output of LES of the fluid phase, \(T_{L,i}\) is a modified time scale obtained by multiplying the Lagrangian time scale, \(T_L\), by the Csanady factor, \(\varepsilon_r\) is the sub grid dissipation and \(b_i = T_{L,i}/T_L\). Finally \(dW_i(t)\) is a Wiener process.

Equations (1) and (2) are discretized through either a first or a second-order scheme, based on stochastic rules.

3. Results and Discussion

As a first test, behaviour is considered of the Lagrangian stochastic model for particles of zero inertia, i.e. fluid tracers. The LES of the fluid flow was carried out on a grid having a resolution of \(32 \times 32 \times 33\), which corresponds to a coarsening factor of 8 in each direction compared to the DNS resolution. Different SGS models have been used, namely no SGS model, the Smagorinsky and the dynamic models. The filtered fluid fields obtained from LES are compared with filtered velocities of zero inertia particles, in this limit case they should be nearly the same. For particle tracking, 40 particles per cell initially randomly distributed was considered. It was checked that increasing the number of particles per cell does not significantly affect the results.

Figure 1 shows the mean streamwise velocity profiles computed from the stochastic model for tracer particles coupled with LES with different SGS models; they are compared with the corresponding LES fluid mean streamwise velocity profiles. It can be seen that, as expected, the stochastic model for tracer particles give the same mean velocity field as the one of the LES in which the particle are tracked.

Figure 1: Mean streamwise velocity profiles computed from the stochastic model for tracer particles compared to those obtained for fluid in LES

For the higher order statistics, Fig. 2 shows the streamwise velocity fluctuations on a plane at \(z^+ = 10\) obtained in LES with no SGS model for the fluid part (top panel) and from the stochastic model for tracer particles (bottom panel). It can be seen that there is a good correlation and in particular streaky structures are clearly visible in both cases. The quantitative agreement is not perfect, as could be expected, since the stochastic model implicitly provides SGS terms, which have an impact on the higher order statistics. Hence, the filtered results of the stochastic model in the limit case of tracer particles can be considered as those of a LES of the fluid phase with a SGS model, which does not exactly correspond to any of the classical SGS models.

Figure 2: Streamwise velocity fluctuations on a plane at \(z^+ = 10\); LES (top panel) and stochastic model for tracer particles (bottom panel)

The presented stochastic model was also used for tracking particles having different inertia in LES fluid velocity fields. The results will be shown in the final presentation.

References


