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FRACAM

Cell Phone Application to Measure Box Counting Dimension

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Abstract. There are two kinds of algorithms: those that are 'better' with respect to accuracy and those that are 'faster'. In the past, fractal analysis by means of box-counting - including both, binary and greyscale analysis - has been focused on the former. In our work, however, we want to aim at the second category: algorithms that are fast and easy to use, without losing view on significance. To this end we have devised a cell phone application which let users grasp and analyse photographs regarding the box-counting dimension of e.g. facades. The application includes two measurement methods for binary images, based on threshold conversions, and one for greyscale images. Accuracy has been tested on deterministic fractals with known fractal dimension. As a matter of fact we are able to produce what was formerly constraint to scientific implementations or discourse on every day's hardware.

Keywords. Fractal analysis; Differential Box-counting; Fractal dimension; Cell phone application.

1. Motivation

Fractal dimension is an indication for the roughness or the irregularity of natural or artificial objects. Among others, box-counting, which is a fractal analysis method, is one of the easiest ways to estimate the fractal dimension and suitable for the study of complex objects. Since Carl Bovill applied box-counting for the first time to architecture (Bovill 1996), it has become widely used in this field (Capo 2004; Zarnowiecka 2002; Rian et al. 2007; Ostwald & Vaughan 2016). It enables the possibility to analyse and compare geometric properties (Ostwald & Vaughan 2016). From a methodological perspective, it enables to visualize the development of roughness across multiple scales, which allows, for example, to draw reliable conclusions about whether or not a harmonic relation between the whole and it parts exists (Lorenz 2013). Studies have shown that box-counting provides an objective comparison method between different design solutions (Lorenz

2013) or give a more precise picture of design intentions (Lorenz 2014). Fractal analysis has also been used to test hypotheses about social, stylistic and personal trends in design (Ostwald & Vaughan 2016). Since all these studies are focused on binary box-counting, they only take plans (line graphics) into account. However, there are two main disadvantages about a binary implementation: first, because elevations have to be prepared for calculation beforehand (e.g. redrawing as vector graphics, applying edge detection or line width reduction) the method is time consuming and second it is used, so to speak, “in the office” but not “on the street”, which much better corresponds to the natural environment of buildings. Unlike other techniques, which are based on fractal analysis methods, the main goal of the here described approach is both, convenience and speed of the computer program. Moreover, everydays usage is an important matter, which is the reason why a cell phone application has been developed. The application comprises the box-counting algorithm for binary images - which includes threshold conversion of photographs beforehand - as well as the differential box-counting algorithm, which is applied to a greyscale image right from a camera shot. Differential box counting-based approaches are so far used for analyzing texture features and texture classification (Filho 2008; Sankar & Thomas 2010; Gao et al. 2016), which will enlarge the previous insights of geometric properties in architecture. Due to the fact that fractal dimension, on the one hand, is (to a certain extent) sensitive to the method used and, on the other hand, can be the same for different textures, the authors propose different fractal analysis methods, which will be implemented step by step. This enlarges the possibility to answer research questions about harmony, design intention and possibilities of comparison. The webpage with the current state of implementations is available for test purposes under: www.iemar.tuwien.ac.at/processviz/fracam/ (Wurzer & Lorenz 2016). The target groups are researchers that use fractal analysis methods for classification and scholars to recognize and to get a feeling for the connection between scale and harmony.

2. Background

Benoît Mandelbrot formulated the theory of fractals, generally speaking, in order to understand and describe the irregularity of nature: the roughness in the shapes of clouds, mountains, coastlines and bark (Mandelbrot 1975). A core message of the “new” theory is that natural objects, such as coastlines, cannot be described by simple primitives (cones, spheres, cubes) and that natural objects rather follow the concept of self-similarity. When B. Mandelbrot (1967) asked for the length of Britain, he thus pointed out the lack of Euclidean geometry to analyse and describe natural forms. As consequence and in order to give irregularity a proper value he introduced the term fractal dimension (Mandelbrot 1967), which is based on the definition proposed by Felix Hausdorff (1919). The Hausdorff dimension of a fractal can be a non-integer value and is strictly greater than its topological dimension (Mandelbrot 1983; Falconer 1990). In 1983 B. Mandelbrot (1983) described a fractal analysis method in order to estimate the fractal dimension (capacity dimension) of objects located in the two dimensional plane; today the method is known as box-counting. According to C. Bovill (1996) the method provides a possibility for quantifying characteristic visual complexity. Since C. Bovill published the first results of applying box-counting to architecture most of the researches in this field uses the method as described there (Bovill 1996): A grid is placed over a black and white (binary) line graphics whereby all boxes (cells) that contain a part of the image are counted; in the next step, the grid size is reduced and boxes that contain a part of the image are counted again; this results in data points with the number of covered boxes versus the grid size; finally, the characteristical value - the box-counting dimension (see formula 1) - is determined by the slope of the regression line in a double logarithmic graph.

$$D_B = \lim_{\epsilon \to 0} \frac{\log N_\epsilon}{\log\left(\frac{1}{\epsilon}\right)} \tag{1}$$

Although the method is an efficient approach to estimate fractal dimension, it is subject to various influences, such as linewidth, starting box size, grid position, grid reduction factor and selection of representing lines (Fourtan-Pour et al. 1999; Lorenz 2012; Ostwald 2013). Most important with the described arrangement is that not the original building is analysed, but a two dimensional representation of it, e.g. the elevation. It is a line graphics that represents the design intention of the building rather than the actual status. Due to the influence factors coming along with the method itself, a meaningful statement is only possible if measurements of different settings are analysed. That leads to a set of significant (statistic) values of the set of measurements: the median; the interquartile range (IQR, which describes the accuracy of the whole set); the smallest and the average coefficient of determination (R^2, which describes the deviation of single measurements); and the range of scale (which equals the range of coherence) (Lorenz 2016). In order to minimize the influence of line width, vector graphics (e.g. AutoCAD files) can be used instead of raster graphics (Lorenz 2012). However, both, to analyse vectorized images as well as to calculate a whole set of measurements leads to an increased time consumption. Unlike the described approach the authors introduce a fast method applied to photographs.

3. Greyscale Box-Counting

So far, architectural research used fractal analysis methods exclusively in order to examine two dimensional representations of buildings, mainly facades. Such an approach takes primarily the design intention into account, but not the is-in-use state. The authors want to fill this gap and go a step further by expanding the binary information using greyscale images. Thus additional information is taken into account, mainly expressed by contrast (light and shadowing, intensity of colours). Since this approach still analyses 2-dimensional representations it is only one more aspect on a comprehensive holistic analysis of architecture using fractal analysis methods. It is a part of the chain of possibilities also including rescaled range analysis that determines the fractal characteristics of rhythmic structures (Bovill 2000). While binary images or line graphics (elevations) allow a quantitative analysis of the shape, greyscale analysis rather concentrates on the texture. In order to analyse greyscale images, basically, two different methods are possible: either the representation is converted into a black and white image, using the same algorithm as described before in “background”; or the “grey” intensity inside the boxes is taken into account instead of just measuring the covered boxes. The latter method means, subsequently, to adapt the algorithm itself.

4. Box-counting and Threshold

In order to analyse greyscale images with the original box-counting method, the value of the threshold used for conversion is crucial. In practise, the value divides the image into the set that is important and its background (black and white points). Following this approach, a first implementation of the program (called *Single threshold*), simply said, converts the greyscale image at different thresholds (e.g. 12.5%, 25%, 37.5% and so on) and measures in each case the box-counting dimension.

Figure 1. Threshold conversion of a greyscale image and box-counting measurement as cell phone application. 1a: settings of the basic implementation (‘Single threshold’); 1b: double logarithmic graph with regression line and slope (DB); 1c: settings of the advanced implementation (‘Different thresholds’); 1d: results (DB) of different thresholds.

With focus on a fast result, the considered photograph is, at first, converted into an image of 512 Pixels width. The user then chooses the favored threshold from a list of possibilities (50% by default), which converts the original photograph into a black and white binary image (figure 1a). Concerning the measurement itself, the algorithm starts with a grid size of 128 Pixels (which is 1/4th of the overall width). This first grid is placed over the image and the boxes that cover the foreground are counted. Thereafter the grid size is halved (according to the default reduction factor of one half), leading to a box size of 64 Pixels and the boxes are counted again. This procedure is repeated until the smallest allowed box size of 2 Pixels is reached. Not only the smallest and the largest box size can be adjusted, but also the reduction factor can be changed to predefined values (which may slow down calculation because of additional box sizes). The result of calculation itself is visualised as a double logarithmic graph with *log(box size)* versus *log(number of covering boxes)*. Finally, the slope of the regression line defines the box-counting dimension (figure 1b).

The modification of the first implementation, called *Different thresholds*, automatically repeats the previously described algorithm for different predefined thresholds (currently running from 12.5% to 87.5%). The outcome is therefore not a single value, but a set of box-counting dimensions (of different threshold values T_i; $i \epsilon 1 \ldots N$). This specific set characterises how roughness changes as threshold increases. Following from that, it is the tendency that characterises the texture signature (see formula 2; Backes & Bruno 2008).

$$\psi(A) = [D_B\ of\ T_1,\ D_B\ of\ T_2,\ \ldots,\ D_B\ of\ T_N] \tag{2}$$

Both implementations (the *Single threshold* and the *Different thresholds*) were tested with deterministic fractals, i.e. fractals that are constructed in an algorithmic way and which are defined as the attractors of certain iterated function systems (Barnsley et al 2005): the Sierpinski gasket, the Minkowski curve and the Koch curve. With Sierpinski gasket, for example, an equilateral triangle (the initiator) is subdivided into four smaller triangles of half the size. The middle part, which arises from connecting the midpoints of each side, is removed. These steps are repeated on the three remaining triangles and so forth. It is obvious that after infinite iterations (applying the rules repeatedly on the output) the final object is exact self-similar. That means zooming in on a detail offers the same structure as the whole. It is thus possible to calculate the so-called self-similar dimension (see formula 3):

$$D_s = -\frac{\log(N)}{\log(\epsilon)} \tag{3}$$

with 'N' equals the number of remaining pieces and 'ϵ' equals the reduction factor. For Sierpinski gasket the number of remaining pieces is three and the reduction factor is one-half. Substituting these values into equation (3) results in the self-similar dimension D_s=1.58. For the test case seven iterations of the Sierpinski gasket were drawn in a CAAD software and exported as an black and white image with 512 pixels width and 72 dpi resolution. Based on default settings, the basic implementation *Different thresholds* results in constant box-counting dimensions of D_B=1.56 for all thresholds, which is only slightly lower than the calculated D_s=1.58. The same holds true for a drawing of six iterations of the Koch curve (D_B=1.23 and D_s=1.26) and a drawing of six iterations of the Minkowski curve (D_B=1.44 and D_s=1.50). Testing all measurements possibilities, the best results are achieved for Sierpinski gasket with the following settings: scale reduction = 1.8, smallest box size = 4 Pixels and largest box size = 256 Pixels. In this case all values equal the self-similar dimension (D_B=1.58 and D_s=1.58) (figure 2).

Standard examples which are well known from image analysis and image compression act as greyscale test images: Baboon, Bird, Cameraman and Lena. As expected, this time the results increase from the first (smallest) to the last (largest) threshold, since more and more grey values are turned into black points and hence counted. What can be observed is that the curves differ from each other, which, in turn, characterises the texture signature (figure 2).

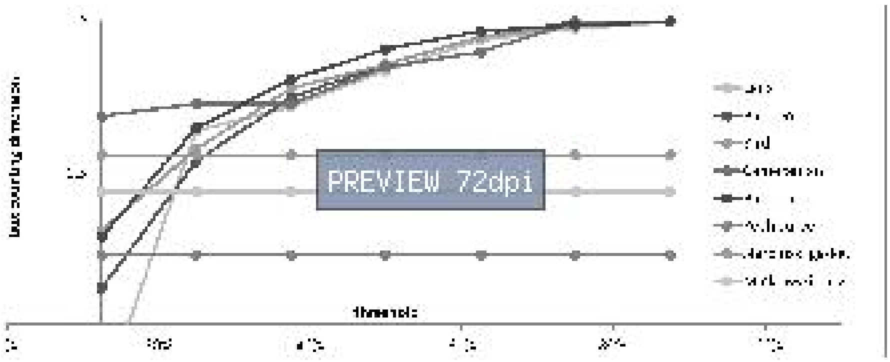

Figure 2. Results for different predefined thresholds of deterministic fractals: Koch curve, Sierpinski gasket and Minkowski curve. And examples of texture signatures for standard examples: Baboon, Bird, Cameraman, Barbara and Lena.

5. Differential Box-counting

An extension of the standard box-counting method directly takes greyscale images into account and by that makes a conversion to binary black and white images obsolete. The so-called differential box-counting (Sarker & Chaudhuri 1994) is currently used for discrimination between textures from urban areas (Filho 2008) and in medicine, e.g. to detect cancer (Sankar &Thomas 2010; Gao et al. 2016). Basically, differential box-counting regards an image of size M x M pixels (x-, y-coordinates) as a three dimensional landscape, whereby the z-coordinate indicates the grey level: with lighter pixels forming mountains and darker pixels defining valleys. Similar to the "standard" box-counting method a grid is placed over the image, with the box size (s) limited to the given range M/2≤s≤1 (with s as an integer). Consequently, the ratio equals ϵ =s/M. Each cell of the grid consists of s x s x s' boxes with G/s'=M/s (with G equals the total number of grey levels). Unlike the binary analysis, which only checks whether or not a cell contains a part (a pixel) of the image, differential box-counting calculates for each cell (i, j) of the grid the difference between the box number in which the minimal grey level falls and the box number in which the maximum grey level falls (Figure 3a). The difference is calculated by

$$n_\epsilon(i,j) = l - k + 1 \quad (4)$$

with 'k' equals the box number of the minimal grey level and 'l' equals the box number of the maximum grey level. Finally, the total number of counted boxes for each grid size covering the full image surface is given as

$$N_\epsilon = \sum_{i,j} n_\epsilon(i,j). \quad (5)$$

The differential box-counting dimension D_{BC} is, finally, calculated as given in equation (1) and determined by the slope of the regression line in a double logarithmic graph. Figure 3b shows the related regression line of plotting $\log(N_\epsilon)$ versus $\log(\epsilon)$, as coming out from the cell phone application *Greyscale*. Since the difference between intensity is calculated, in some sense, the method uses an approximation of the thickness of the covering surface.

Following from above, differential box-counting regards a greyscale image of size M x M pixels. Thus the algorithm starts with a conversion of the image into 8 bit greyscale (256 grey levels) and a reduction and/or cropping in order to fulfil the requirement of a squared image size. As with the aforementioned applications the calculation method encompasses a default range of box sizes from 128 to 2 Pixels. However, the user interface offers, once more, the possibility to change the smallest and largest box size (figure 3b).

Figure 3. Left 3a: Schema of the differential box-counting method for s=3 and s'=3, determining 'l' (box number of the maximum grey level) and 'k' (box number of the minimal grey level). Right 3b: Settings and result of the 'Greyscale' application.

The applied method has been tested with *experimental* cases, and, once more, with standard examples (Baboon, Bird, Cameraman and Lena). *Experimental* cases are those that fulfil certain *extreme* characteristics. Theoretically, the differential box-counting dimension is always between two (very smooth) and three (very rough); i.e. the rougher an image is, the closer the value is to three. Hence it is expected that D_{BC} for a monochrome image (completely filled with white, grey or black) is close to two. Whereas a black and white chess field like image (with a field size of 1 Pixel) offers the greatest possible difference of N_ϵ leading to a theoretically result of three.

The size M x M of all test images equals 512 x 512 Pixels, while the total number of greyscale *G* equals 256. As can be seen from figure 4 all *experimental* cases achieve the previously described values. Whereas those of the standard examples offer different results between two and three, depending on their roughness.

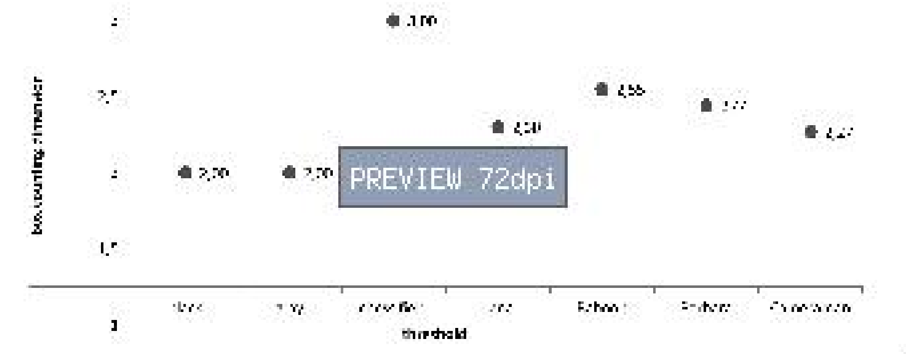

Figure 4. Results for experimental cases and examples of texture signatures for standard examples (Baboon, Bird, Cameraman, Barbara and Lena).

6. Interpretation of the Results

The building's fractal dimension, as a characteristic value of roughness is, in any case, only reasonable if the result is compared with those of other buildings. Apart from this, since all used methods are subject to influences inherent to the technique itself, it is important that only results of the same method are examined. As a consequence we recommend not to look at each individual measurement in isolation, but to consider it as a part of a "self-contained" comparison system. Concerning the cell phone application the result should not only immediately be available from a taken photo, but should also provide a quick possibility of comparison as well. This will be achieved by access to a set of results for iconic buildings as reference. The idea is to display three references taken from the data set with values closest to the calculated one. This gives the possibility of a visual classification of the considered building.

In contrast, if the result is examined in isolation, we nevertheless see a possibility of analysation. This time the information about the changing thickness of the covering greyscale blanket may serve as characterization. Thickness strongly or hardly changes from one iteration to the next (iteration terms the calculation procedure for a certain grid size). Areas that display the most significant change are then highlighted by color.

7. Outlook

Although the application shows promising results, there are still some points which must be taken into consideration. First of all, since the object of focus is a building, it is to be questioned whether or not solely this object should be considered; i.e. whether or not the building should be extracted from its surrounding (the sky with clouds, trees and adjoining buildings). In the current state we forego the implementation of an appropriate algorithm, not least because of time performance and usability, but also because a building in real life is never looked at without its environment (including foreground and background).

Further studies also relate to the handling of influences. E.g. in order to reduce quantization effects and to improve accuracy the starting position of the grid may be changed (Lorenz 2012). Again computation time is the reason why this is not implemented yet. One additional aspect in this context is the discussion about the outcome with respect to accuracy. Therefore statistic values such as standard deviation should be included in the application.

Another focus of future work will be laid on additional applications. Two immediate proposed implementations concern a variation of the threshold segmentation and an improved differential box-counting method for colour images (Nayak & Mishra 2016). Currently threshold segmentation uses ranges from zero to a certain upper limit. However, segmentation of thresholds can also range from one limit to another: e.g. 0-12.5%, 12.5-25%, 25-37.5% and so on. This will strengthen the attention on changes of characteristics between separate ranges of grey intensities. Since colour plays an important role in perception, the improved differential box-counting method will take this part into account. In the current stage colour is only included in form of different intensity.

8. Conclusion

The here presented web application offer two fast solutions to estimate the fractal dimension of photographs taken with a mobile phone. The first one is based on the standard box-count for binary images (using threshold conversion), while the second one is based on the differential box-count algorithm applied to greyscale images. Accuracy has been tested on deterministic fractals with known fractal dimensions. It has been demonstrated that not only results (box-counting dimensions D_B) are close to the expected ones (self-similar dimensions D_s) but also that the application is a rapid and effective implementation. For test purposes the current version of the webpage is available under: www.iemar.tuwien.ac.at/processviz/fracam/ (Wurzer & Lorenz 2016).

References

Backes, A.R. and Bruno, O.M.: 2008, A new approach to estimate fractal dimension of texture images, *Proceedings of 3rd International Conference, ICISP 2008, LNCS 5099, Image and signal processing*, Cherbourg-Octeville, France, 136-143.

Barnsley, M., Hutchinson, J. and Stenflo, Ö: 2005, A fractal valued random iteration algorithm and fractal hierarchy, *Fractals*, **13**, 111–146.

Bovill, C.: 1996, *Fractal Geometry in Architecture and Design*, Birkhäuser, Boston, Mass..

Bovill, C.: 2000, Fractal Geometry as Design Aid, *Journal for Geometry and Graphics*, **4**, 71-78.

Capo, D.: 2004, The Fractal Nature of the Architectural Orders, *Nexus Network Journal*, **6**(1), 30-40.

Falconer, K.: 1990, *Fractal Geometry: Mathematical Foundations and Applications*, John Wiley & Sons, Chichester.

Filho, M.N.B. and Sobreira, F.J.A.: 2008, Accuracy of Lacunarity Algorithms in Texture Classification of High Spatial Resolution Images from Urban Areas, *XXIst ISPRS Congress The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Science. XXXVII.*, Beijing, 417-422.

Fourtan-Pour, K., Dutilleul, P. and Smith, D.L.: 1999, Advances in the implementation of the boxcounting method of fractal dimension estimation, *Applied Mathematics and Computation*, **105**, 195-210.

Gao, W., Zakharov, V.P., Myakinin, O.O., Bratchenko, I.A., Artemyev, D.N. and Kornilin, D.V.: 2016, Medical images classification for skin cancer using quantitative image features with optical coherence tomography, *Journal of Innovative Optical Health Sciences*, **9**(2), 9 pages.

Hausdorff, F.: 1919, Dimension und äußeres Maß, *Mathematische Annalen*, **79**(1–2), 157–179.

Lorenz, W.E.: 2012, Estimating the Fractal Dimension of Architecture: Using two Measurement Methods implemented in AutoCAD by VBA, *30th eCAADe Conference Proceedings*, Prague, Czech Republic, 505–514.

Lorenz, W.E.: 2013, Combining Complexity and Harmony by the Box- Counting Method – A comparison between entrance façades of the Pantheon in Rome and Il Redentore by Palladio, *Proceedings of eCAADe 2013 Vol. 2*, Delft, 667-676.

Lorenz, W.E.: 2014, Measurability of Loos' rejection of the ornament – Using box-counting as a method for analysing facades, *Proceedings of eCAADe 2014 Vol. 1*, Newcastle upon Tyne, 495-504.

Lorenz, W.E.: 2016, Complexity across scales in the work of Le Corbusier Using box-counting as a method for analysing facades, *CAADence in Architecture*, Budapest, Hungary.

Mandelbrot, B.B.: 1967, How long is the coast of britain? Statistical self-similarity and fractional dimension, *Science*, **May 5**, 636–638.

Mandelbrot, B.B.: 1975, *Les Objects Fractals: Forme, Hasard et Dimension; In Englisch: Fractals: Form, Chance and Dimension (1977)*, W.H. Freeman & Co.

Mandelbrot, B.B.: 1983, *The fractal geometry of nature*, W.H. Freeman & Co, New York.

Nayak, S.R. and Mishra, J.: 2016, An improved method to estimate the fractal dimension of colour images, *Perspectives in Science*, **8**, 412–416.

Ostwald, M.J.: 2013, The fractal analysis of architecture: calibrating the box-counting method using scaling coefficient and grid disposition variables, *Environment and Planning B: Planning and Design*, **40**(4), 644-663.

Ostwald, M.J. and Vaughan, J.: 2016, *The fractal Fractal Dimension of Architecture*, Birkhäuser.

Rian, I.M., Park, J.H., Ahn, H.U. and Chang, D.: 2007, Fractal geometry as the synthesis of Hindu cosmology in Kandariya Mahadev temple, Khajuraho, *Building and Environment*, **42**(12), 4093-4107.

Sankar, D. and Thomas, T.: 2010, Fractal features based on differential box counting method for the categorization of digital mammograms, *International Journal of Computer Information System and Industrial Management Applications*, **2**, 11-19.

Sarker, N. and Chaudhuri, B.B.: 1994, An efficient differential box-counting approach to compute fractal dimension of image, *IEEE Transaction on Systems, Man, and Cybernetics*, **24**, 115-120.

Wurzer, G. and Lorenz, W.E.: 2016, "FRACAM, Open Source Test Version" . Available from <www.iemar.tuwien.ac.at/processviz/fracam/> (accessed 18th December 2016).

Zarnowiecka, J.C.: 2000, In search of new computer tools: What does Bovill really measure in architecture?, *20th eCAADe Conference Proceedings*, Warsaw, Poland, 342-345.