Performance Evaluation and Resource Allocation in HetNets Under Joint Offloading and Frequency Reuse

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***Abstract*—Offloading users from an overloaded macro base station (BS) to a lightly loaded small cell BS is critical to alleviate the congestion of the macro BSs. However to overcome the signal-to-interference-plus-noise ratio (SINR) degradation of the offloaded users, offloading should be done in conjunction with an efficient interference management technique. Strict fractional frequency reuse (Strict-FFR) is an attractive interference management technique due to its bandwidth efficiency and its fitness to orthogonal frequency division multiple access based cellular networks. We provide a mathematical framework for evaluating the performance of the users under a flexible user association and a Strict-FFR interference management mechanism. Furthermore, we propose a novel resource allocation mechanism based on the BSs bias values and FFR thresholds that achieves higher user throughput and rate coverage probability. The accuracy of analysis and the effectiveness of the Strict-FFR mechanism is verified by numerical experiments in different scenarios.**

***Index Terms*—Heterogeneous cellular networks; resource allocation; Poisson point process; frequency reuse.**

I. INTRODUCTION

Cellular network operators are forced to increase the network capacity to cope with the growing demand of the user traffic. Small low-power base stations (BSs), are one of the key technology enablers to provide higher end-user throughput in future cellular networks. Because of the uncontrolled distribution nature of small cell BSs in the heterogeneous cellular networks (HetNets), conventional models such as Wyner [1] and hexagonal grid are considered to be obsolete for modeling of such cellular networks. Stochastic geometry and Poison point processes (PPPs) provide a tractable and accurate tool for modeling and analyzing HetNets[2].

Performance evaluation of cellular networks using PPPs was first investigated in [2, 3]. It is shown that in an interference-limited networks with identical target signal-to-interference-plus-noise ratio (SINR) for all tiers, the coverage probability is independent from the BS density and number of tiers.

Since small cell BSs generally have lower transmission power compared to the macro BSs, most of the users tend to associate to the macro BS. The congestion of an overloaded macro BS can be alleviated by pushing the users to be associated with small cell BSs. Bias user association follows the idea of encouraging users to be associated with the lower tier(s) BSs by adding an artificial bias value to the received power of small cell BSs, and is included in 3rd Generation Partnership Project (3GPP) since Release 10 [4]. The performance of HetNets by offloading users traffic to the small cell BSs is investigated in [5, 6]. In [5], it is shown that biasing of the users without an efficient interference coordination mechanism always reduces the overall coverage probability of HetNets. In [7] the authors exploit a model to evaluate performance of a two-tier cellular network under a simple resource partitioning mechanism. They put small cell users into biased and unbiased users groups and allocate $(1-\eta)$ fraction of resources to the biased users of small cell BSs while the unbiased users of small cell BSs share the rest of the resources with the macro cell users. Since, the interference level in the fraction of resources used for the biased users is lower than the shared part, the biased users experience a higher coverage probability.

Strict fractional frequency reuse (Strict-FFR) is a simple and efficient method to mitigate interference for cellular networks and is included in the 3GPP-LTE standard since Release 8 [8]. Performance of cellular network under a Strict-FFR interference coordination mechanism is investigated in [9], when the users are associated to the *closest* BSs of each tier.

Contributions: The main contribution of this work is the development of a general analytical model to evaluate the performance of HetNets downlink transmissions under offloading and Strict-FFR interference coordination. We extend the work in [9] by employing the biased user association and presenting the per-tier coverage probability of the cell edge and the cell interior users. While [9] just provide the average ergodic rate of networks, in this paper, we derive the per-tier average rate of the cell edge and the cell interior users as well as average number of cell edge and cell interior users of each tier, rate coverage probability, and the minimum achievable user rate. The mathematical results in general scenario are not in closed form, however we provide closed form expressions for the coverage probability of users under a special case. It in turn allows a deeper insight in the performance dependency on various parameters. Last but not least a contribution of this paper is proposing a novel resource allocation mechanism based on the cell edge and the cell interior load of BSs and the user SINR distribution that achieves higher minimum user throughput and the rate coverage probability. Our analysis shows that even in an interference limited network when all tiers experience the same path loss exponent and we employ an unbiased user association, the coverage probability of the network is not independent of the BSs density. This result contradicts recent observations which show the coverage probability is independent of the BS density and the number of tiers [2, 3, 5]. Due to space limitations, we will state some of the theorems without detailed proof. The complete proofs will be published in a forthcoming journal paper.

II. SYSTEM MODEL

Consider the downlink of a K-tier cellular network where $\mathcal{K} = \{1, ..., K\}$ is the tiers set of network. The BSs in k-th tier are spatially distributed as a PPP $\phi_k \in \mathbb{R}^2$ of density λ_k and

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the locations of users are modeled by another independent PPP $\phi_u \in \mathbb{R}^2$, with the non-zero density λ_u. The maximum transmit power of the k-th tier BSs is shown as P_k, τ_k denotes the tier SINR threshold, and noise power is represented by σ^2. A signal traveling from the BS to user experiences path loss and small scale fading. The downlink SINR of a user when it connects to a BS $x \in \phi_k$, is abstracted as

$$\mathrm{SINR}\left(k, P_k\right) = \frac{P_k h_{kx} y_{kx}^{-\alpha_k}}{\sum_{j=1}^{K}\sum_{z\in\phi_j\backslash x} P_j h_{jz} {y_{jz}}^{-\alpha_j} + \sigma^2}, \quad (1)$$

where y_{jz} and h_{jz} represent the distance and the small scale fading between the BS z in j-th tier and the user under consideration, respectively and α_j is the path loss exponent of tier j.

In this paper we adopt the Strict-FFR interference coordination mechanism. In a Strict-FFR mechanism the network users are divided into the cell edge and the cell interior users based on a certain frequency reuse threshold $\tau_{i,\mathrm{FR}}$. A user in k-th tier is considered as a interior user if its received SINR exceeds threshold $\tau_{i,\mathrm{FR}}$, otherwise it is considered as an edge user. The entire frequency band W is partitioned into a common part $W_{\mathrm{In}}^{\mathrm{FFR}} = \beta W$ and a reuse part $W_{\mathrm{e}}^{\mathrm{FFR}} = (1-\beta)W$, where $0 \le \beta \le 1$. The common part of resources is shared by the cell interior users of all tiers. The reuse part is divided among the network tiers and a fraction $W_{\mathrm{e},k}^{\mathrm{FFR}}$ of resources is utilized by the k-th tier edge users, where $W_{\mathrm{e}}^{\mathrm{FFR}} = \sum_{k=1}^{K} W_{\mathrm{e},k}^{\mathrm{FFR}}$. Hence, the cell edge users of a different tier use a disjoint set of resources. Furthermore, the reuse part of the k-th tier is further partitioned into Δ_k sub-bands and each BS randomly chooses one sub-band to transmit to the cell edge users.

A. User Association Probability and Distance Distribution

Each user chooses a BS as its serving BS, if it provides the maximum long-term biased received power [5]. The user association policy is given by:

$$u \in \mathcal{U}_i \quad \text{if} \quad i = \arg\max_{k\in\mathcal{K}} B_k P_k {R_k}^{-\alpha_k} \quad , \quad (2)$$

where $\mathcal{U}_i$ denotes the users set of i-th tier BSs, R_k is the distance between the user and its nearest BS in k-th tier, and B_k is a positive bias value which is identical for all the BSs of tier k. Employing a bias value $B_k > 1$ by the BSs of the k-th tier, extends its coverage area.

The tier association probability A_i, the number of user associated to tier i and the probability distribution function (PDF) of the distance between the user and its associated BS $f_i(x)$ are obtained via [5, Lemma 1], [5, Lemma 2] and [5, Lemma 3], respectively.

$$A_i = \int_0^\infty \exp\left(\sum_{k=1}^{K} -\lambda_k \pi \left(\frac{B_k P_k}{B_i P_i}\right)^{2/\alpha_k} r^{\frac{2\alpha_k}{\alpha_i}}\right) 2\pi r \lambda_i dr, \quad (3)$$

$$N(i) = \int_0^\infty \exp\left(\sum_{k=1}^{K} -\lambda_k \pi \left(\frac{B_k P_k}{B_i P_i}\right)^{2/\alpha_k} r^{\frac{2\alpha_k}{\alpha_i}}\right) 2\pi r \lambda_u dr, (4)$$

$$f_i(x) = \frac{2\pi\lambda_i x}{A_i} \exp\left(-\pi \sum_{k=1}^{K} \lambda_k \left(\frac{B_k P_k}{B_i P_i}\right)^{2/\alpha_k} x^{\frac{2\alpha_k}{\alpha_i}}\right). (5)$$

III. Coverage Probability

A. General Results

This section provides the per-tier coverage probability of the cell edge and the cell interior users under Strict-FFR interference coordination mechanism. The coverage probability is the probability that the SINR of a typical user is greater than a predefined target SINR value. The coverage probability of network is computed using the law of total probability.

Theorem 1. *The coverage probability of the i-th tier cell edge user in a K-tier cellular network with the biased user association and Strict-FFR mechanism is*

$$S_{\mathrm{e}}^{\mathrm{FFR}}(i,\tau_i) = \int_0^\infty \Bigg[\delta_i(\tau_i)\exp\left(-2\pi\lambda_i \int_r^\infty \left(1 - Q'(y)\right) y dy\right) - \delta_i(\tau_i)\delta_i\left(\tau_{i,\mathrm{FR}}\right) \prod_{k=1,k\neq i}^{K} Q\left(\Psi_{k,i}\left(\tau_{i,\mathrm{FR}}\right), \omega_{k,i}\right) \exp\left(-2\pi\lambda_i \int_r^\infty \left[1 - \frac{Q'(y)}{1+\Psi_{i,i}\left(\tau_{i,\mathrm{FR}}\right) y^{-\alpha_i}}\right] y dy\right)\Bigg] f_i(r) dr,$$

where

$$\Psi_{k,i}\left(x\right) = \frac{P_k r^{\alpha_i}}{P_i} x, \;\; Q'(y) = 1 - \frac{1}{\Delta_i}\left(\frac{1}{1+\Psi_{i,i}\left(\tau\right)^{-1} y^{\alpha_i}}\right),$$

$$Q\left(a_k, \omega\right) = \exp\left(-2\pi\lambda_k \int_\omega^\infty \frac{y}{1+\left(a_k\right)^{-1} y^{\alpha_k}} dy\right),$$

$$\delta_i\left(x\right) = \exp\left(-\frac{r^{\alpha_i}\sigma^2}{P_i} x\right), \quad \omega_{k,i} = \left(\frac{r^{\alpha_i} B_k P_k}{B_i P_i}\right)^{1/\alpha_k}$$

Proof. See Appendix A. □

Theorem 2. *The coverage probability of i-th tier cell interior user under Strict-FFR mechanism is*

$$S_{\mathrm{In}}^{\mathrm{FFR}}(i,\tau_i) = \int_0^\infty \Bigg[\delta_i\left(\tau_i + \tau_{i,\mathrm{FR}}\right) \prod_{k=1}^{K} Q\left(\Psi_{k,i}\left(\tau_i\right), \omega_{k,i}\right) \prod_{k=1}^{K} Q\left(\Psi_{k,i}\left(\tau_{i,\mathrm{FR}}\right), \omega_{k,i}\right)\Bigg] f_i(r) dr. \quad (6)$$

Proof. The proof is carried out similar to Theorem 1. □

Combining the results of Theorem 1 and Theorem 2, the overall coverage probability under Strict-FFR mechanism is

$$S^{\mathrm{FFR}} = \sum_{i=1}^{K} A_i \left(S_{\mathrm{e}}^{\mathrm{FFR}}(i,\tau_i) + S_{\mathrm{In}}^{\mathrm{FFR}}(i,\tau_i)\right). \quad (7)$$

B. Special Case of Interest

The general coverage probability obtained in the previous part is not in a closed form. This sub-section provides a closed form expression for the coverage probability of users in a special case where $\sigma^2 = 0$ and $\alpha = 4$ for all the tiers. Because of the high BS density in HetNets, in general the noise power is negligible compared to the interference power. Besides, the choice of the path loss exponent $\alpha = 4$ is commonly accepted in practice as long as users are not too close to the BS.

Corollary 1. *Consider $\sigma^2 = 0$ and $\alpha = 4$. The coverage probability of i-th tier cell edge user under Strict-FFR mechanism in Theorem 1 is*

$$S_{\mathrm{e}}^{\mathrm{SFR}}(i,\tau_i) = \frac{\lambda_i\left(A_i\right)^{-1}}{\left(C_2\left(i\right)+C_4(i)\right)} - \frac{\lambda_i\left(A_i\right)^{-1}}{\left(C_1(i)+C_3(i)+C_4(i)\right)}, \quad (8)$$

$$\begin{aligned}
D_1(i) &= \left(\Delta_i\left(\tau_{i,\mathrm{FR}}-\tau_i\right)+\tau_i\right)\arctan\left(\sqrt{\tau_{i,\mathrm{FR}}}\right),\\
D_2(i) &= \frac{\tau_i\sqrt{\tau_i}}{\sqrt{\tau_{i,\mathrm{FR}}}}\arctan\left(\sqrt{\tau_i}\right),\\
C_1(i) &= \frac{\lambda_i\left(D_1(i)-D_2(i)\right)}{\Delta_i\sqrt{1/\tau_{i,\mathrm{FR}}}(\tau_{i,\mathrm{FR}}-\tau_i)},\\
C_2(i) &= \frac{\lambda_k}{\Delta_i}\sqrt{\tau_i}\arctan\left(\sqrt{\tau_i}\right),\\
C_3(i) &= \sum_{k=1,k\neq i}^{K}\frac{\lambda_k}{\Delta_k}\sqrt{\frac{P_k\tau_{i,\mathrm{FR}}}{P_i}}\arctan\left\{\sqrt{\frac{B_i\tau_{i,\mathrm{FR}}}{B_k}}\right\},\\
C_4(i) &= \sum_{k=1}^{K}\lambda_k\left(\frac{B_kP_k}{B_iP_i}\right)^{1/2}.
\end{aligned}$$

Corollary 2. *Consider $\sigma^2 = 0$ and $\alpha = 4$. The coverage probability of i-th tier cell interior user under a Strict-FFR interference coordination mechanism in Theorem 2 is*

$$S_{\mathrm{In}}^{\mathrm{SFR}}(i,\tau_i) = \frac{\lambda_i\left(A_i\right)^{-1}}{C_5(i,\tau_i)+C_5(i,\tau_{i,\mathrm{FR}})+C_4(i)}, \quad (9)$$

$$C_5(i,x) = \sum_{k=1}^{K}\frac{\lambda_k}{\Delta_k}\left(\frac{P_kx}{P_i}\right)^{1/2}\arctan\left\{\left(\frac{B_ix}{B_k}\right)^{1/2}\right\}.$$

This result is opposing [2, 3, 5], where it is argued that in interference limited networks when all tiers have the same path loss exponent, the coverage probability is independent of the BS density and the number of tiers. The model used in mentioned works does not employ any interference coordination mechanism among the users. However as evident from the coverage probability expressions (8) and corollary 2, by employing Strict-FFR interference coordination, even in an unbiased user association mechanism, the overall coverage probability depends on the BS distribution density. Besides, we observe that by using a larger value for reuse factor Δ_k, the coverage probability increases as well. Hence, by sufficient increment of frequency reuse factor Δ_k of all tiers, we can achieve the coverage probability $S^{\mathrm{FFR}}(i,\tau_i) = 1$ for the users of tier i.

IV. SPECTRAL EFFICIENCY

The average ergodic rate (*average cell throughput*) constitutes an upper bound on the average data rate that can be achieved in a mobile network per bandwidth, and is defined as $\mathcal{R} = \mathbb{E}_\gamma[\ln(1+\gamma)]$, where γ denotes the instantaneous SINR.

The overall average ergodic rate of a typical user under a Strict-FFR interference management $\mathcal{R}^{\mathrm{FFR}}$ is given as

$$\mathcal{R}^{\mathrm{FFR}} = \sum_{i=1}^{K}A_i\left(\mathcal{R}_{\mathrm{e}}^{\mathrm{FFR}}(i)+\mathcal{R}_{\mathrm{In}}^{\mathrm{FFR}}(i)\right). \quad (10)$$

where $\mathcal{R}_{\mathrm{e}}^{\mathrm{FFR}}(i)$ and $\mathcal{R}_{\mathrm{In}}^{\mathrm{FFR}}(i)$ are the rate of the cell edge users and the rate of the cell interior users of tier i under a Strict-FFR interference coordination mechanism, respectively.

Lemma 1. *The average ergodic rate of a user under a Strict-FFR interference coordination mechanism is formulated in terms of coverage probability as*

$$\begin{aligned}
\mathcal{R}^{\mathrm{FFR}} &= \sum_{i=1}^{K}A_i\int_0^\infty S_{\mathrm{e}}^{\mathrm{FFR}}\left(i,\exp\left(\nu\right)\ -1\right)d\nu\\
&+\sum_{i=1}^{K}A_i\int_0^\infty S_{\mathrm{In}}^{\mathrm{FFR}}\left(i,\exp\left(\nu\right)\ -1\right)d\nu. \quad (11)
\end{aligned}$$

Another important metric considered in the design of the cellular networks is the *average user throughput*. Assuming the resources are fairly shared between the users associated to a BS, the average cell edge user throughput of tier i under Strict-FFR is given as

$$\mathcal{R}_{\mathrm{u,e}}^{\mathrm{FFR}}(i) = \frac{W_{\mathrm{e},i}^{\mathrm{FFR}}\mathcal{R}_{\mathrm{e}}^{\mathrm{FFR}}(i)}{\Delta_iWN_{\mathrm{e}}^{\mathrm{FFR}}(i)}, \quad (12)$$

where $N_{\mathrm{e}}^{\mathrm{FFR}}(i)$ is the average number of cell edge users associated to a BS of tier i under a Strict-FFR mechanism

$$\begin{aligned}
N_{\mathrm{e}}^{\mathrm{FFR}}(i) = \frac{\lambda_uA_i}{\lambda_i}\Bigg[1-\int_0^\infty\delta_i\left(\tau_{i,\mathrm{FR}}\right)\\
\prod_{k=1}^{K}Q\left(\Psi_{k,i}\left(\tau_{i,\mathrm{FR}}\right),\omega_{k,i}\right)f_i(r)dr\Bigg]. \quad (13)
\end{aligned}$$

The average cell interior user throughput under a Strict-FFR interference coordination is given as

$$\mathcal{R}_{\mathrm{u,In}}^{\mathrm{FFR}}(i) = \frac{W_{\mathrm{In}}^{\mathrm{FFR}}\mathcal{R}_{\mathrm{In}}^{\mathrm{FFR}}(i)}{WN_{\mathrm{In}}^{\mathrm{FFR}}(i)}, \quad (14)$$

where $N_{\mathrm{In}}^{\mathrm{FFR}}(i)$ is the average number of cell interior users associated to a BS of tier i under a Strict-FFR mechanism

$$\begin{aligned}
N_{\mathrm{In}}^{\mathrm{FFR}}(i) = \frac{\lambda_uA_i}{\lambda_i}\int_0^\infty\delta_i\left(\tau_{i,\mathrm{FR}}\right)\\
\prod_{k=1}^{K}Q\left(\Psi_{k,i}\left(\tau_{i,\mathrm{FR}}\right),\omega_{k,i}\right)f_i(r)dr. \quad (15)
\end{aligned}$$

Finally, using (16), we compute the minimum rate achievable by each user

$$\mathcal{R}_{\mathrm{min}}^{\mathrm{FFR}} = \min_{i\in\mathcal{K}}\left(\mathcal{R}_{\mathrm{u,e}}^{\mathrm{FFR}}(i),\mathcal{R}_{\mathrm{u,In}}^{\mathrm{FFR}}(i)\right). \quad (16)$$

We now compute the rate coverage probability of a user located at the origin with the same approach as before for computing the coverage probability.

Lemma 2. *The rate coverage probability of tier i under a Strict-FFR interference coordination for a rate threshold γ_i is*

$$\mathcal{P}^{\mathrm{FFR}}\left(i,\gamma_i\right) = A_i\mathcal{P}_{\mathrm{e}}^{\mathrm{FFR}}\left(i,\gamma_i\right)+A_i\mathcal{P}_{\mathrm{In}}^{\mathrm{FFR}}\left(i,\gamma_i\right), \quad (17)$$

$$\begin{aligned}
\mathcal{P}_{\mathrm{e}}^{\mathrm{FFR}}\left(i,\gamma_i\right) &= S_{\mathrm{e}}^{\mathrm{FFR}}\left(i,\exp\left(\frac{\gamma_i\Delta_iN_{\mathrm{e}}^{\mathrm{FFR}}(i)}{W_{\mathrm{e},i}^{\mathrm{FFR}}}\right)-1\right),\\
\mathcal{P}_{\mathrm{In}}^{\mathrm{FFR}}\left(i,\gamma_i\right) &= S_{\mathrm{In}}^{\mathrm{FFR}}\left(i,\exp\left(\frac{\gamma_iN_{\mathrm{In}}^{\mathrm{FFR}}(i)}{W_{\mathrm{In}}^{\mathrm{FFR}}}\right)-1\right).
\end{aligned}$$

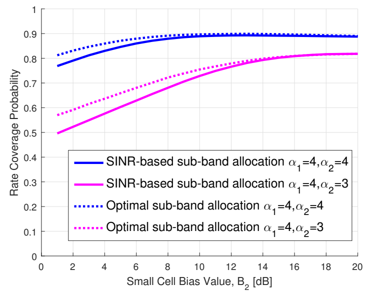

Fig. 1: Performance of resource allocation mechanism.

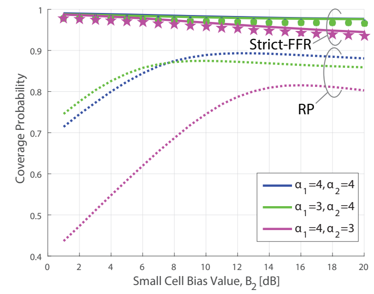

Fig. 2: Effect of bias value on coverage probability.

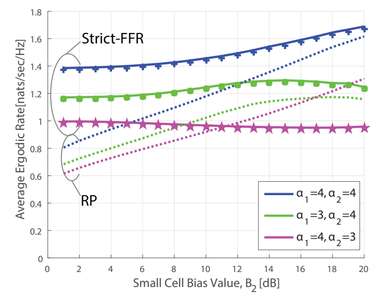

Fig. 3: Effect of bias value on average ergodic rate.

Proof. To compute the rate coverage probability of tier i, let us first concentrate on the rate coverage probability of the cell edge users of tier i, i.e., $\mathcal{P}_{\rm e}^{\rm FFR}(i,\gamma_i)$

$$\begin{aligned}\mathcal{P}_{\rm e}^{\rm FFR}(i,\gamma_i) &= \mathbb{P}\Bigg[\frac{W_{{\rm e},i}^{\rm FFR}}{\Delta_i N_{\rm e}^{\rm FFR}(i)}\log\left(1+\mathrm{SINR}'(i,P_i)\right) \geq \gamma_i \\ &\quad, \mathrm{SINR}(i,P_i) < \tau_{i,\rm FR}\Bigg| u \in \mathcal{U}_i\Bigg], \\ &= \mathbb{P}\Bigg[\mathrm{SINR}'(i,P_i) \geq \exp\left(\frac{\gamma_i \Delta_i N_{\rm e}^{\rm FFR}(i)}{W_{{\rm e},i}^{\rm FFR}}\right) - 1 \\ &\quad, \mathrm{SINR}(i,P_i) < \tau_{i,\rm FR}\Bigg| u \in \mathcal{U}_i\Bigg], \\ &= S_{\rm e}^{\rm FFR}\left(i, \exp\left(\frac{\gamma_i \Delta_i N_{\rm e}^{\rm FFR}(i)}{W_{{\rm e},i}^{\rm FFR}}\right) - 1\right),\end{aligned}$$

the rate coverage probability of the cell interior users, $\mathcal{P}_{\rm In}^{\rm FFR}(i,\gamma_i)$ is obtained by following the same approach. □

V. Resource Allocation

Resource allocation among the cell edge and the cell interior users is one of the main concerns in FFR literature. Most of the former works determined the optimal values of FFR system parameters by utilizing advanced techniques such as convex optimization [10].

We determine the number of sub-bands of the cell edge and the cell interior users depending on the chosen bias values and FFR thresholds of different tiers. Considering $\rho_{\rm total}$ as the total number of available sub-bands, $\rho_{\rm e}$ is the number of cell edge user sub-bands and $\rho_{\rm In}$ the number of cell interior user sub-bands, the optimal resource allocation of Strict-FFR is

$$\beta = \frac{\sum_{k=1}^{K} N_{\rm In}^{\rm FFR}(k)}{\sum_{k=1}^{K} N(k)}, \tag{18}$$

$$\rho_{\rm In}^{\rm FFR} = \left\lceil \beta \rho_{\rm total} \right\rceil, \tag{19}$$

$$\rho_{\rm e}^{\rm FFR}(i) = \left\lfloor \frac{\left(\rho_{\rm total} - \rho_{\rm In}^{\rm FFR}\right) N_{\rm e}^{\rm FFR}(i)}{\Delta_i \sum_{k=1}^{K} N_{\rm e}^{\rm FFR}(k)} \right\rfloor. \tag{20}$$

Intuitively, applying this resource allocation mechanism will enforce a more efficient resource allocation among the users based on the number of the cell edge and the cell interior users and the SINR distribution. Hence, the user will experience a higher throughput and the rate coverage probability.

Fig. 1 compares the proposed resource allocation mechanism with the optimal values of the resource allocation parameters. The simulation results show that employing the proposed resource allocation mechanism provides a good approximation of the optimal values of the resource allocation parameters, particularly in the high bias value regime.

VI. Numerical Results

This section provides numerical results for evaluating the performance of system under consideration. In particular, we consider a two tier cellular network but the results can be extended for a general K-tier cellular network as well. In the numerical evaluation, for both tiers we set the SINR threshold to -4 dB and the frequency reuse threshold to -1 dB. We compare the performance of Strict-FFR mechanism with the performance of resource partitioning (RP) mechanism [7]. In the figures lines denote the results from simulation, and markers refer to the results from mathematical analysis.

Fig. 2 and Fig. 3 reveal the effects of bias value on the coverage probability and the average ergodic rate of the users. In these figures, we observe a constant increase in the average ergodic rate and the coverage probability of RP mechanism while we do not observe the same increase in Strict-FFR mechanism. In RP mechanism a fraction of resources is allocated to the biased users of small cell BSs, hence the effect of biasing is more considerable for the RP mechanism. However even in the high bias value regime, Strict-FFR mechanism outperforms the RP mechanism in terms of coverage probability and average ergodic rate by allocating the reuse fraction of resources to the users with low SINR distribution and not just the biased users. Besides, we observe that the overall average ergodic rate of user is higher when the user experiences a larger path loss exponent from the small cell BSs. In dense deployment of

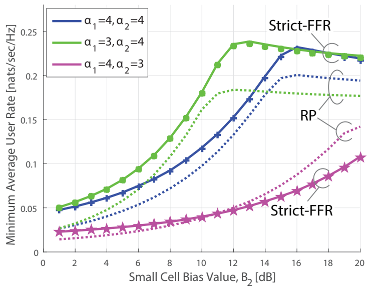

Fig. 4: Effect of bias value on minimum achievable user rate.

small cell BSs the interference level from the small cell BSs outweighs the macro cell BSs interference. Hence, we observe a higher performance when the user experiences a larger path loss exponent from the small cell BSs.

The minimum achievable rate of the users under different bias values is shown in Fig. 4. By employing a higher bias value, more users are pushed from highly loaded macro BSs to lightly loaded small cell BSs. Therefore, the minimum rate of users increases with increasing bias value until it reaches a certain optimal point where a further increase in the bias value decreases the minimum achievable rate of the users by overloading the small cell BSs. The simulation results in Fig. 4 shows that the user under Strict-FFR mechanism experiences higher minimum average achievable user rate compared to RP mechanism. Finally, we observe that our analytical evaluations accurately fit with simulation results in all numerical experiments.

VII. Conclusion

This paper proposes a mathematical framework for performance evaluation of HetNets with flexible user association and an Strict-FFR interference coordination mechanism. We show that even in interference limited networks with unbiased user association and same path loss exponent for all tiers the coverage probability is not independent of the BSs distribution density. The presented model can be utilized by the system developers in adjusting the the system parameters.

Appendix A
Proof of Theorem 1

Using the law of total probability, the coverage probability of cell edge user under the Strict-FFR system is defined as,

$$
\begin{aligned}
S_{\mathrm{e}}^{\mathrm{FFR}}(i,\tau_i) &= \mathbb{P}\left[\mathrm{SINR}'(i,P_i)>\tau_i, \mathrm{SINR}(i,P_i)<\tau_{i,\mathrm{FR}} \middle| u\in\mathcal{U}_i\right] \\
&= \mathbb{E}_{\Phi,h}\left[\exp\left(-\frac{r^{\alpha_i}\tau_i}{P_i}\sigma^2\right)\right]\mathbb{E}_{\Phi,h}\left[\exp\left(-r^{\alpha_i}\tau_i\hat{I}_i\right)\right] \\
&\quad - \mathbb{E}_{\Phi,h}\left[\exp\left(-\frac{r^{\alpha_i}}{P_i}\left(\tau_i P_i\hat{I}_i+\tau_{i,\mathrm{FR}}\sum_{k=1}^{K}P_k I_k\right)\right)\right] \\
&\quad \mathbb{E}_{\Phi,h}\left[\exp\left(-\frac{r^{\alpha_i}\sigma^2}{P_i}\left(\tau_i+\tau_{i,\mathrm{FR}}\right)\right)\right], \\
&= \delta_i(\tau_i)\mathcal{L}_{\hat{I}_i}\left(\Psi_{i,i}(\tau_i)\right)-\delta_i\left(\tau_i+\tau_{i,\mathrm{FR}}\right) \\
&\quad \underbrace{\mathcal{L}_{\hat{I}_i,I_1,\ldots,I_K}\left(\Psi_{i,i}(\tau_i),\hat{\Psi}_{1,i}(\tau_{i,\mathrm{FR}}),\ldots,\hat{\Psi}_{K,i}(\tau_{i,\mathrm{FR}})\right)}_{A},
\end{aligned}
$$

where $\Psi_{k,i}(x)=\frac{P_k r^{\alpha_i}}{P_i}x$ and $\delta_i(x)=\exp\left(-\frac{r^{\alpha_i}\sigma^2}{P_i}x\right)$. The terms I_i and $\hat{I}_i$ denote the interference level in common and reuse bands, respectively. The second Laplace transform is

$$
\begin{aligned}
A =& \mathbb{E}_{\Phi,h}\Bigg[\exp\Bigg(-\Psi_{i,i}(\tau_i)\sum_{z\in\Phi_i\setminus x}\mathbf{1}(\rho_x=\rho_z)\hat{h}_{iz}{y_{iz}}^{-\alpha_i}\Bigg) \\
& \exp\Bigg(-\Psi_{i,i}(\tau_{i,\mathrm{FR}})\sum_{z\in\phi_i}h_{iz}{y_{iz}}^{-\alpha_i}\Bigg)\Bigg] \\
& \prod_{k=1,k\neq i}^{K}\mathbb{E}_{\Phi,h}\Bigg[\exp\Bigg(-\Psi_{k,i}(\tau_{i,\mathrm{FR}})\sum_{z\in\phi_k}h_{kz}{y_{kz}}^{-\alpha_k}\Bigg)\Bigg], \\
\overset{a}{=}& \mathbb{E}_{\Phi}\Bigg[\prod_{z\in\Phi_i\setminus x}\frac{\left(1-\frac{1}{\Delta_i}\left(\frac{1}{1+\Psi_{i,i}(\tau)^{-1}{y_{iz}}^{\alpha_i}}\right)\right)}{1+\Psi_{i,i}(\tau_{i,\mathrm{FR}})\,{y_{iz}}^{-\alpha_i}}\Bigg] \\
& \prod_{k=1,k\neq i}^{K}\mathbb{E}_{\Phi}\Bigg[\prod_{z\in\Phi_k}\frac{1}{1+\Psi_{k,i}(\tau_{i,\mathrm{FR}})\,{y_{kz}}^{-\alpha_k}}\Bigg].
\end{aligned}
$$

The term $\mathbf{1}(\rho_x=\rho_z)$ takes the value 1 when BS x and BS z use the same sub-band and (a) is obtained by assuming that fading follows exponential distribution with unit mean $h\sim\exp(1)$. Finally, using the probability generating functional (PGFL) of PPP, we find the coverage probability of cell edge users under the Strict-FFR mechanism.

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