Parametric PID Controller Tuning for a Fast Steering Mirror
Ernst Csencsics and Georg Schitter

Abstract—Even though the design and tuning of proportional-integral-derivative (PID) controllers appears to be conceptually simple it can be difficult in practice, especially when competing control objectives are present. This paper presents a tuning method for PID controllers applied to low stiffness mechatronic systems that allows a direct and intuitive trade-off between the robustness and the performance of the resulting system. With the required system bandwidth typically determined by the targeted application and an according parametrization, the controller tuning is reduced to the selection of the cross-over frequency ωc and the variation of a single parameter α. It is demonstrated how the α-value influences the resulting system properties, while a larger α increases robustness but also diminishes control quality. The tuning method is experimentally verified on a fast steering mirror (FSM) system by implementing controllers with α-values of 2, 3 and 4.5. It is shown that the settling time for α = 2 is 4-times smaller than for α = 4.5, when applied to the nominal plant. On the other hand the stability margins for α = 2 are also significantly smaller, diminishing robustness and increasing oscillating transients when plant uncertainties are present. An α-value of 3 yields a good trade-off between robustness and performance of the closed-loop operated system.

1. INTRODUCTION

More than 90% of all control loops today employ proportional-integral-derivative (PID) controllers with applications ranging from motor drives, mechatronic systems, and instrumentation to process control, magnetic memories, flight control and automotive [1]. The main reasons for its success are clearly its robustness, simplicity and wide applicability [2].

Even though the design and tuning of PID controllers appears to be conceptually simple and straightforward it can be difficult in practice, especially when competing control objectives (e.g. robustness and short transients) are present. Thus basically all text books from scientific fields that utilize feedback control, such as process control [3], [4], mechatronics [5], [6], or control engineering [7], [8] provide a chapter on tuning PID controllers. Most early but still widely spread tuning methods were proposed by Ziegler-Nichols [9] and Cohen-Coon [10]. These more heuristic tuning rules are, however, reported to give rather poor results in many cases [1] and result in relatively poor closed-loop robustness [11]. There are many more comparable [2] as well as analytic tuning methods [12] reported and publications on comparing the robustness and performance of these well-known PID tuning formulas are available [13].

Due to increased application of PID hardware modules in commercial and industrial applications [14] a major research focus in recent years has been on the development of various optimization based [15] and automated PID tuning approaches [16]. Resulting optimization based tuning methods mainly rely on the maximization of stability margins [15], shaping the loop transfer function (TF) [17], or the optimization of closed-loop properties like bandwidth [18] or control activity [19].

Particularly in mechatronic positioning systems PID controllers are employed and tuned for a wide variety of applications and tasks, including motion control of fast steering mirrors (FSMs) [20] and atomic force microscopes (AFMs) [16], and disturbance rejection in active vibration isolation systems [21], [22]. There are also analytic methods for the design of PID controllers for motion systems reported, that tune the controller to the requirements of a particular trajectory [23]. In practice often a trade-off between robustness and performance of the closed-loop system has to be made. With many of the reported tuning methods it is, however, hardly possible to perform this tradeoff in an analytic and intuitive way, that clearly shows the effects of the parameter tuning on these two aspects.

This paper presents a novel loop shaping based tuning approach for PID controllers, which allows a direct and
intuitive tradeoff between robustness and performance (e.g. tracking performance) of the closed-loop controlled system by tuning a single parameter. The method is applicable for tuning controllers of low stiffness mechatronic systems that are showing a double integrator characteristic above the suspension mode and are typically controlled on their mass line (see Fig. 1). This system class includes many mechatronic positioning applications, ranging from wafer scanners [24] over fast steering mirrors [20] to CD player pickup heads and voice coil actuator based linear motion drives [25]. With the bandwidth typically determined by the specific target application, the parametrization of the tuning method reduces the number of tuning parameters of the PID controller to one. After a description of the controller parametrization and the effects of tuning the variable parameter, the method is experimentally demonstrated and evaluated by designing three differently tuned controllers and applying them to a low stiffness fast steering mirror (FSM) system.

II. PID Alpha Tuning Method

The targeted class of mechatronic motion systems is characterized by low or quasi-zero stiffness dynamics and is typically controlled on the mass line with a bandwidth above the suspension mode of the system (see Fig. 1). When designing PID controllers for this system class, three tuning parameters have to be adjusted:

- The P-gain is used to shift the magnitude slope of the loop gain to place the intersection between mass- and 0 dB line at the targeted cross-over frequency.
- The D-gain is tuned to ensure sufficient phase lead around the unity cross-over and is usually tamed at higher frequencies to reduce the control effort.
- The I-gain increases the loop gain at low frequencies, to achieve zero steady state error in the closed-loop system.

In practice the closed-loop system specifications of interest are typically bandwidth (speed, performance) and robustness (stability margins, parameter variation). It is, however, not intuitively clear how the three tuning parameters affect these specifications, such that an intuitive and simplified tuning procedure is desirable. Further, when tuning the controller gains independently, the different control actions may overlap and interfere with each other, mutually diminishing their desired action and the overall system performance.

The Alpha Tuning Method enables such an intuitive tuning and uses a parametrization for the controller gains that is based on the cross-over frequency $\omega_c$ and the tuning parameter $\alpha$ only (see Section II-A). The cross-over frequency $\omega_c$ is usually either maximized for high performance, typically limited by structural modes of the positioned mass (see Fig. 1), or fixed by the requirements of the respective application, e.g. targeted trajectory in a scanning system (highest harmonics) or required disturbance rejection performance in active vibration isolation systems (disturbance frequency components). The tuning parameter $\alpha$ adjusts the spectral distance between the corner frequencies that separate the control actions in the frequency domain and enables a direct tradeoff between performance and robustness of the closed-loop system.

For demonstration of the tuning method the low stiffness system from Fig. 1 is considered as plant (the structural modes are omitted) and the target cross-over frequency is fixed to $\omega_c = 400$ Hz. The system is a second order mass-spring-damper system

$$G(s) = K \cdot \frac{\omega_0}{s^2 + 2\zeta\omega_0 \cdot s + \omega_0^2}$$

with $K = 12.45$, $\omega_0 = 130.1$ rad/s and $\zeta = 0.06$.

A. Controller Parametrization

The parallel PID controller structure

$$C_{\text{pid}}(s) = k_p + \frac{k_i}{s} + k_d s$$

is considered as starting point for the controller tuning. The P-gain is used to vertically shift the loop gain in order to cross the 0 dB line at the targeted cross-over frequency $\omega_c$. It is thus found by using the inverse of the plant gain at the targeted cross-over frequency. To provide sufficient phase lead at cross-over a D-control part will, however, be required. The starting frequency of the D-action can be parametrized by using the cross-over frequency $\omega_c$ and the $\alpha$-value:

$$\omega_d = \frac{\omega_c}{\alpha}$$

Above this starting frequency the D-control introduces a phase lead and reduces the -2 mass line to a -1 slope, so that the P-gain needs to be reduced by this factor $\alpha$ in order to retain a loop gain of 1 at the targeted $\omega_c$. This yields a P-gain of

$$k_p = \frac{1}{\alpha \cdot |G(s)|_{s=j\omega_c}}$$

The D-gain is needed to add phase lead (phase margin) and damping to the loop TF around $\omega_c$. Looking only at the PD-part of the initial controller in (2), it can be seen that the D-term starts to dominate for $\omega > k_p / k_d$. Using this relation together with the previously mentioned parametrization of $\omega_d$ (3) the D-gain results to

$$k_d' = k_p \cdot \frac{\alpha}{\omega_c}$$

To limit the D-control part to frequencies around $\omega_c$ in order to provide a steeper roll-off of the loop TF at higher frequencies and to limit the control effort, the D-action is tamed. The D-control part is terminated by introducing an additional pole above the cross-over frequency at $\omega_d = \alpha \cdot \omega_c$. This results in an entire D-gain of

$$k_d = k_p \cdot \frac{\alpha}{\omega_c} \cdot \frac{1}{\pi \cdot \omega_c + 1}$$

The I-gain is introduced to increase the loop gain at low frequencies, in order to achieve zero steady-state error in the response to the reference and to disturbances. It can be neglected for quasi-zero stiffness systems in which the plant itself already provides sufficient loop gain at low frequencies. The I-control part adds 90° phase lag to the loop TF. To

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not affect the phase lead introduced by the D-control part the I-control needs to be terminated well below the starting point $\omega_d$ of the D-action. At the termination of the I-control, parametrized by $\omega_i = \omega_d/\alpha$, the term $k_i/s$ from (2) should be equal and at larger frequencies smaller than $k_p$. The I-gain can thus be found by

$$k_i = k_p \cdot \frac{\omega_d}{\alpha^2}. \quad (7)$$

**B. Variation of $\alpha$-Value**

By tuning the only remaining independent parameter $\alpha$ it is possible to directly tradeoff the robustness and performance of the closed-loop system. According to the previous parametrization, a value of $\alpha = 3$ places the I-control termination to $\omega_i = \omega_d/3$, and the start and the end of the D-control to $\omega_d = \omega_c/3$ and $\omega_i = 3\omega_c$, respectively. The resulting controller TF is depicted in Fig. 2 and the loop TF with the model from (1) results in a phase margin (PM) of $54^\circ$, according to Table I. Increasing the $\alpha$-value (e.g. $\alpha = 4.5$, see Fig. 2) leads to

- an increase of the D-control range and of the PM ($66^\circ$)
- a decrease of the controller gain at low frequencies and
- an increase of the controller gain at high frequencies.

This means that for large values of $\alpha$ the system becomes more robust, but the disturbance rejection and tracking performance at low frequencies is reduced. The increased gain at high frequencies additionally leads to an increased sensor noise feedback. Decreasing the $\alpha$-value (e.g. $\alpha = 2$, see Fig. 2) on the other hand results in

- a decrease of the D-control range and of the PM ($37^\circ$)
- an increase of the controller gain at low frequencies and
- a decrease of the controller gain at high frequencies.

This means that for small values of $\alpha$ the robustness of the system is reduced, but the performance at low frequencies is increased. The reduced gain at high frequencies also leads to a reduced sensor noise feedback.

**C. Robustness**

To demonstrate the previously mentioned effects of different $\alpha$-values on the robustness of the system the controllers with $\alpha$-values of 2, 3 and 4.5 are applied to the low stiffness system from (1). Values smaller than $\alpha = 1.5$ lead to PMs that are smaller than $20^\circ$, while at $\alpha$-values from 4.5 upwards the conjugate complex zeros terminating the I-control and starting the D-control are separating into two real valued zeros.

The effects of different $\alpha$-values on the loop TF is depicted in Fig. 3a. All loop TFs show the same cross-over frequency of $400 \text{ Hz}$, with $\alpha = 2$ showing the highest gain at low frequencies, the steepest slope at the cross-over and the lowest gain at high frequencies. The phase lead at the cross-over is significantly lower as compared to the higher $\alpha$-values. Additionally the maximum phase lead is not at the cross-over but at a higher frequency. This is due to the I-control part which is for $\alpha < 2.5$ still influencing the resulting phase lead at the cross-over frequency, shifting the maximum phase lead of the loop TF to increasingly higher frequencies with decreasing alpha value. The reduced phase margin clearly indicates that the system with a controller with $\alpha = 2$ is less robust to plant variations than when a controller with $\alpha = 4.5$ is applied. As an indicator for the robustness of the controlled system the resulting gain margins (GM) and PMs for different $\alpha$-controllers are listed in Tab. I.

**Table I**

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>PM [°]</th>
<th>GM [dB]</th>
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</thead>
<tbody>
<tr>
<td>1.5</td>
<td>21</td>
<td>8.2</td>
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<tr>
<td>2</td>
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<td>13.1</td>
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<td>58.8</td>
<td>23.7</td>
</tr>
<tr>
<td>4</td>
<td>62.6</td>
<td>27</td>
</tr>
<tr>
<td>4.5</td>
<td>65.3</td>
<td>30.3</td>
</tr>
</tbody>
</table>

As consequence to a smaller PM the peaking in the complementary sensitivity function of the closed-loop system (see Fig. 3b) is 5 dB for $\alpha = 2$. With $\alpha$-values of 3 and 4.5 the peaking can be reduced below 2 dB. The resulting -3dB bandwidth of the system varies from 615 Hz for $\alpha = 4.5$, to 706 Hz for $\alpha = 3$ and 770 Hz for $\alpha = 2$.

**D. Performance**

From the sensitivity function of the closed-loop system in Fig. 3c it can be seen that with a controllers with $\alpha = 2$ the disturbance rejection performance up to about 200 Hz is clearly superior to controller with lower $\alpha$-values. As a consequence of the Waterbed-effect [5] the disturbance rejection performance is however diminished from 200 Hz up to 1.5 kHz with a maximum disturbance amplification...
of 4.4 dB at 550 Hz. A more robust controller design with an $\alpha$ of 3 or 4.5 shows less performance at low frequencies but also a reduced disturbance amplification at higher frequencies.

The effects of different $\alpha$-values on the performance in the time domain is investigated by evaluating the systems step response as measure for a set-point change (see Fig. 4a). While the system with controllers with an $\alpha$ of 3 and 4.5 shows moderate overshoot of 25% and 15%, respectively, and aperiodic transients, the system with the controller with $\alpha = 2$ shows large overshoot of 50% and longer transients. The shortest settling time was achieved with $\alpha = 3$.

Considering a plant with up to 50% mass uncertainty, the benefits of a robust controller design can be demonstrated. A plant with a 50% larger inertial mass leads to a decrease of the loop gain at frequencies above the suspension mode due to a lowered mass line. This means that the actual crossover frequency is smaller than the one the controller was designed for, leading to a decreased PM. Fig. 4b depicts the step response with the designed controllers and the plant with increased mass. It shows that with the least robust controller ($\alpha = 2$) a significant increase of the oscillating transients can be observed, while with the more robust controllers, apart from a slightly increase in settling time and overshoot, the transient behavior of the step response remains basically unchanged.

Similar investigations in the time domain with comparable results can be done for e.g. the tracking error of a raster.
trajectory in scanning systems or evaluation or the remaining position uncertainty in disturbance rejection applications.

III. PID ALPHA CONTROLLER FOR FSM

A. Experimental setup

To experimentally investigate the performance with controllers of different $\alpha$-values a single axis of a commercial FSM (Type: OIM101, Optics in Motion LLC, Long Beach, USA) with a maximum range of +/-26.2 mrad (+/-1.5 deg) is used as system plant. To measure the mirror rotation for closed-loop operation the FSM has an internal optical sensor system. The system axis is actuated by two voice coil actuators (moving magnet type), which are operated in a push-pull configuration. The actuator coils are driven by a custom made current amplifier (OPA544T, Texas Instruments Inc., Dallas, TX, USA) with a bandwidth of 10 kHz. The controller implementation is done on a dSpace platform (Type: DS1202, dSPACE GmbH, Germany) running with a sampling frequency of $f_s = 50$ kHz.

To identify the system dynamics a system analyzer (3562A, Hewlett-Packard, Palo Alto, CA, USA) is used. The input of the amplifier is considered as the system input and the signal of the internal sensor represents the system output. The mirror, the amplifier, and the internal sensor are thus together considered as the plant. To model the measured frequency response a second order model

$$P(s) = G(s) \cdot e^{-T_s},$$  

with $G(s)$ from (1) and $K = 12.45$, $\omega_0$=130.1 rad/s and $\zeta=0.06$ is fitted. The dead time $T_d=1e-4$ is used to model the phase loss due to the internal sensor and the sampling of the digital system.

B. Controller Design

To demonstrate the explained performance tradeoffs three controllers with different $\alpha$-values are designed and implemented for the experimental plant, following the method presented in Section II. As in Section II the smallest $\alpha$-value is chosen to be 2 ($\alpha = 1.5$ would already result in a PM of less than $5^\circ$). Additionally $\alpha = 3$ is chosen as a moderate and $\alpha = 4.5$ as a higher value. For reasons of implementation the I-control is tamed below 1 Hz. The resulting controllers are of the form

$$C_{\alpha}(s) = K \frac{s^2 + 2\zeta\omega_n s + \omega_n^2}{(s + \omega_{n1}) \cdot (s + \omega_{n2})},$$

with parameters according to Table II.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$K$</th>
<th>$\omega_0$ [rad/s]</th>
<th>$\zeta$</th>
<th>$\omega_{n1}$ [rad/s]</th>
<th>$\omega_{n2}$ [rad/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>40.2</td>
<td>795</td>
<td>0.71</td>
<td>5.03</td>
<td>6.28</td>
</tr>
<tr>
<td>3</td>
<td>34.7</td>
<td>409</td>
<td>0.88</td>
<td>7.54</td>
<td>6.28</td>
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<tr>
<td>4.5</td>
<td>27.9</td>
<td>257</td>
<td>1.08</td>
<td>1.18</td>
<td>6.28</td>
</tr>
</tbody>
</table>

C. Experimental Validation

For evaluation of the system performance and robustness with the three designed controllers, the resulting stability margins, the complementary sensitivity function and the response to a set-point change are investigated and compared.

Fig. 5 depicts the gain and phase margins of the open loop system resulting from a simulation using the plant model $P(s)$ from (8) and controllers with $\alpha$-values of 1.5, 2, 2.5, 3, 3.5, 4, and 4.5. Additionally the measured margins of the FSM with the implemented controllers are shown, demonstrating good agreement with the simulated curves. The slight deviations in the gain margin are results of unmodeled structural modes of the system. It can be seen that below an $\alpha$-value of 3 the stability margins decrease very quickly, while above $\alpha = 3$ there is only a comparably slight increase of GM and PM observable, suggesting $\alpha = 3$ (PM = 40°) as a good tradeoff value for a robust controller.

In Fig. 6 the complementary sensitivity functions of the closed-loop systems are shown. As discussed in Section II-C the peaking of the TF increases for lower $\alpha$-values due to the reduced PM of the open loop and is 7.4 dB for $\alpha = 2$ ($\alpha = 4.5$ yields 1.9 dB). The -3dB bandwidths are ranging from 785 Hz for $\alpha = 4.5$ to 813 Hz for $\alpha = 2$. Above 1 kHz unmodeled structural modes of the plant are observable, which have, however, no influence on the performance.
The responses to a set-point change of the closed-loop system are shown in Fig. 7a. It can be seen that the system with the less robust controller for \( \alpha = 2 \) yields the largest overshoot (40%) but also the shortest settling time of 5 ms. The system with the controller for \( \alpha = 3 \) shows significantly less overshoot (11%) at a still moderate settling time of 10 ms. The system with the controller for \( \alpha = 4.5 \) shows only an overshoot of 5% but has also a significantly longer settling time of more than 20 ms. To demonstrate the robustness of the three systems with respect to uncertainties of the system mass, the measured step responses of a plant with 50% larger nominal mass and the same controllers is depicted in Fig 7b. It can be seen that the system with the less robust controller shows an enlarged overshoot of about 70% and a significant increase of oscillating transients, that also increases the settling time to more than 13 ms. The system with the other controllers shows overshoots well below 25% and reduced settling times due to the reduced PMs. Both step responses show excellent agreement with the simulated responses in Fig. 4.

![Fig. 7. Measured step responses using controllers with \( \alpha \)-values of 2, 3, and 4.5. (a) shows the nominal plant with the accordingly designed nominal controllers applied. (b) shows a plant with a 50% increased inertial mass, still applying the same nominal controllers.](image)

An evaluation of the positioning uncertainty after the transients also revealed that the noise in the system output signal increases from 5.33e-4 Vrms for \( \alpha = 2 \) and 4.97e-4 Vrms for \( \alpha = 3 \) to 3.6e-3 Vrms for \( \alpha = 4.5 \) (Section II-B).

In summary it is shown that the presented tuning method for PID controllers can be applied to directly and intuitively tradeoff the robustness and performance of a feedback controlled mechatronic low stiffness motion system.

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REFERENCES