Proceedings of the Conference on Mathematical Foundations of Informatics
MFOI2017, November 9-11, 2017, Chisinau, Republic of Moldova

P Systems with Random RHS Exchange

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Abstract

P systems are a model of hierarchically compartmentalized multiset rewriting. We present a novel kind of P systems in which rules are dynamically constructed in each step by non-deterministic pairing of left-hand and right-hand sides. It turns out that this variant enables non-cooperative P systems to generate exponential (and thus non-semilinear) number languages.

1 Introduction and Preliminaries

For a comprehensive overview of different variants of P systems and their expressive power we refer the reader to the handbook [3], and for a state of the art snapshot of the domain to the P systems website [5] as well as to the Bulletin series of the International Membrane Computing Society [4].

Dynamic evolution of the set of available rules has been considered from the very beginning of membrane computing. Already in 1999, generalized P systems were introduced in [2]. We remark, however, that the previous studies on dynamic rule sets either treated the rules as atomic entities (symport/antiport of rules, operators in generalized P systems), or allowed virtually unlimited possibilities of tampering with their shape (polymorphic P systems). In the present work, we propose a yet different approach which can be seen as an intermediate one.

In *P systems with randomized rule-right-hand sides* (or with randomized RHS, for short), the available left-hand sides and right-hand sides of rules are fixed, but the associations between them are *re-evaluated in every step*: a left-hand side may pick a right-hand side arbitrarily (randomly).

In this extended abstract, we focus on the expressive power of P systems with randomized RHS, as well as on comparing them to the classical model with or without cooperative rules. One of the central conclusions of the present work is that non-cooperative P systems with randomized RHS can generate *exponential* number languages, thus (partially) surpassing the power of conventional (transitional) P systems. More details about the possible definitions of P systems with randomized RHS as well as about their expressive power can be found in the original article [1].

2 P Systems with Random RHS Exchange

In this variant of transitional P systems, rules randomly exchange right-hand sides at the beginning of every evolution step. This variant was the first to be conceived and is the closest to the classical definition [1].

A *transitional P system with random RHS exchange* is a construct

$$\Pi = (O, T, \mu, w_1, \dots, w_n, R_1, \dots R_n, h_o),$$

where the components of the tuple are defined as in the classical model.

As different from conventional transitional P systems, Π does not apply the rules from R_i directly. Instead, Π *non-deterministically permutes* the right-hand sides of rules in each membrane i, and then applies the obtained rules according to the maximally parallel semantics.

The conventional (total) halting condition for P systems can be naturally lifted to randomized RHS: a P system Π with randomized RHS halts on a configuration C if, however it permutes rule right-hand sides, no rule can be applied in C, in any membrane.

Example 1. *Consider the P system* $\Pi_2 = (\{a, b\}, \{b\}, [_1\ ]_1, a, R, 1)$ *with the rule set* $R = \{a \to aa, c \to b\}$*.* Π_2 *is graphically represented in Figure 1.*

$$\boxed{\begin{array}{c} a \to aa \\ c \to b \\ a \end{array}}_1$$

Figure 1. The P system Π_2 with random RHS exchange generating the number language $\{2^n \mid n \in \mathbb{N}\}$.

The number language generated by Π_2 *(the set of numbers of instances of* b *that may appear in the skin after* Π_2 *has halted) is exactly* $\{2^n \mid n \in \mathbb{N}^+\}$*. Indeed, while* Π_2 *applies the identity permutation on the right-hand sides,* $a \to aa$ *will double the number of symbols* a*, while the rule* $c \to b$ *will never be applicable. When* Π_2 *exchanges the right-hand sides of the rules, the rule* $a \to b$ *will rewrite every symbol* a *into a symbol* b*. After this has happened, no rule will ever be applicable any more and* Π_2 *will halt with* 2^n *symbols* b *in the skin, where* $n+1$ *is the number of computation steps taken.*

We will use the notation

$$OP_n(rhsExchange, coo)$$

to denote the family of transitional P systems with random RHS exchange, with at most n membranes, with cooperative rules.

The following statement is one of the central results of the article [1].

Theorem 1.

$$\{2^m \mid m \in \mathbb{N}\} \in NOP_n(rhsExchange, ncoo) \setminus NOP_n(ncoo).$$

Proof. The statement follows (for $n \geq 1$) from the construction given in Example 1 and from the well-known fact that non-cooperative P systems operating under the total halting condition cannot generate non-semilinear number languages (for example, see [3]). □

References

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