Value (in)definiteness and contextuality

http://tph.tuwien.ac.at/∼svozil/publ/2018-Svozil-Prague2018-pres.pdf

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Some questions one could ask, and answers one might expect

- Does a(n empirical) structure of propositions (logic) uniquely induce a probability? No!
- What kind of non-Boolean (non-classical) structure of propositions can one imagine?
  - Partition logic (Svozil 1993, Dvurečenskij, Pulmannová&Svozil, 1995); models include
    - Wright's generalized urn model as well as (Wright, 1978, 1990)
    - Moore's finite automaton state identification problem (Moore, Svozil 1993, Svozil&Schaller, 1995,1996).
  - quantum logic (Hilbert lattices)
  - general logics constructed by the pasting of Boolean subalgebras (contexts, blocks)
- What criteria/axioms to assume for probabilities? Classical Kolmogorov's probability theory for classical bits & pieces or blocks. "Stitching" or "pasting" of these blocks. Eg, Gleason-type frame functions on quantum contexts: additivity of mutually exclusive events, totally (im)probable events have probability 0 and 1, respectively

(Geometric) Strategies to classical probabilities

- Froissart (1981), Pitowsky (1986), Tsirelson (1993): geometric interpretation of probability distributions as the surface of a convex polytope "spanned" by vertices aka "mutually exclusive extreme cases."
  - The vertices are encoded by two-valued states on the logic.
  - The Bell-type face (in)equalities indicating "inside-outside relations" are very similar to Boole's "conditions of possible experience" (1854,1862). The *hull problem* of finding these faces is NP-complete in the number of vertices.
- Wright (1978,1990): Quasiclassical probabilities are in the convex hull of the dispersion-free two-valued states (aka classical truth assignments).

Heuristic use of the terms value indefiniteness and contextuality

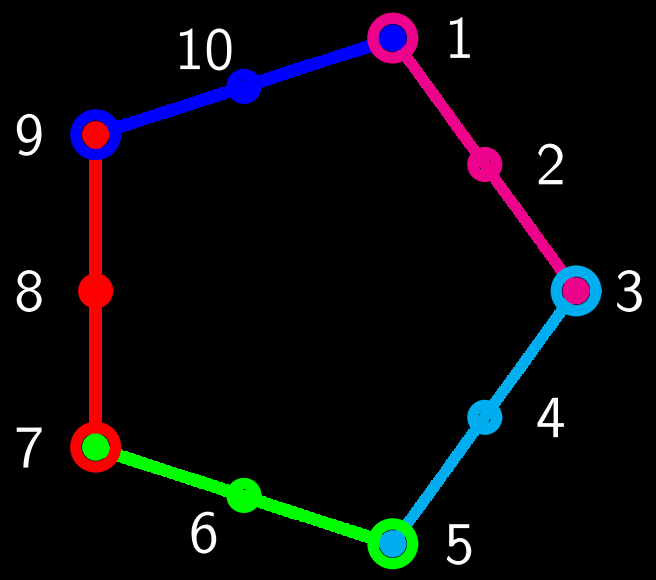
Heuristically, a collection of observables (hypothetical propositions) can be called *contextual* if (in order of severe deviation from classicality) it

- "somehow" is not classical; eg. exhibits complementarity (non-distributivity);
- has no (quasi)classical probability interpretation in terms of the convex hull of the (Kochen&Specker 1967, Theorem 0: separating) set of classical truth assignments;
- is partial (unless inconsistent) and thus value indefinite (Gleason 1957, Zierler&Schlessinger 1965, Kochen&Specker 1967, Pitowsky 1998, ACCS, ACS, 2012-2015).

Trigger warning

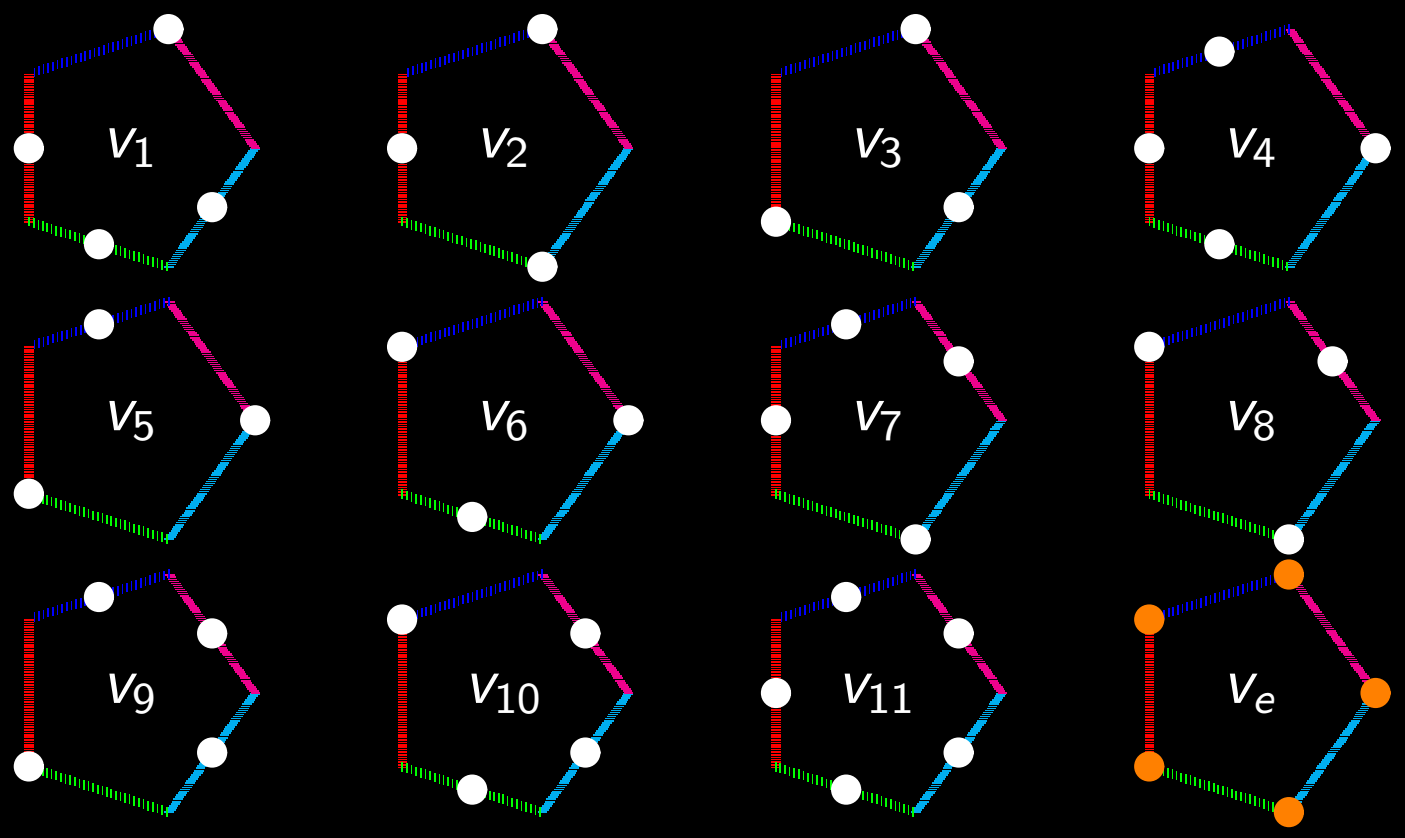
The term contextuality has been used by the realist John Bell to enforce value definiteness on intertwined collections of observables (grouped into maximal classical subalgebras called "contexts," "cliques" or "blocks") at the price of their dependency on the measurement context – thereby effectively discarding the assumption of an "isolated" observable (from whatever is effectively co-measured alongside of it). This is in contrast to the quantum (logical) formalism, in particular to elementary propositions identified with perpendicular projection operators.
Thus the current wide use of the term contextuality is often-times confusing and distractive: many researchers using the term would not like it to imply realism and context dependence of quantum observables.

Example I: Pentagon logic

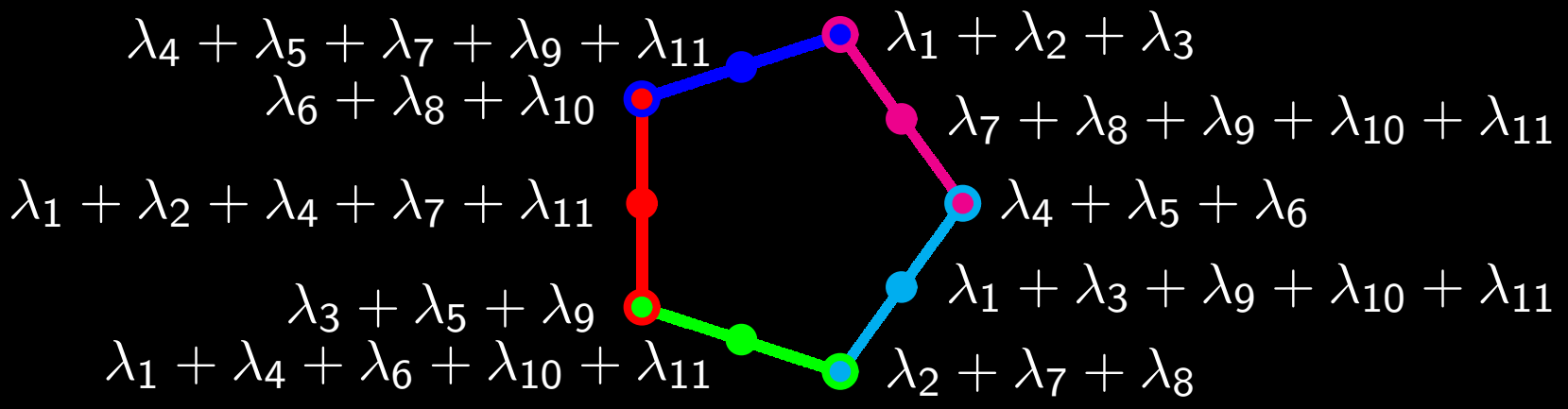


#	1	2	3	4	5	6	7	8	9	10
v_1	1	0	0	1	0	1	0	1	0	0
v_2	1	0	0	0	1	0	0	1	0	0
v_3	1	0	0	1	0	0	1	0	0	0
v_4	0	0	1	0	0	1	0	1	0	1
v_5	0	0	1	0	0	0	1	0	0	1
v_6	0	0	1	0	0	1	0	0	1	0
v_7	0	1	0	0	1	0	0	1	0	1
v_8	0	1	0	0	1	0	0	0	1	0
v_9	0	1	0	1	0	0	1	0	0	1
v_{10}	0	1	0	1	0	1	0	0	1	0
v_{11}	0	1	0	1	0	1	0	1	0	1
v_e	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0

Example I: two-valued states on the pentagon logic (Wright, 1978)



Example I: Probabilities on partition logics from two-valued states on the pentagon logic – with $\lambda_i \geq 0$, $i = 1, \ldots 11$, $\sum_{i=1}^{11} \lambda_i = 1$



Example I: hull computation on the pentagon logic

The full hull computations for the probabilities $p_1, \ldots, p_{10}$ on all atoms $1, \ldots, 10$ reduces to 16 inequalities, among them

$$\begin{aligned} p_4 + p_8 + p_9 &\geq +p_1 + p_2 + p_6, \\ 2p_1 + p_2 + p_6 + p_{10} &\geq 1 + p_4 + p_8. \end{aligned} \tag{1}$$

If one considers only the five probabilities on the intertwining atoms, then the Bub-Stairs) inequalitiy (Bub, 2009)

$$p_1 + p_3 + p_5 + p_7 + p_9 \leq 2 \tag{2}$$

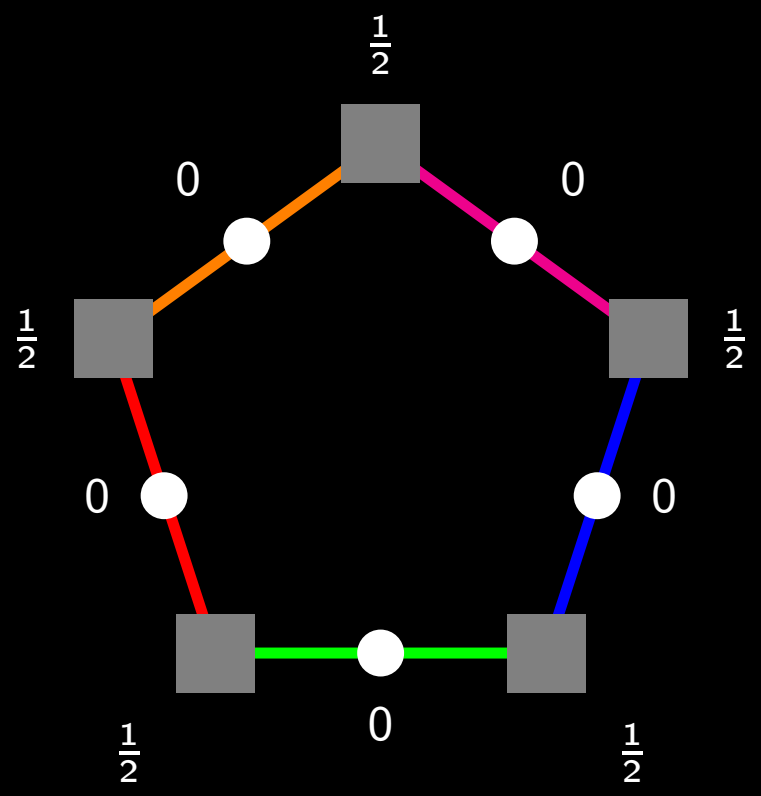
result. Concentration on the four non-intertwining atoms yields

$$p_2 + p_4 + p_6 + p_8 + p_{10} \geq 1. \tag{3}$$

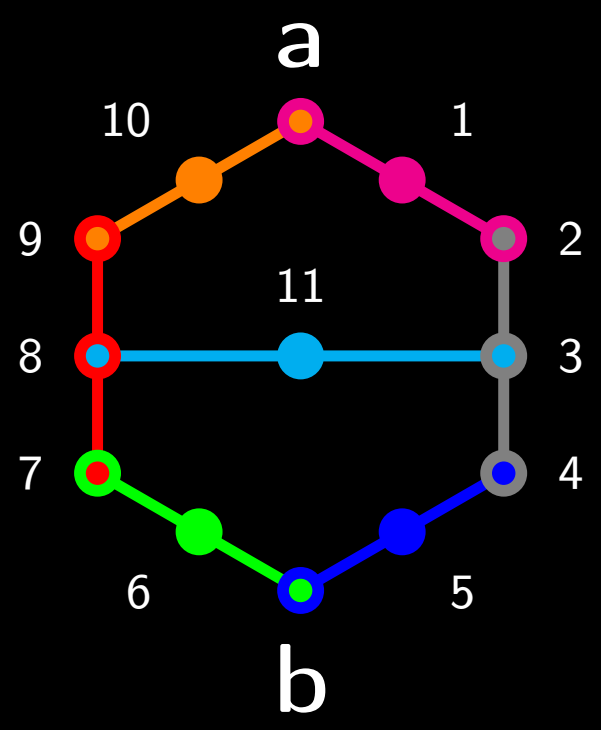
Limiting the hull computation to adjacent pair expectations of dichotomic ± 1 observables yields the Klyachko-Can-Biniciogolu-Shumovsky inequality (2008)

$$E_{13} + E_{35} + E_{57} + E_{79} + E_{91} \geq 3. \tag{4}$$

Examples I: Wright's 12th dispersionless measure on the pentagon: neither quasi-classical nor quantum



Examples II: Specker's "Käfer" (bug) combologic (Kochen&Specker, 1965, 67) - true (1) implies false (0) / true (1) / inseparable logics



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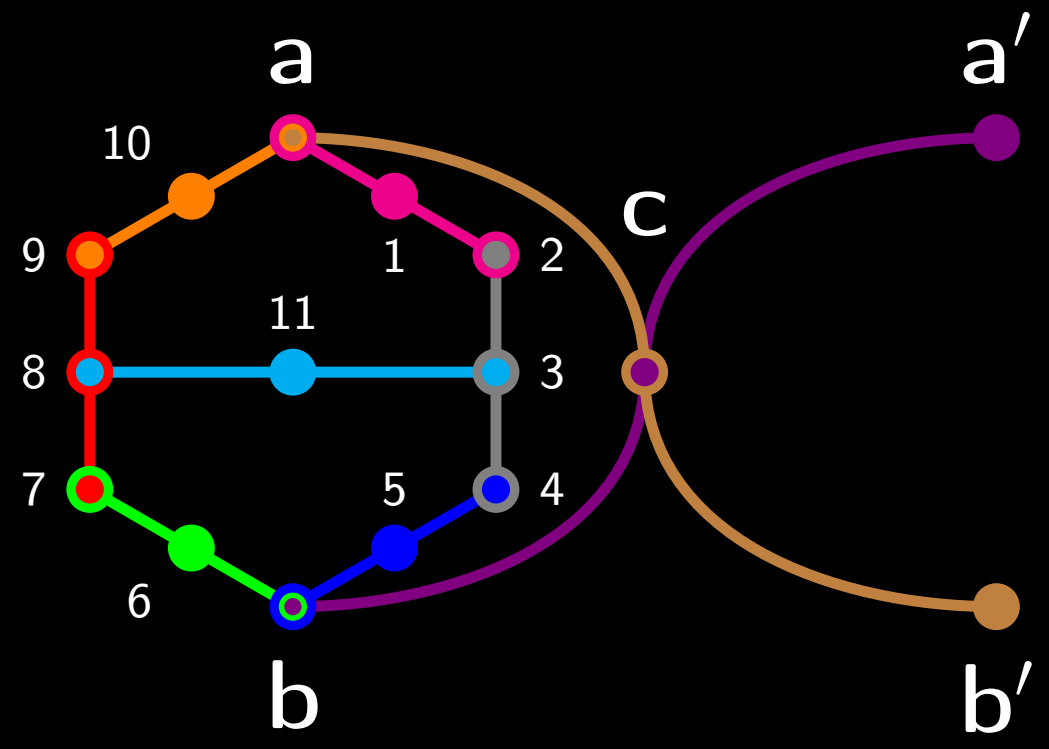

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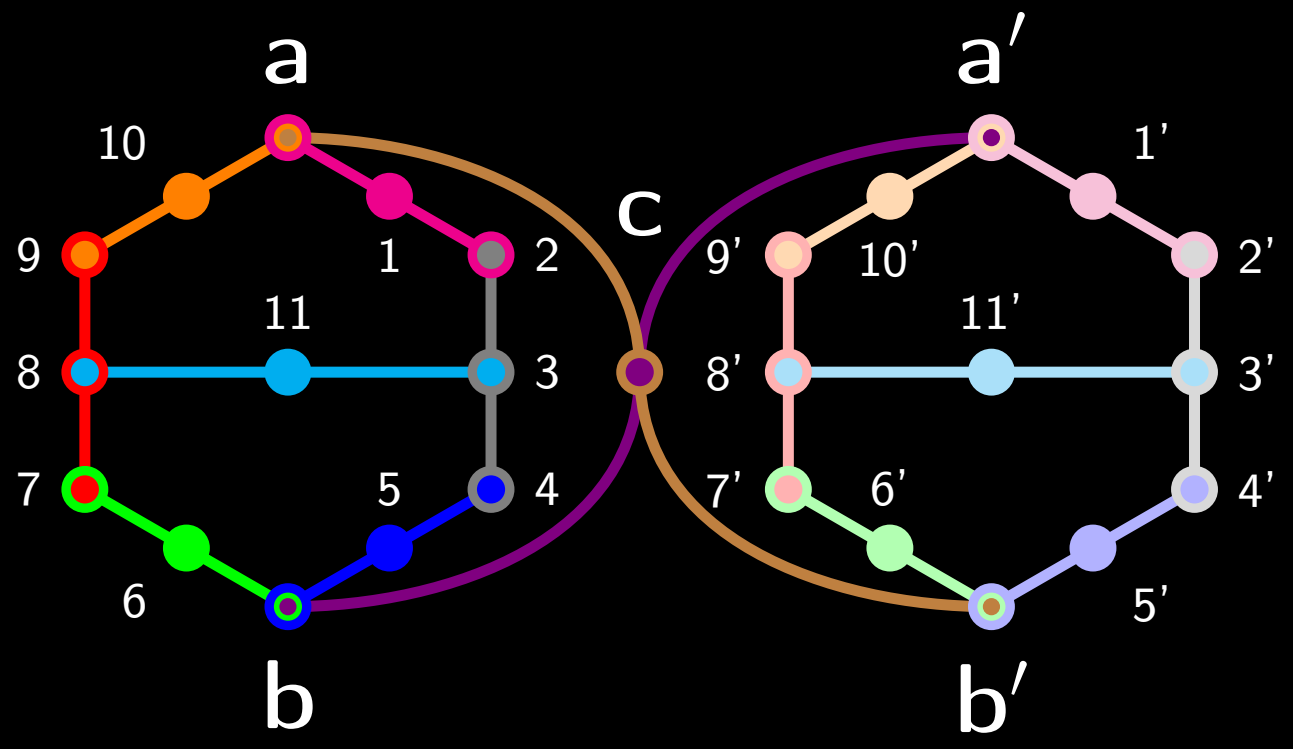
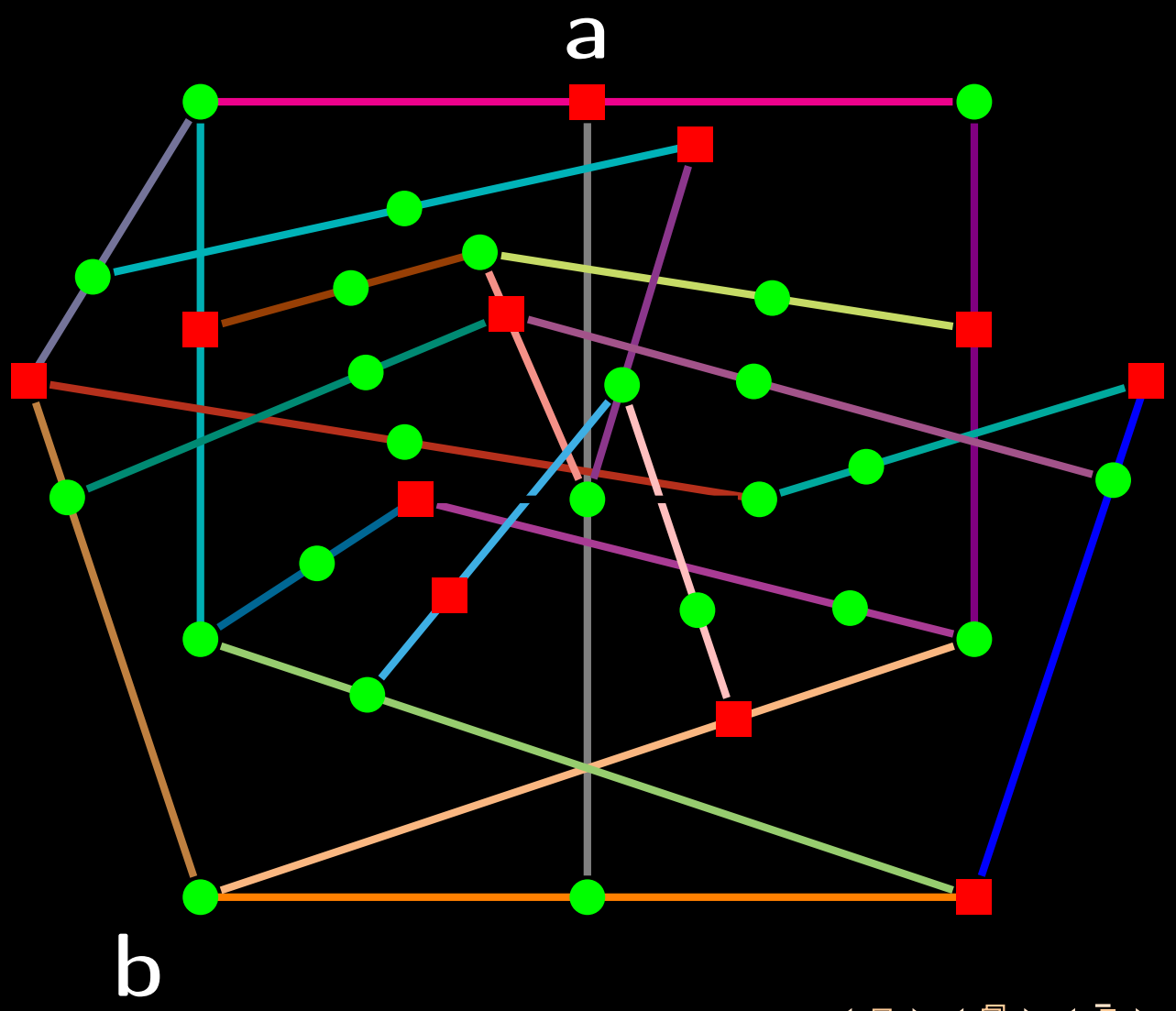


Example III: True (1) implies false (0) / true (1) and value indefinite / partiality logics (Abbott, Calude, Svozil 2015, Svozil 2018)



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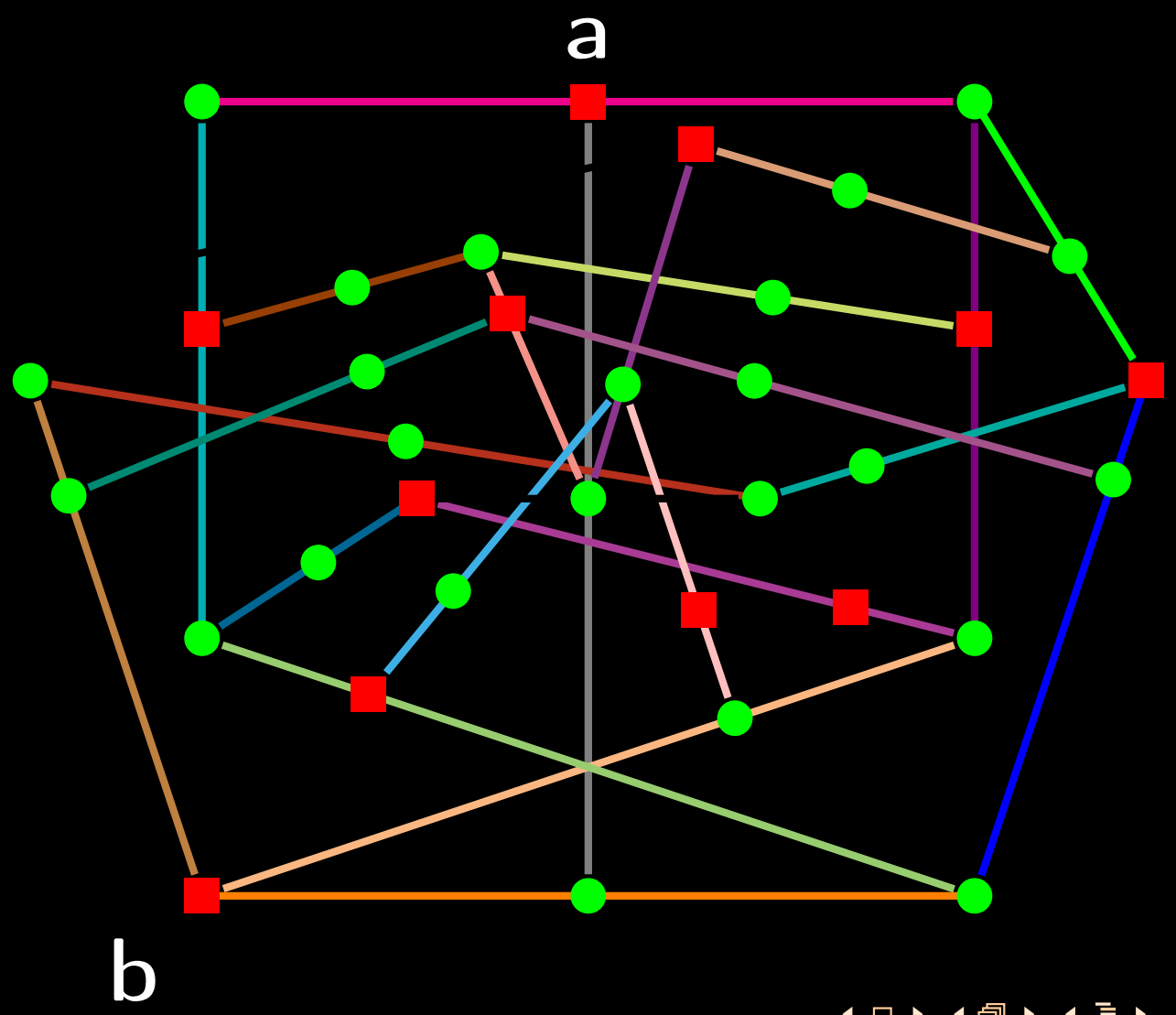

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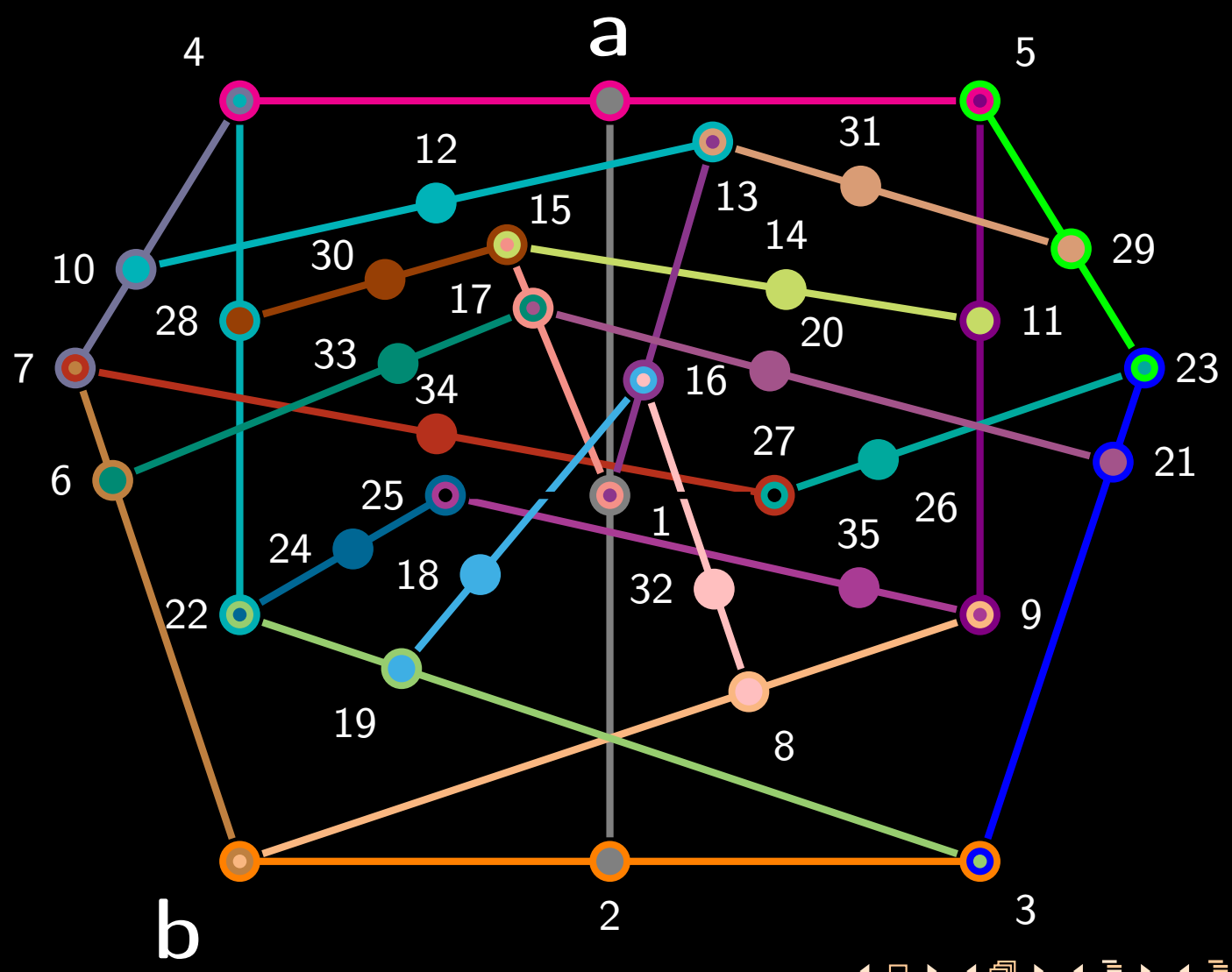

Strategies to obtain value indefiniteness / partiality

The scheme of the construction & proof of partiality of value assignments is as follows:

(i) Find a logic (collection of intertwined contexts of observables) exhibiting a true-implies-false property on the two atoms **a** and **b**.

(ii) Find another logic exhibiting a true-implies-true property on the same two atoms **a** and **b**.

(iii) Then join (paste) these logics into a larger logic, which, given **a**, neither allows **b** to be true nor false. Consequently **b** must be value indefinite.

ps: the Abbott, Calude, Svozil 2015 logic introduced earlier has a quantum (Hilbert space) realization of $|\mathbf{a}\rangle = (1,0,0)^{\mathsf{T}}$, and $|\mathbf{b}\rangle = \left(\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}\right)^{\mathsf{T}}$; Thus the probability to observe **b** given **a** is 50:50.

pps: partiality /value indefiniteness /strong contextuality can be extended to any vector non-collinear and non-orthogonal to **a**.

Meaning of value indefiniteness/partiality of the truth assignments in view of the arbitrariness in the choice of logics connecting **a** and **b**

Given **a**, there exist a continuum of possible pathways to **b**; many with very different "contextual" properties.
Since the selection of these pathways (or logics connecting them) is merely hypothetical, personal and thus epistemic, "I am inclined to believe" (cf. Born 1926) that in the quantum world there does not exist an observable **b**, given **a**, unless both are orthogonal or collinear. What we effectively do when we allegedly "measure **b**" are just "translations" of properties which are relational (via entanglement throuh interaction) to **a**, mediated through our measurement devices; thereby we fapp introduce stochasticity because of the many uncontrollable degrees of freedom of the latter.
Cf. https://arxiv.org/abs/1804.10030

Thank you for your attention!