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HARMONIC SIGNAL ESTIMATION IN THE TIME DOMAIN FOR ON-LINE ELECTROCHEMICAL IMPEDANCE SPECTROSCOPY

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Abstract **– On-line monitoring of the electrochemical impedance typically involves the buffering of streaming data and subsequent use of the fast Fourier transform. This leads to an inherent trade-off between time and frequency resolution and biased amplitude estimates due to frequency leakage. In this work, time-domain estimation techniques, namely recursive least squares and Kalman filtering, are utilized for the estimation of amplitude and phase of harmonic output responses and its advantages are discussed. Furthermore, it is shown how the Kalman filter can be parameterized to filter out disturbances, present in any industrial application, which would otherwise lead to distorted estimates when applying the fast Fourier transform.**

Index Terms **– electrochemical impedance spectroscopy, on-line harmonic signal estimation, recursive least squares, Kalman filter**

I. Introduction

The electrochemical impedance spectroscopy (EIS) is a potent tool in fuel cell diagnostics. Complete impedance spectra are typically obtained by single sinusoidal excitations with increasing frequency and subsequent off-line estimation of amplitude and phase of the harmonic output response. When real-time impedance information during dynamic operating conditions are desired, it is advantageous to continuously monitor the impedance at fixed frequencies of interest [1-2]. Thereby, streaming current and voltage data is buffered into windowed segments for which the fast Fourier transformation (FFT) is applied. The inherent trade-off, when using the sliding window FFT, between time and frequency resolution is well known, as an increase of the frequency resolution is only obtained by increasing the window length, which also increases the time it takes to fill the data buffer. Additionally, when the frequency of a sampled harmonic signal is not coinciding with a point on the frequency resolution grid of the FFT, spectral leakage is present leading to distorted estimations. Although, by applying windowing functions or using overlapping data segments the influence of the aforementioned problems can be reduced, the question if there are alternatives better suited for the on-line estimation of harmonic signal parameters (and subsequently the electrochemical impedance) is of importance. In this work, time-domain techniques such as recursive least squares (RLS) and the Kalman filter [3] are investigated for the harmonic signal estimation.

II. Methodology

A. Harmonic Signal in Discrete Time Domain

A multi-harmonic signal with known angular frequencies $\omega_1 \dots \omega_N$ is described as

$$y(k) = \sum_{n=1}^{N} A_n \cos(\omega_N k\Delta t) + \sum_{n=1}^{N} B_n \sin(\omega_N k\Delta t). \qquad (1)$$

Thereby Δt denotes the sampling time and k the discrete time index. The amplitude and phase of the n-th harmonic signal content are given by

$$V_n = \sqrt{A_n^2 + B_n^2}, \qquad \varphi_n = \tan^{-1}\left(\frac{B_n}{A_n}\right). \qquad (2)$$

B. Recursive Least Squares

Note that the Eq. (1) is linear in its parameters A_n and B_n, which facilitates the use of computationally inexpensive linear parameter estimation techniques. After obtaining k samples and rearranging Eq. (1) in matrix form,

$$\boldsymbol{Y}(\mathrm{k}) = \boldsymbol{X}(\mathrm{k})\boldsymbol{\theta}(\mathrm{k}),$$
$$\boldsymbol{Y}(\mathrm{k}) = \begin{bmatrix} y(1) \\ \vdots \\ y(k) \end{bmatrix}, \boldsymbol{\theta}(\mathrm{k}) = \begin{bmatrix} A_1 \\ B_1 \\ \vdots \\ A_N \\ B_N \end{bmatrix} \qquad (3)$$
$$\boldsymbol{X}(k) = \begin{bmatrix} \cos(\omega_1 1\Delta t) & \sin(\omega_1 1\Delta t) & \cdots \\ \vdots & \vdots & \\ \cos(\omega_1 k\Delta t) & \sin(\omega_1 k\Delta t) & \cdots \end{bmatrix}$$

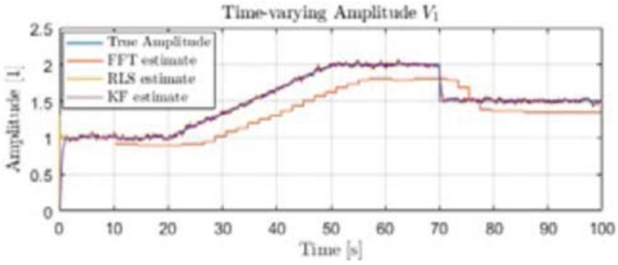

Fig. 1: Comparison of time-varying amplitude V_1 with estimates

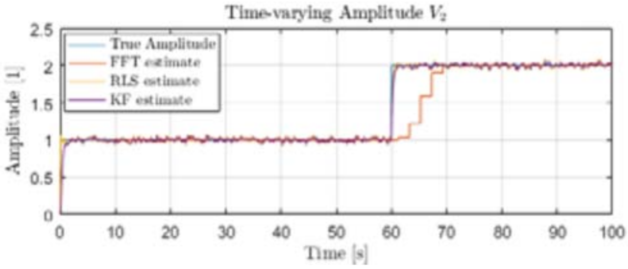

Fig. 2: Comparison of time-varying amplitude V_2 with estimates

the harmonic signal parameters can be estimated using least squares:

$$\boldsymbol{\theta}(\mathrm{k}) = \left(\boldsymbol{X}(\mathrm{k})^{\mathrm{T}}\boldsymbol{X}(\mathrm{k})\right)^{-1}\boldsymbol{X}(\mathrm{k})^{\mathrm{T}}\boldsymbol{Y}(\mathrm{k}). \quad (5)$$

To avoid the inefficient re-estimation of the parameters at the next sampling instance using the complete dataset, the recursive least squares algorithm is used:

$$\boldsymbol{\theta}(\mathrm{k}+1) = \boldsymbol{\theta}(\mathrm{k}) + \mathbf{K}_{\mathrm{R}}(k,\lambda)\left(y(k+1) - \boldsymbol{x}(k+1)^{\mathrm{T}}\boldsymbol{\theta}(\mathrm{k})\right). \quad (6)$$

Thereby, $y(k+1)$ and $\boldsymbol{x}(k+1)^{\mathrm{T}}$ is the measured signal and regressor line at the next time step, $\mathbf{K}_{\mathrm{R}}(k,\lambda)$ is the time dependent RLS filter gain, and λ is a weighting parameter on historic data, e.g. tuning parameter for the responsiveness and variance of the RLS algorithm.

C. Kalman Filter

As an alternative to the aforementioned RLS algorithm, the Kalman filter can be used. By defining Eq. (1) as the output equation, and by assuming a random walk model for the parameter evolution in the Kalman filter prediction step

$$\widehat{\boldsymbol{\theta}}(\mathrm{k}+1) = \widehat{\boldsymbol{\theta}}(\mathrm{k}) + \boldsymbol{w}(k), \quad (7)$$

where $\boldsymbol{w}(k)$ is a Gaussian process noise with zero mean and $\mathrm{cov}(\boldsymbol{w}) = \mathbf{Q}$, the parameter update equation has in general the same form as the RLS filter:

$$\boldsymbol{\theta}(k+1) = \boldsymbol{\theta}(k) + \mathbf{K}_{\mathrm{K}}(k,\mathbf{Q})\left(y(k+1) - \boldsymbol{x}(k+1)^{\mathrm{T}}\boldsymbol{\theta}(k)\right). \quad (8)$$

The process noise covariance matrix is thereby used as the tuning parameter in deriving the Kalman filter gain $\mathbf{K}_{\mathrm{K}}(k,\mathbf{Q})$.
By assuming

$$\mathbf{Q} = \mathrm{q}\mathbf{I}, \quad (9)$$

where $\mathbf{I}$ denotes the identity matrix**,** the performance of the Kalman filter is identical to the RLS algorithm.
The advantage of the Kalman filter approach lies in the fact, that by including a constant offset term in Eq. (1) and using the additional degrees of freedom in weighting, the Kalman filter can be parameterized to filter out additional disturbances present in the signal.

III. Results

A bi-tonal signal sampled with $\Delta t = 0.01[s]$, $\omega_1 = 2\pi 10\ [rad/s]$ and $\omega_2 = 2\pi 25\ [rad/s]$ is used to evaluate the performance of the RLS and Kalman filter as opposed to the FFT. The time varying amplitudes and its estimates can be observed in Fig.1 and 2.
The sliding window FFT utilized 1024 data points per windowed segment, 80% overlap and a Hann windowing function. In Fig. 1, a biased FFT estimate due to spectral leakage is observed, as the excitation frequency in ω_1 does not coincide with a discrete frequency on the FFT grid. Amplitudes estimated by the time-domain filter approaches are unbiased. Even in the unlikely event that the excitation frequency happens to exactly coincide with a point on the FFT grid (Fig. 2) it can be observed, that the convergence of the time-domain filters is significantly faster.

IV. Conclusion

In this work, time-domain filters are proposed to be used when on-line estimating the electrochemical impedance from multi-harmonic output signals as opposed to the sliding-window FFT. As the excitation frequencies in the time-domain approach can be chosen freely, the issue of biased estimates due to spectral leakage is resolved. Additionally, faster convergence to time-varying amplitudes is achieved, making this approach more suitable in dynamic operating conditions of the fuel cell

Acknowledgment

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