A shear stress driven rotary wave in a cylindrical container: stability of higher order modes and experimental results

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In vertical cylinder partially filled with water, a rotary gravity wave can be excited by rotating the top lid. Such waves have been observed in the draft tube of Francis turbines in phase condenser mode.

To study the instability mechanism the two-phase flow problem has been decomposed into flow problems for the air above the water and the water. For the base flow, the air flow is the well-studied problem of the flow in a cylinder with rotating lid. For the flow in the water, it is assumed that the air flow is given and exerts a given shear stress distribution in radial and azimuthal direction on the free water surface.

The stability problem of the base flow is controlled by the Ekman Ek number of the base flow and the Reynolds number Re of the rotary wave. For an asymptotic expansion of the rotary eigenmodes an expansion with respect to the Froude number $Fr = (Ek\,Re)^{-1}$, which is here a measure for the ratio of suitable reference values of the angular wave speed and the angular velocity of the base flow, is more appropriate.

To leading order, the inviscid first order Stokes waves are recovered. In a previous analysis[1] the the grow the rate of a wave mode is of the form $g \sim -g_{1/2}/\sqrt{Re} + Fr^2 g_2(Ek)$. The first term represents damping of the wave due to the viscous boundary-layers at the wall and bottom. The second term represents the interaction with the base flow. However, the stability analysis was limited to the lowest wave mode which resembles the shape of the fully developed wave very well. It was proposed that this lowest mode is unstable if there is an inward component of the rotary base flow. This is the case when the radial inward surface shear stress exceeds the azimuthal shear stress. However, under laminar flow conditions, the shear stresses are too small to excite a rotary gravity wave. Under turbulent flow conditions, CFD simulations of the air flow show that the azimuthal shear stress dominates and thus the flow on the water surface is directed outward, and the ground mode is stable. If the applied shear stress corresponds to an air flow in a relatively narrow gap between the water surface and the rotating disc, the coefficient g_2 of the growth rate is positive for higher order wavy modes.

We will discuss the stability of higher modes depending on the applied shear stress distribution. Moreover, we show experimental results which indicate that first, a wavy mode is excited. Then the wave form changes to lower order modes until finally the ground mode with high amplitude develops, see Figure 1.

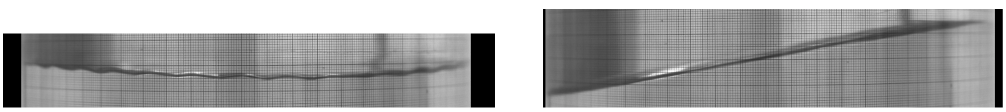

Figure 1: Initial wavy perturbation (left), fully developed wave (right)

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