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## STATISTICAL ENTROPY – AN INDICATOR FOR RESOURCE SYSTEMS

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### Introduction

In 1971 Nicholas Georgescu-Roegen introduced the term “low entropy materials” for valuable natural resources and “high entropy materials” for valueless waste and emissions. He explained the economy as a process which irreversibly transforms low entropy into high entropy materials by providing and using a flow of natural resources (Georgescu-Roegen 1971). Since then many others used the term entropy to describe the characteristics of the anthropogenic metabolism in a qualitative way. One of the first to make the entropy concept applicable to material systems was Szargut (1988), who developed a way to quantify exergy, a thermodynamic term introduced by Rant in 1956 to address available energy, for materials. Since entropy and exergy are directly coupled (the destruction of exergy causes entropy) this allowed to calculate exergy budgets for coupled energy/material systems and therefore indirectly consider entropy budgets. In this way exergy has been used by a growing community to evaluate and optimize technical processes and anthropogenic systems. Another way to employ entropy, Statistical Entropy Analysis (SEA), was developed by Rechberger and colleagues (Rechberger 1999, Rechberger & Brunner, 2002, Rechberger & Graedel 2002, Sobańtka et al. 2012, Laner et al. 2017). They used the statistical interpretation of entropy and made it applicable to results of Material Flow Analyses (MFA). The combination of MFA and SEA to analyse and evaluate systems can be seen as a straight forward application of the two Laws of Thermodynamics to material systems. MFA builds on the principle of mass conservation and SEA on the entropy law.

### Statistical Entropy Analysis (SEA) and Material Flow Analysis (MFA)

Statistical Entropy (SE) derives from Information Theory (Shannon 1948) and is used to describe any kind of distribution of a set of features. Rechberger & Brunner (2002) showed that a process can be perceived as an entity that transform materials, e.g., wood into emissions and ash. This transformation can be described as a transition of the distributions of mass flow and concentration data (see Figure 1). In this way SE can be applied to MFA results and quantifies the power of a process (system) to concentrate or dilute substances. Concentrating of substances is seen as a positive process, since concentrates can be better controlled and are easier to handle, to recycle, to recover or to landfill. Dilution is to some extent a necessary process for all living organisms (hence also for the economy), however, strong dilution reduces the availability to recover a material and might result in environmental damages. While diluting is usually a rather simple process the opposite, concentrating, is difficult and requires technology, energy, and effort. As a

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working hypothesis, which so far has been unrefuted, one can say that all anthropogenic activities should be carried out in a way that entropy generation is minimized.

Figure 1 shows the basics of SEA. The inputs and outputs of a process can be described as a distribution of substance concentrations and masses among the single flows. These distributions can be quantified by SE, resulting in normalized entropies ( $H_{rel}$ ). If the process increases  $H_{rel}$  then dilution takes place. The opposite case stands for concentration.

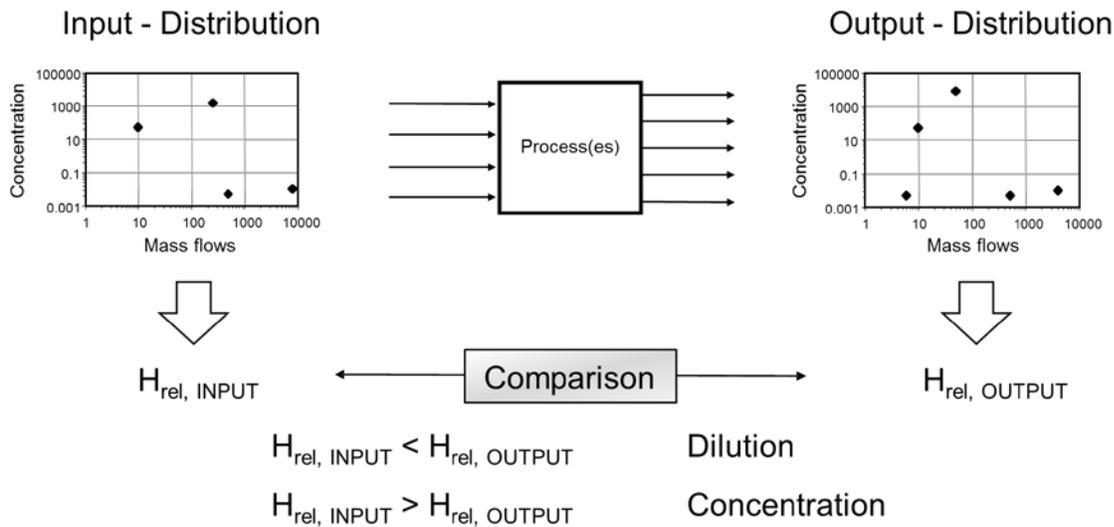


Figure 14 Basics of Statistical Entropy Analysis (after Rechberger & Brunner 2002)

### Theoretical application of SEA to MFA results

In order to apply SE to a MFA result the MFA system has to be translated into a stage diagram (cf. Figure 2). This procedure follows strict rules (cf. Rechberger & Graedel 2002; Laner et al. 2017) and allows following a substance along the system's process chain.

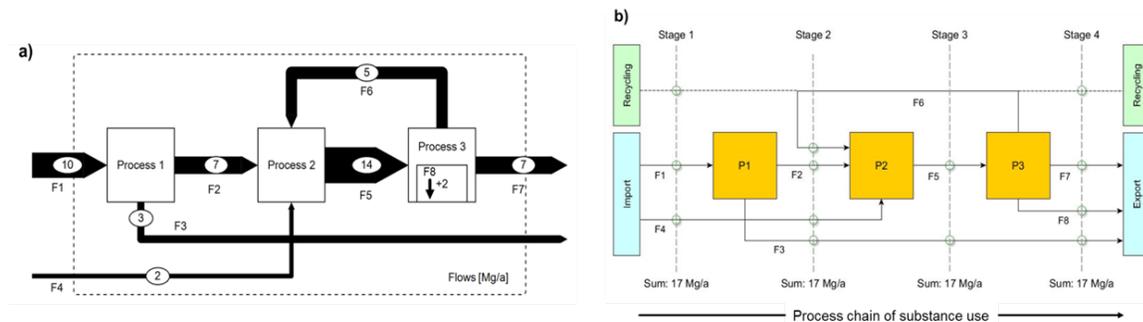
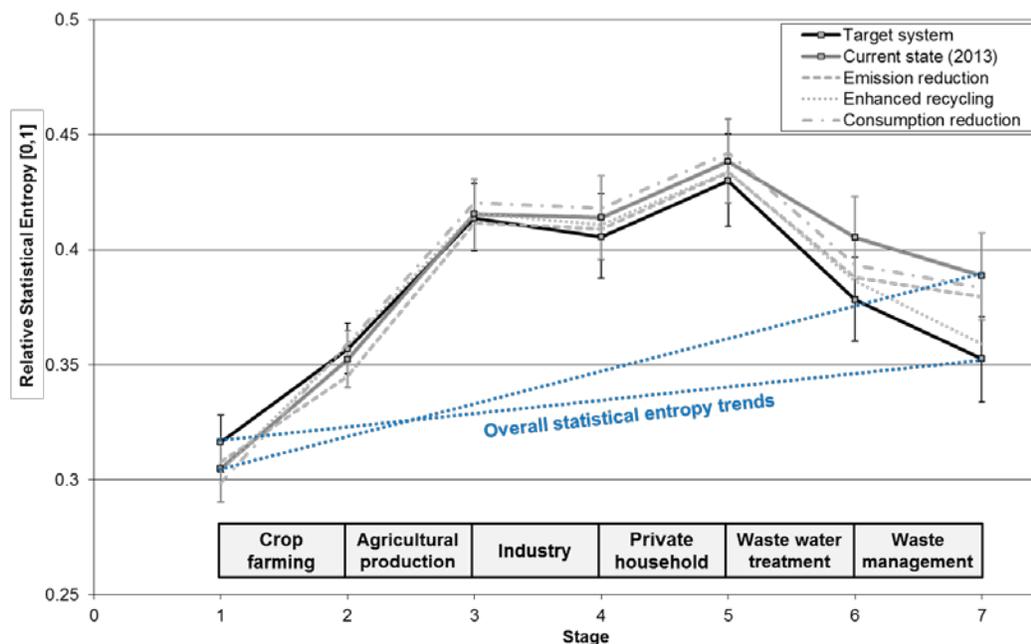


Figure 2 Transformation of a MFA system (a) into a stage diagram (b) (after Rechberger & Graedel 2002 and Laner et al. 2017)

The flows between the processes are organised in stages and the system as a whole can be considered as an entity transforming the throughput form stage to stage. Each stage is characterized by a specific distribution of mass flows and substance concentrations (cf. Figure 1) which can be quantified by SE.

### Selected results: the Austrian budget of Phosphorus

The Austrian budget for Phosphorus (P) was analysed in detail by Zoboli and colleagues (2016ab) and resulted in a rather complex system consisting of 56 processes, 122 flows, and 8 stocks. The system has been meanwhile analysed over a period of 24 years beginning with year 1990, thereby generating profound system understanding, which resulted in the identification of 15 measures to optimize the system. The measures were implemented in the model of the status quo and a hypothetical target system (optimized system) was compiled. Figure 3 shows the entropy trends for the current state (2013) and the target system. The optimization measures are grouped into three categories, namely reduction of consumption (1) and emissions (2) as well as enhanced recycling (3). One can see that the overall entropy trends are ascending indicating that even in the target system P is used in a dissipative manner overall. However, inefficiencies and losses could be reduced significantly and the main fields of action are in wastewater treatment, waste management and to some extent in industry. The comparison between current state and target system shows that the entropy increases in the agriculture are to a large extent unavoidable and they have to be compensated as much as possible by optimizations at the end of the P cycle.



**Figure 3** SEA of the Austrian Phosphorus household. The difference between the inclinations of the dotted lines (overall trends) indicates the potential for optimization (Laner et al. 2017).



One major advantage of applying SEA to MFA results is that one gets one single metric per stage, per process and per system, always indicating if dilution or concentration is dominating. However, SEA also allows for assessing the contribution of each single flow to the overall trend and so serves as a practical tool to analyse the relevance of single flows and specific measures of optimization. Generally the experience with the application of SEA shows that it enhances the system understanding, making clear which processes and flows are of particular relevance and it can also be used to decide about data quality requirements. For instance, the results displayed in Figure 3 show that the differences between current and target system are not significant for all stages indicating, that the MFA results suffer from uncertain basic data. This is a typical employment for Sensitivity Analysis to find out which data (uncertainties) have the largest impacts on the reliability of the final SEA results. In a following step it can then be checked how the identified data weaknesses could be resolved, which is valuable feedback to data producers and providers.

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