# About Accuracy and Influence of the Pole Coverage for Eddy Current Losses within Permanent Magnets of Electrical Machines 

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#### Abstract

In particular with surface mounted permanent magnets, eddy current losses within the magnets are one of the most significant portion of losses in permanent magnet exited synchronous machines. These losses are generated by asynchronous components of the air-gap field caused by either higher harmonic waves or higher time harmonics. On one hand, there is no interaction of the various harmonics with regard to these losses. On the other hand, the pole coverage shows a significant impact on these losses. Thus, detailed numerical analyses with various higher order formulations are carried out in order to show aspects of the accuracy of these eddy current losses, too.


Index Terms - Eddy current losses, Permanent magnet synchronous machine, High order finite element analysis.

## I. Introduction

TTHE precalculation of eddy current losses arising in surface mounted permanent magnets caused by sub- and superharmonics harmonics of the air-gap field is a matter of particular interest with the design process of permanent magnet excited electrical machines. On one hand by using very fast evaluation methods for the standard design procedures, on the other hand by using highly accurate calculation methods for reference purposes [1]-[3]. These eddy current losses may always lead to an excessive partial heating [4]-[7] and subsequently can cause the magnets to get partially or even fully demagnetised [8]-[11].

As shown in Fig. 1 and Fig. 2, both planar and cylindrical arrangements are described with only few parameters, such as air-gap $\delta$, ratio of pole pitch and air-gap $\tau_{p} / \delta$, ratio of magnet height and air-gap $h_{M} / \delta$ as well as the pole coverage as ratio of magnet width and pole pitch $b_{M} / \tau_{p}$. A surface current sheet in axial direction $K_{z}(x, t)$ at the inner stator boundary perpendicular to the cross section of the conducting region as

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\begin{equation*}
K_{z}(x, t)=\hat{K}_{z} \operatorname{Re}\left(\mathrm{e}^{j \omega t} \mathrm{e}^{-j \nu \pi x / \tau_{p}}\right) \tag{1}
\end{equation*}
$$

can cover for any harmonics generated from either the stator currents, the slotting as well as the saturation. Therein, $\omega=2 \pi f$ denotes the exciting circular frequency with respect to the moving region, $\nu$ the harmonic order and $-1 \leq x / \tau_{p} \leq 1$ being the region of two pole pitches along the circumferential direction, respectively.

Referring to the total eddy current losses, there is no interaction between waves with different harmonic orders as well as different frequencies. Therefore, each harmonic wave can be treated separately in particular in terms of its eddy current losses. The total eddy current losses generated from the spectrum of the harmonics are finally obtained by the sum of the losses of each harmonic wave.

In order to compare the various approximation orders of the finite element analyses and their influence on the accuracy of the numerically obtained results, an analytical calculation will be used for the reference results [12]. Additionally, various pole coverages with their effects on the eddy current losses of each harmonic wave are discussed by these numerical analyses.


Fig. 1: Simplified geometry of a pole pitch, planar arrangement.


Fig. 2: Simplified geometry of a pole pitch, cylindrical arrangement.

## II. General numerical results

The finite element analyses carried out with various higher order approximation functions utilise an identical
discretisation with the minimum skin depth as approximately the half of the mesh size in radial direction and the minimum wave length as approximately 7.5 times the mesh size in circumferential direction.

For the direct comparison of the analytical results with those from the numerical analyses, the finite element analyses are carried out with a pole coverage of $b_{M} / \tau_{p}=1$, which occurs practically with Halbach arrays.

Fig. 3 depicts the power losses of one NdFeB magnet in dependence on exciting frequency and ordinal number of the harmonics for a constant current sheet excitation of $\hat{K}_{z}=10^{4} \mathrm{~A} / \mathrm{m}$ as well as air-gap $\delta=2 \mathrm{~mm}$, ratio of pole pitch and air-gap $\tau_{p} / \delta=60$, ratio of magnet height and air-gap $h_{M} / \delta=3$. Fig. 4 shows the respective ratio of the power losses between cylindrical and planar arrangements.


Fig. 3: Power losses of various harmonics versus frequency, order $p=3$, pole coverage 1, planar arrangement.


Fig. 4: Ratio of power losses between cylindrical and planar arrangement, order $p=3$, pole coverage 1 .

With a ratio of wave length to skin depth $\left(2 \tau_{p}\right) /(\nu d) \ll 1$, the power losses versus frequency increase with a power of 2. On the other hand with a ratio of wave length to skin depth $\left(2 \tau_{p}\right) /(\nu d) \gg 1$, the power losses versus frequency increase with a power of 0.5 only. However with very low ordinal numbers, there is a transitional region where the power losses are rather constant.

## III. Accuracy of the numerical results

The relative error $\epsilon=P_{F E A} / P_{\text {ana }}-1$ between the power losses of finite element and analytical analyses with different approximation orders are shown in Fig. 5, Fig. 6, Fig. 7 and Fig. 8. In addition, Fig. 9 and Fig. 10 depict this relative error for $1^{\text {st }}$ and $2^{\text {nd }}$ orders with the half mesh size in both directions.


Fig. 5: Relative error of power losses, planar arrangement, pole coverage 1 , order $p=1$.


Fig. 6: Relative error of power losses, planar arrangement, pole coverage 1 , order $p=2$.

As expected, $1^{\text {st }}$ order elements cannot encounter both for small skin depths as well as short wave lengths. $2^{\text {nd }}$ order elements are better with an exception of short wave lengths and very high frequencies. $3^{\text {rd }}$ and $4^{\text {th }}$ elements give the same results with a relative error less than $0.5 \%$ which means convergence with respect to the higher orders.

In comparison of the default mesh with the half size mesh, of course the results of $1^{\text {st }}$ and $2^{\text {nd }}$ order elements are better with the dense mesh. However, the results of $2^{\text {nd }}$ order elements with the dense mesh are still less accurate than the results of in particular $3^{\text {rd }}$ order elements with the default mesh. Additionally, the latter have approximately only the half number of unknowns.

Consequently, the usage of $3^{\text {rd }}$ or even higher order elements will be strongly suggested by evaluating eddy current losses. In particular with 3D meshes, the possibility of generating a relatively coarse mesh within the conducting regions shows explicit advantages against a dense mesh with $2^{\text {nd }}$ order elements.


Fig. 7: Relative error of power losses, planar arrangement, pole coverage 1 , order $p=3$.


Fig. 8: Relative error of power losses, planar arrangement, pole coverage 1 , order $p=4$.

## IV. Influence of the pole coverage

The finite element calculations very easily allow to encounter for the influence of various pole coverages on the eddy current losses, too. With regard to a practical point of view, the pole coverages as of $5 / 6,3 / 4$ and $2 / 3$ are concerned for both arrangements in more detail.

Fig. 11, Fig. 12 and Fig. 13 depict the respective ratio of the power losses between cylindrical and planar arrangements. Fig. 14, Fig. 15 and Fig. 16 depict the ratio of the power losses with the above mentioned pole coverages in comparison to a full coverage.

Obviously, the pole coverage strongly affects the power losses of the very low harmonics $\nu=1 \ldots 7$ with frequencies above 100 Hz . On the other hand, the power losses of the harmonics $\nu>7$ are rather proportional to the value of the pole coverage with the entire range of frequencies.

## V. Conclusion

The paper discusses the numerical evaluation of eddy current losses in surface mounted permanent magnets of electrical machines. Planar as well as cylindrical arrangements are compared against their results by using identical geometry parameters and various pole coverages. With all harmonic orders along the entire frequency range, there is a deviation only in the range $\pm 5 \%$ between these two arrangements.


Fig. 9: Relative error of power losses with half mesh size, planar arrangement, pole coverage 1 , order $p=1$.


Fig. 10: Relative error of power losses with half mesh size, planar arrangement, pole coverage 1 , order $p=2$.

With both arrangements, it is shown that the pole coverage influences the power losses of the lower harmonic waves very strongly while the higher harmonic waves generate power losses directly proportional to the value of the pole coverage.

Additionally, the finite element analyses utilise different approximation orders with hierarchic shape functions in order to validate the accuracy of the eddy current losses. Thereby, lower order elements always yield significantly lower total losses while higher order elements with $p \geq 3$ can handle these parameters very well.

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Fig. 11: Ratio of power losses between cylindrical and planar arrangement, order $p=3$, pole coverage $5 / 6$.


Fig. 12: Ratio of power losses between cylindrical and planar arrangement, order $p=3$, pole coverage $3 / 4$.


Fig. 13: Ratio of power losses between cylindrical and planar arrangement, order $p=3$, pole coverage $2 / 3$.
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Fig. 14: Ratio of power losses between pole coverages $5 / 6$ and 1, planar arrangement, order $p=3$.


Fig. 15: Ratio of power losses between pole coverages $3 / 4$ and 1 , planar arrangement, order $p=3$.


Fig. 16: Ratio of power losses between pole coverages $2 / 3$ and 1 , planar arrangement, order $p=3$.
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