Consistent Approval-Based Multi-Winner Rules

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This paper is an axiomatic study of consistent approval-based multi-winner rules, i.e., voting rules that select a fixed-size group of candidates based on approval ballots. We introduce the class of counting rules, provide an axiomatic characterization of this class and, in particular, show that counting rules are consistent. Building upon this result, we axiomatically characterize three important consistent multi-winner rules: Proportional Approval Voting, Multi-Winner Approval Voting and the Approval Chamberlin–Courant rule. Our results demonstrate the variety of multi-winner rules and illustrate three different, orthogonal principles that multi-winner voting rules may represent: individual excellence, diversity, and proportionality.

A *multi-winner* rule [1] selects a fixed-size set of candidates—a *committee*—based on the preferences of voters. In this paper, we look at multi-winner rules based on approval ballots [3]; an approval ballot corresponds to a subset of (approved) candidates. Consider the following simple example: There are 100 voters and 5 candidates $\{a,b,c,d,e\}$: 66 voters approve the set $\{a,b,c\}$, 33 voters approve $\{d\}$, and one voter approves $\{e\}$. Assume we want to select a committee of size three. If we follow the principle of proportionality, we could choose, e.g., $\{a,b,d\}$; this committee closely reflects the proportions of voter support. If we aim for diversity and do not consider it important to give voters more than one representative, we may choose the committee $\{a,d,e\}$; it contains one approved candidate of every voter. The principle of individual excellence aims to select the strongest candidates: a, b, and c have most supporters and are thus a natural choice, although the opinions of 34 voters are essentially ignored. We see that these three intuitive principles give rise to very different committees and will explore these principles in a formal, mathematical framework.

The first main result of this paper is an axiomatic characterization of *ABC counting rules*, which are a subclass of multi-winner approval-based rules. ABC counting rules are informally defined as follows: given a real-valued function $f(x,y)$ (the so-called *counting function*), a committee W receives a score of $f(x,y)$ from every voter for whom committee W contains x approved candidates and who approves y candidates in total; the ABC counting rule implemented by f ranks committees according to the sum of scores obtained from all voters. We obtain the following characterization.

Theorem 1. *A multi-winner approval-based rule is an ABC counting rule if and only if it satisfies symmetry, consistency, weak efficiency, and continuity.*

For the full version of the paper we refer the reader to the technical report [2]: `https://arxiv.org/abs/1704.02453`.
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The axioms used in this theorem can be intuitively described as follows: A rule is consistent if the following holds: if two disjoint societies decide on the same set of candidates and if both societies prefer committee W_1 to a committee W_2, then the union of these two societies should also prefer W_1 to W_2. This is a straightforward adaption of consistency as defined for single-winner rules by Smith [4] and Young [5]. We say that a rule is symmetric if the names of voters and candidates do not affect the result of an election. Weak efficiency informally states that candidates that no one approves cannot be "better" committee members than approved candidates. Continuity [4] is a more technical condition that states that sufficiently large majorities can dictate decisions.

Theorem 1 gives a powerful technical tool that allows to obtain further axiomatic characterizations of more specific rules. Indeed, building upon this result, we explore the space of ABC counting rules, and obtain our second main result—the axiomatic explanation of the differences between three important rules: Multi-Winner Approval Voting (AV), Proportional Approval Voting (PAV), and Approval Chamberlin–Courant (CC), which are defined by the following counting functions:

$$f_{\mathrm{AV}}(x,y) = x; \qquad f_{\mathrm{PAV}}(x,y) = \sum_{i=1}^{x} 1/i; \qquad f_{\mathrm{CC}}(x,y) = \begin{cases} 0 & \text{if } x = 0, \\ 1 & \text{if } x \geq 1. \end{cases}$$

These three well-known rules are prime examples of multi-winner systems following the principle of individual excellence, proportionality, and diversity, respectively. Our results imply that the differences between these three voting rules can be understood by studying how these rules behave when viewed as *apportionment methods*. Apportionment methods are a well-studied special case of approval-based multi-winner voting, where the set of candidates can be represented as a disjoint union of groups (intuitively, each group can be viewed as a political party), and where each voter approves all candidates within one of these groups (which can be viewed as voting for a single party). For these mathematically much simpler profiles it is easier to formalize the principles of individual excellence, proportionality and diversity:

Disjoint equality states that if each candidate is approved by at most one voter, then any committee consisting of approved candidates is a winning committee. One can argue that the principle of individual excellence implies disjoint equality: if every candidate is approved only once, then every approved candidate has the same support, their "quality" cannot be distinguished, and hence all approved candidates are equally well suited for selection.

D'Hondt proportionality defines a way in which parliamentary seats are assigned to parties in a proportional manner. The D'Hondt method (also known as Jefferson method) is one of the most commonly used methods of apportionment in parliamentary elections.

Disjoint diversity states that as many parties as possible should receive one seat and, if necessary, priority is given to stronger parties. In contrast to D'Hondt proportionality, there are no guarantees for strong parties to receive more than one seat.

We show that Multi-Winner Approval Voting is the only ABC counting rule satisfying disjoint equality, Proportional Approval Voting is the only ABC counting rule satisfying D'Hondt proportionality and that Approval Chamberlin–Courant is the only one satisfying disjoint diversity.

For a complete version of this paper we refer the reader to the technical report [2].

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