Simulation of Sheet Metal Induction Heating Processes with Harmonic Balance FEM

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***Abstract*—We perform simulations of an induction heating process of a moving thin steel sheet by solving the nonlinear eddy current problem in frequency domain using harmonic balancing and an algebraic multigrid preconditioner. The computed Joule losses are then applied as heat sources to a convection-diffusion equation to solve for the temperature distribution in the plate.**

***Index Terms*—Algebraic Multigrid, Convection-Diffusion Equation, Electromagnetic-Heat Coupling, Harmonic Balance FE**

I. Introduction

The efficient simulation of induction heating processes for thin structures is a challenging task, due to the large time-scale differences of the electromagnetic (EM) and thermal field quantities [1]. Solving the coupled (nonlinear) EM-heat system in time-domain results in large and unnecessary computation times because we are only interested in the steady state solution.

In our work, we solve the nonlinear eddy current equation with solution-dependent reluctivity in frequency-domain with a harmonic balancing finite element method (HBFEM) from [2], together with an algebraic multigrid (AMG) preconditioner, evaluate the Joule-losses in the plate and apply them as sources in a convection-diffusion equation to compute the thermal field in the plate.

II. Magnetic Field

Neglecting the displacement currents in Maxwell's equation, we obtain the following problem for the magnetic vector potential $\mathbf{A}$

$$\nabla \times \nu(|\nabla \times \mathbf{A}|)\nabla \times \mathbf{A} + \sigma\frac{\partial \mathbf{A}}{\partial t} = \mathbf{J}_i, \tag{1}$$

where ν is the reluctivity, σ the electric conductivity and $\mathbf{J}_i$ the impressed current density. Expanding ν, $\mathbf{A}$ and $\mathbf{J}_i$ into a complex Fourier as in [2], we obtain a diagonally dominant nonlinear system, which we solve with a fixed point iteration and an AMG-preconditioned GMRES for the linear system. As basis for the local element subspace, edge elements with different polynomial order can be chosen. Since we are using edge elements, an adapted (auxiliary mesh) AMG [3] must be used to preserve the discrete gradient fields across the different hierarchy levels.

III. Thermal Field

The considered steel sheet is moving with velocity $\mathbf{u}$ and we solve (2) for the steady state temperature distribution T

$$\nabla \cdot \mathbf{u}T = c^2\nabla \cdot (\lambda(T)\nabla T) + \frac{1}{\rho c_v}\dot{W}, \tag{2}$$

with c as the diffusion coefficient, ρ the density, λ the conduction coefficient and c_v the specific heat capacity. The right hand side term with $\dot{W}$ represents the heat sources, which are Joule losses in our case. This equation is solved together with Dirichlet- and heat transfer boundary conditions. On the downstream edge of the plate, the boundary condition is chosen to be a Dirichlet boundary, mapped to infinity [5], in order not to force a certain temperature or temperature-gradient at the edge.

IV. Coupling

The coupling of the magnetic with the thermal field is realized by computing the Joule losses from the magnetic simulation and applying it as the source density $\dot{W}$ in (2). Since we solve (1) in frequency domain and (2) in steady state, we have to average the Joule and eddy current losses over one period to obtain consistent heat sources

$$\dot{W} = \frac{1}{\tau}\int_0^\tau \left(\sigma\frac{\partial \mathbf{A}}{\partial t} \cdot \frac{\partial \mathbf{A}}{\partial t} + \mathbf{J}_i \cdot \frac{\partial \mathbf{A}}{\partial t}\right) dt, \tag{3}$$

where τ is the period length of the base harmonic. The HBFEM method for the electromagneic field and convection-diffusion equation for the thermal field is implemented in the finite element framework CFS++ [4] and applied to a generic example to study the effect of different parameters, like number of harmonics or thermal boundary conditions.

References

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