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Analytical modelling of the motion of an oscillating roller during soil compaction assuming pure rolling contact

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1! INTRODUCTION

Continuous Compaction Control (CCC) (Adam 1996) has become the standard technology for controlling the soil compaction induced by *vibratory* rollers. This control technique is based on the dynamic response of the interacting drum-soil system recorded during the roller passage, and thus, allows an instant continuous assessment of the degree of compaction. Recently, also for *oscillating* rollers an efficient roller integrated compaction measurement method has been developed (Pistrol 2016). Pistrol has shown that the vertical acceleration component of the drum centre plotted against the corresponding horizontal acceleration component generates a figure in the shape of a “recumbent eight”. According to this study, the area enclosed in the “recumbent eight” is a characteristic quantity of the compaction degree of the subsoil. Since this CCC technique has been found primarily in an empirical manner, both analytical and numerical studies are required for its theoretical verification. To fill partially this gap, in the present contribution a semi-analytical parametric study of the motion of an oscillating drum during soil compaction is conducted. The obtained results allow to detect, to understand and to explain better the dynamic response of the interacting oscillating drum-soil system, and ultimately to validate this novel compaction measurement method for oscillating rollers.

2! ANALYTICAL MODELLING

The present study is based on the simplified analytical model of the interacting oscillating roller – subsoil system shown in Fig. 1 (Pistrol 2016). This model allows to studying the motion of the oscillating drum in its settlement trough. The drum is represented as a rigid circular plate of radius r, mass m and mass moment of inertia I. It is assumed that the settlement trough has a circular shape with constant radius R, and thus it is rigid as well. At the bottom of the settlement trough the effect of the subsoil is captured through two Kelvin-Voigt bodies, one in vertical (subscript “V”) and another in horizontal (subscript “H”) direction. As such, the settlement trough exhibits a translational motion in both horizontal and vertical direction. Rotation of the settlement trough is not permitted. The so-called cone model of Wolf (Wolf 1994) is used to derive stiffness coefficients k_H and k_V of the springs, damping coefficients c_H and c_V of the dashpot dampers, and the trapped soil mass Δm, considering the measured contact length between drum and soil (Pistrol 2016). Between drum and circular guiding track continuous rolling contact is assumed, i.e. rolling without slipping. Hence, the arc length between support point A and contact point P_{cont} is equal to the length between contact point P_{cont} and point P_D on the drum, see Fig. 1. The relationship between the absolute rotation angle φ of the drum and the centre angle δ can be expressed as follows:

$$\varphi = \frac{R-r}{r}\delta = \frac{l}{r}\delta \tag{1}$$

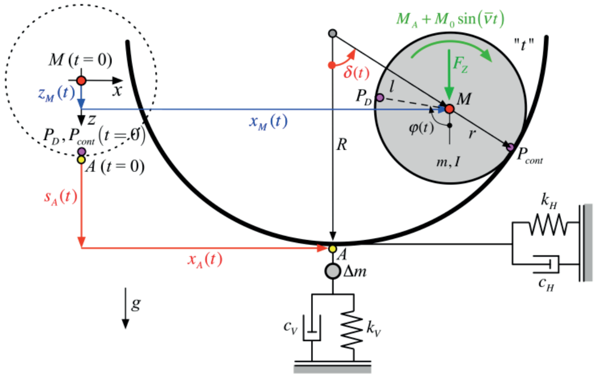

Figure 1. Mechanical model of the oscillating drum in its settlement trough (deformed position).

The motion of the three-degree-of-freedom model is described by centre angle δ, and settlement s_A and horizontal displacement x_A of support point A. Application of Lagrange's equations (Ziegler 1995), conservation of momentum and angular momentum yields the following coupled equations of motion of this model:

$$m\ddot{x}_A + c_H\dot{x}_A + k_H x_A + ml\cos\delta\ddot{\delta} - ml\sin\delta\dot{\delta}^2 = 0 \tag{2}$$

$$(m+\Delta m)\ddot{s}_A + c_V\dot{s}_A + k_V s_A - ml\sin\delta\ddot{\delta} - ml\cos\delta\dot{\delta}^2 = (m+\Delta m)g + F_z \tag{3}$$

$$\left(1+\frac{I}{mr^2}\right)ml\ddot{\delta} + m\cos\delta\ddot{x}_A - m\sin\delta\ddot{s}_A + (mg+F_z)\sin\delta = \frac{M_A}{r} + \frac{M_0}{r}\sin(\bar{\nu}t) \tag{4}$$

Variable l in Eq. (1) is equal to $R-r$, g in Eqs (3) and (4) denotes the acceleration of gravity, F_z in Eqs (3) and (4) represents the axle load of the drum (static load in vertical direction with point of application M), M_A in Eq. (4) denotes a constant driving torque, and M_0 and $\bar{\nu}$ in Eq. (4) represent amplitude and angular frequency of the oscillating moment (harmonic excitation), respectively. The second order nonlinear ordinary differential equations (ODEs), Eqs (2) to (4), are solved numerically by the MATLAB function *ode45* (Mathworks 2017) for δ, s_A and x_A. Then, the governing response quantities for the investigated CCC technique, i.e. the horizontal ($\ddot{x}_M$) and vertical ($\ddot{z}_M$) acceleration components of the drum centre (point M in Fig. 1), are calculated using the following relations:

$$\ddot{x}_M = l\left(\cos\delta\ddot{\delta} - \sin\delta\dot{\delta}^2\right) + \ddot{x}_A\,, \quad \ddot{z}_M = -l\left(\sin\delta\ddot{\delta} - \cos\delta\dot{\delta}^2\right) + \ddot{s}_A \tag{4}$$

3! RESULTS

For the subsequent studies the properties of a HAMM HD+ 90 VO tandem roller with the following parameters are utilized: radius of the drum $r = 0.6$ m, mass of the drum $m = 1{,}851$ kg, mass moment of inertia of the drum $I = 411.78$ kgm^2, static axle load $F_z = 27{,}066$ N, amplitude of the oscillating moment $M_0 = 54{,}947$ Nm, excitation frequency of 39 Hz (HAMM AG 2017). It is assumed that all considered subsoil conditions exhibit a Poison's ratio ν of 0.3 and a density ρ of 1,900 kg/m^3. The shear modulus G of the subsoil is varied between 5 and 70 MN/m^2. Based on experience (Kopf 1999), the damping coefficients c_H and c_V resulting from the cone model according to Wolf are multiplied by 2. In the following, selected results derived from the acceleration components of the drum centre (point M in Fig. 1) are presented and discussed. In particular, Fig. 2 shows for one subsoil and two different radii R of the settlement trough the vertical steady state acceleration component $\ddot{z}_M$ in the frequency domain. In Fig. 3, $\ddot{z}_M$ is plotted against horizontal acceleration component $\ddot{x}_M$ for various subsoil conditions, yielding figures in shape of a "recumbent eight". Finally, in Fig. 4 the area enclosed in the "recumbent eight" is shown as a function of the soil stiffness.

- Simulations with varying radius R of the settlement trough reveal that the function $\ddot{z}_M$ over $\ddot{x}_M$ ($\ddot{z}_M - \ddot{x}_M$ plot) only exhibits an intersection if R is slightly larger than the drum radius, i.e. r (= 0.60) < $R \leq 0.61$ m. In this range, vertical acceleration $\ddot{z}_M$ is dominated by a frequency that is two times the excitation frequency, as shown in Fig. 2, left subplot. In contrast, for $R = 0.7$ m the dominant frequencies of $\ddot{z}_M$ are close to the excitation frequency (Fig. 2, right subplot).
- The horizontal drum centre acceleration $\ddot{x}_M$ is dominated by the excitation frequency for both radii of the settlement trough, i.e. $R = 0.606$ m and $R = 0.7$ m.
- Thus, the shape of the $\ddot{z}_M - \ddot{x}_M$ plot is similar to a "recumbent eight" (Fig. 3) only if radius R of the circular settlement trough is slightly larger than drum radius r (here $r < R \leq 0.61$ m).
- Initially, the horizontal peak acceleration $\ddot{x}_M$ increases with increasing soil stiffness, its maximum is attained at about $G = 40$ MN/m^2, and it decreases subsequently with further increasing soil stiffness (Fig. 3).
- Initially, the vertical peak acceleration $\ddot{z}_M$ increases with increasing soil stiffness, its maximum is attained at $G = 22.5$ MN/m^2, and it decreases subsequently with further increasing soil stiffness (Fig. 3). Thus, the maximum of $\ddot{z}_M$ is related to a lower shear modulus compared to the maximum of $\ddot{x}_M$.
- The shape of the $\ddot{z}_M - \ddot{x}_M$ plot represents a so-called Lissajous (or Bowditch) curve (Klotter 1981). Depending on the phase shift between the vertical and horizontal accelerations it is either similar to the lemniscate of Gerono (i.e. a "recumbent eight") (Lawrence 1972) or to a general besace.

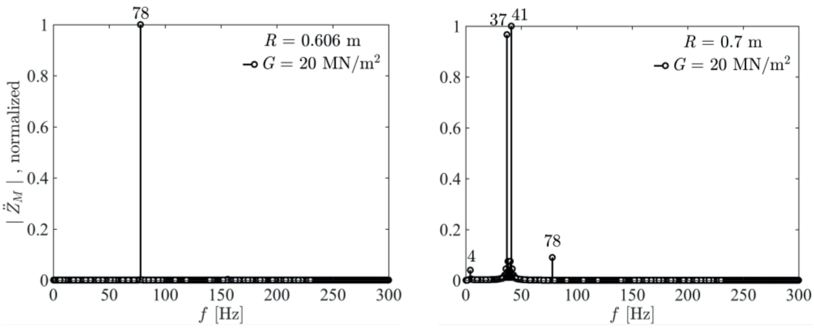

Figure 2. Vertical accelerations of the drum centre in the frequency domain for $R = 0.606$ m (left subplot) and $R = 0.70$ m (right subplot).

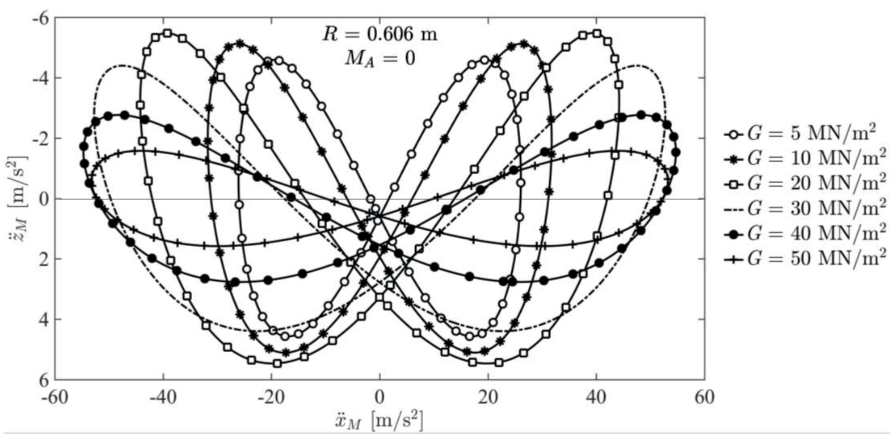

Figure 3. Vertical acceleration plotted against horizontal acceleration of the drum centre.

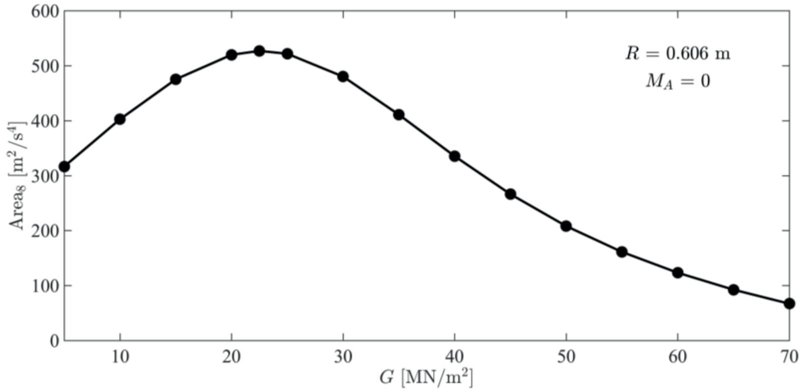

Figure 4. Area enclosed by the "recumbent eight" as a function of soil stiffness *G*.

- With increasing soil stiffness (in the range G = 10-30 MN/m^2), the "recumbent eight" degenerates to a general besace similar to a parabolic segment. When further increasing the soil stiffness in the range G = 40-70 MN/m^2 the curve resembles the lemniscate of Bernoulli (Lawrence 1972).
- The shape of the $\ddot{z}_M - \ddot{x}_M$ plot is symmetric with respect to the vertical axis if no driving torque M_A is applied ($M_A = 0$).
- Initially, the area enclosed by the "recumbent eight" increases with growing soil stiffness and it attains its maximum at G = 20-25 MN/m^2. Subsequently the area decreases with further increasing soil stiffness (Fig. 4).

4! CONCLUSIONS

Semi-analytic simulations of the motion of a selected oscillating roller have demonstrated that the simplified mechanical model reproduces the findings from in situ tests if the radius of the circular settlement trough is slightly larger than the drum radius. The vertical steady state acceleration of the drum centre ($\ddot{z}_M$) is dominated by a frequency that is two times the excitation frequency, whereas the dominant frequency of the corresponding horizontal acceleration ($\ddot{x}_M$) is equal the excitation frequency. It has been confirmed that function $\ddot{z}_M$ over $\ddot{x}_M$ is in the shape of a "recumbent eight", and the area enclosed by this figure is related to the soil stiffness.

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