OntoREA© Accounting and Finance Model: Including Option Contracts in the MDD-Context

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Abstract. OntoREA© is a specification of the Accounting and Finance domain in the OntoUML language [1] which includes derivative instruments as well. The authors use a forward contract as running example to demonstrate the validity of the OntoREA© Accounting and Finance model within the design science resource methodology (DSRM) [2]. They claim but they do not proof that the model not only holds for forward but also for option contracts. In this article the missing proof is given by refining the relational MySQL model. In the model driven development (MDD) context [3] this model constitutes the platform specific model (PSM) which is derived from the platform independent model (PIM) in form of the OntoREA© model. The refined PSM model adequately represents European option contracts and it should be especially useful for business analysts as well as for educational purposes.

Keywords: OntoREA© Accounting and Finance Model, Design Science Research Methodology DSRM, Model Driven Development MDD, Conceptual Modeling, Derivative Instruments, Dynamic Hedge Portfolio

1 Introduction

The no-arbitrage pricing theory also holds for options. For options a dynamic hedge portfolio exactly replicates (duplicates) the option value over time. In a dynamic replication policy fractions of the stock are bought and they are partially financed by a loan liability. Over time the investment and financing has to be continuously adjusted according to the revealing stock prices. At the expiration date the hedge portfolio gives the same result as the initial buying of an option contract.[1]

In contrast to options the hedge portfolio composition of a forward contract does not change over time and that's why it is called a *static hedge portfolio*. The static hedge portfolio representation was used in [1] to demonstrate the suitability of the OntoREA© Accounting and Finance model as conceptual platform independent model (PIM) for the development of a platform specific MySQL relational database model (PSM) within the model-driven software development context (MDD).

The hedge portfolio representation of derivative instruments is one of the core features of the OntoREA© model and it is expressed in the upper left part of Figure 1 in form of the *Collective* class Derivative Instrument and its *MemberOf* relationship to the *Kind* class Economic Resource. In simple terms the meta-physical stereotypes of the OntoUML language have the following meaning (for a full description of the conceptual and meta-physical OntoUML details see [4]): A derivative instrument is represented as a rigid and identity-providing portfolio *collective* that *consists of* two economic resources that are themselves rigid and identity providing *kinds*.

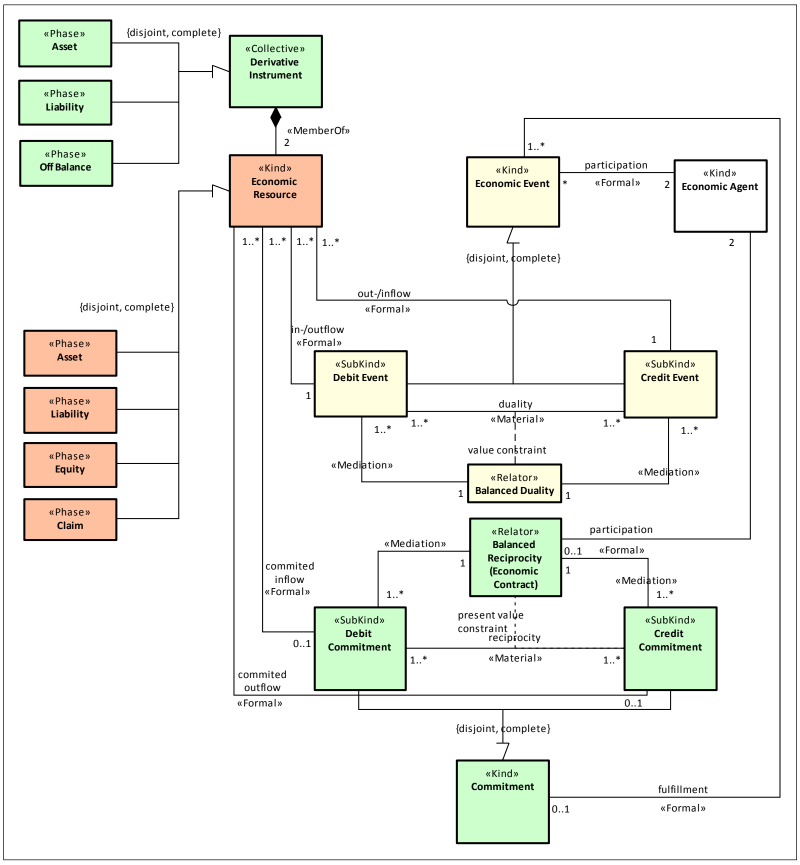

Figure 1: OntoREA© Accounting and Finance Model - Conceptual PIM Level

The hedge portfolio [5] representation of derivative instruments was originated by the Nobel laureates Black/Scholes [6] and Merton [7] who developed to the *no-arbitrage pricing theory*. This representation holds true for *unconditional derivatives* (e.g. forward contracts) as well as for *conditional derivatives* (e.g. option contracts). The main difference between un- and conditional derivates lies in the dynamic behavior of the hedge portfolio composition. Unconditional derivatives include the obligation for the buyer of the contract to buy the underlying asset in the future. Due to this obligation the hedge portfolio composition does not change over time. Conditional derivatives include the right for the buyer of the contract to buy the asset in the future.

As the probability of executing the option is changing over time, the hedge portfolio composition changes as well.

The research objective of this article is to provide the proof that the conceptual OntoREA© PIM model incorporates not only the static but also the dynamic hedge portfolio representation of derivative instruments. For this purpose a refined MySQL database PSM model is constructed that includes the peculiarities of dynamic hedging portfolios as well, so that un- and conditional derivative instruments are covered in the proposed PSM model.

This paper is organized as follows: In the next section the no-arbitrage pricing theory and its hedge portfolio foundation are presented. In following section the refined relational database model (PSM) for un- and conditional derivative instruments is presented and its applicability to conditional option contracts is demonstrated with a stock call running example. The last section concludes the paper and shows directions for further research.

2 No-Arbitrage Pricing: Hedge Portfolio Representation

The no-arbitrage pricing theory was – as already mentioned – developed by the Nobel laureates Black/Scholes [6] and Merton [7]. They show that there is only one price for the derivative instruments, i.e. the no-arbitrage price that does not allow arbitrage possibilities. They derive the no-arbitrage price for European stock call options. European stock calls have the peculiarity that the right to buy refers to a stock asset, which is the underlying of the contract, and that the right can be exercised by the buyer of the contract only at expiration date (European style). The no-arbitrage price for the European stock call is given by the Black/Scholes formula:

$$Fair\ Value_{\ Call,t} = \overbrace{+P_{A,t} \cdot \underbrace{N\left(d_{1,t}\right)}_{asset\ weight}}^{\substack{value \\ (asset/left\ leg)}} \overbrace{-X_{0,T} \cdot N\left(d_{2,t}\right) \cdot e^{\left(-ln(1+R_{0,T}) \cdot T_{t,T}\right)}}^{\substack{present\ value \\ (liability/right\ leg)}} \qquad (1)$$

As can be seen by the annotations in equation 1, the no-arbitrage price, which is called *fair value*, corresponds according to the hedge portfolio of two parts: The value of the asset (*asset value*) on the left side (left leg) and the present value of the liability (*loan liability*) on the right side.

The *asset weight* , i.e. $N(d_{1,t})$ gives the fraction of the underlying stock that is hold in the hedge portfolio. It is calculated by evaluating the standard normal distribution function $N()$ at the value of $d_{1,t}$. The $d_{1,t}$-value (for further details see [6]) is a function of the stock *Price* $P_{A,t}$ and the time to maturity $T_{t,T}$. Consequently this value changes over the life cycle of the call option. The asset weight is a probabilistic term that expresses the probability of a stock option execution. It ranges between zero and 100 %.

The present value of the loan liability is calculated by weighting the exercise price $X_{0,T}$ with the weighting factor $N(d_{2,t})$ and discounting the resulting product by multiplying it with the discount factor $exp(-ln(1+R_{0,T})*T_{t,T})$. The discount factor is calculated in form of a continuous compounding by inserting the interest rate $R_{0,T}$ over the whole life time of the option, i.e. from 0 to T, and the time to maturity $T_{t,T}$ into the Euler exponential.

Finally, by using the t variable for the pricing date, the Black/Scholes formula is generically defined so that it can be applied for the initial (i.e. t = 0) and the subsequent (i.e. t > 0) pricing.

Table 1: European Stock Call (running example) – Specification and Pricing

Contracting date:	01.01.
Expiration date:	31.12.
Exercise price:	100
Initial stock price:	100
Volatility:	20%
Initial interest rate:	5%
Asset weight N(d1):	63.68%
Liability weight N(d2):	55.96%
Stock Asset:	63.68
Loan Liability:	53.23
Fair value: = A – L	10.45 (A)

Table 1 contains the specification of a European stock call and its initial pricing at the beginning of the year (01.01.) according to the Black/Scholes formula, which is evaluated at the contracting date, i.e. t = 0. The *fair value* of the stock option, i.e. its no-arbitrage price, amounts to 10.45 and it is calculated by subtracting the present value of the *loan liability* (53.23) from the value of the *stock asset* (63.68).

Due to its generic definition the Black/Scholes formula can be used for pricing purpose not only at the call's contracting date but also at subsequent dates. The big advantage of the formula is that it not only gives the call price, but also the composition of the hedge portfolio at all pricing dates.

The composition information is of special importance in the case of a *dynamic call replication policy*, where the call is not bought initially but instead it is synthetically created by implementing and rebalancing the dynamic hedge portfolio over time. In this case the asset weights $N(d_{1,t})$ are of special importance. They indicate the fractions of the underlying stock assets in the hedge portfolio.

For demonstrative purposes a pricing after each quarter is assumed. Furthermore it is assumed that the stock price from initially 100 does not change after the first and second quarter and then increases to 120. In this constellation – as can be seen in the first column of Table 2 – the asset weights start decreasing from 63.68 % (01.01.: 100) to 61.91 % (31.03.: 100) and to 59.77 % (30.06.: 100) and consequently increase to 97.72 % (30.09.: 120). The initial decrease at the stable price of 100 indicates a decreasing execution probability and consequently a smaller stock position is hold in the hedge portfolio. The stock price increase increases the execution probability and consequently the stock position in the hedge portfolio. The changing asset weights over the call's life time demonstrate what is meant by saying that the composition of the option's hedge portfolio is changing over time. This changing composition in the dynamic hedge portfolio is contrasted to the stable composition in the static hedge portfolio of stock forwards where at each point in time exactly one unit of the underlying stock is held in the hedge portfolio.

3 Hedge Portfolio: Switching from PIM- to PSM-Level

After having a deeper understanding of the hedge portfolio in the Black/Scholes formula, the transformation of its conceptualization in the OntoREA© Accounting and Finance model – as the *Collective* class Derivative Instrument with a *MemberOf* relationship to the *Kind* class Economic Resource – into a MySQL database model can be addressed. In the MDD context this transformation corresponds to the switch from an abstract conceptual PIM model into a specific database PSM model. Associated with this concretization step is an informational extension that is accomplished by adding additional attributes and tables in the PSM model in order to capture the more detailed contents at the PSM model level.

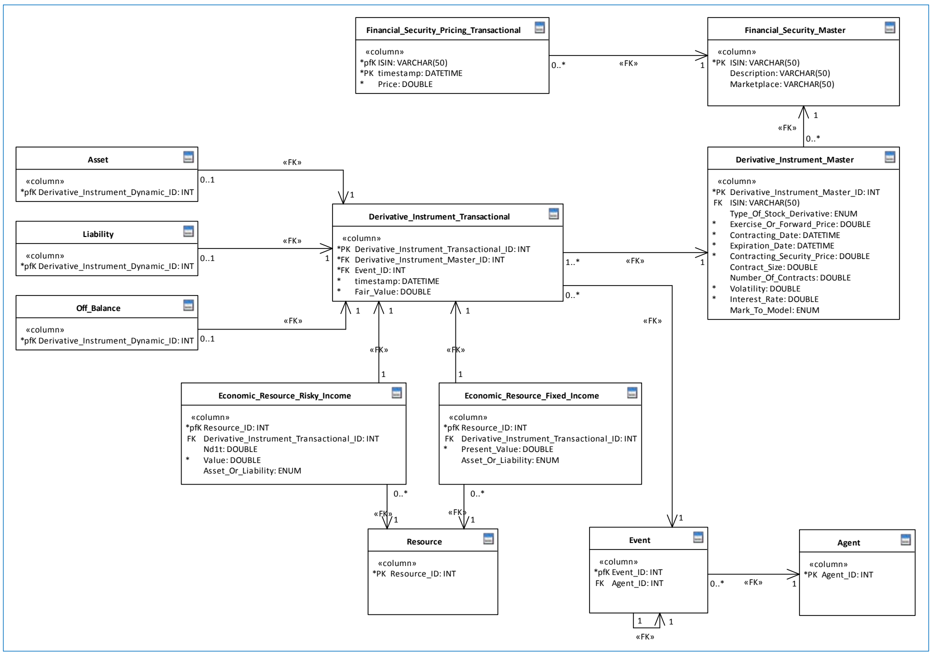

Figure 2: MySQL Relational Database Model – Database PSM Level

Figure 2 contains the refined database PSM model that concretizes the OntoREA© conceptualization of the derivative instruments' hedge portfolio representation. It covers not only unconditional but also conditional derivative instruments. Compared to the development of the MySQL database (PSM) model related to the static hedge portfolio representation in [1], the dynamic hedge portfolio peculiarities for the stock options are now explicitly incorporated for the:

1. *Collective* class Derivative Instrument,
2. *MemberOf* relationship between *Collective* class Derivative Instrument and *Kind* class Economic Resource and
3. *Formal* relationship in-/outflow (out-/inflow) between *Kind* class Economic Resource and *SubKind* class Debit Event (Credit Event).

Ad 1) The *Collective* class Derivative Instrument is transformed via the four tables in the right upper corner of Figure 2. The splitting into four tables allows a clear distinction of information that is stable over time (*master information*) and information that changes (*transactional information*).

- The table Derivative_Instrument_Master contains the stable information which specifies the derivative instruments. Its attribute Type_Of_Stock_Derivative is of ENUM type so that un- and conditional derivatives are covered in the MySQL database model. It can be seen that the table also contains all information from Table 1 which are used to specify the stock call.
- The table Derivative_Instrument_Transactional contains the pricing information which is associated to the initial and subsequent pricing dates measured with the Attribut timestamp. In the case of a dynamic replication policy the hedge portfolio composition adjustments are connected with capital market transactions. That's why the table has the *Transactional* suffix.
- The two tables Financial_Security_Pricing_Master and Financial_Security_Pricing_Transactional are included in order to allow the separate specification of the derivative's underlying asset (i.e. financial security) which is fully defined by its international security identification number (ISIN). The usage of two tables is due to the intended separation of master and transactional information.

Ad 2) The *MemberOf* relationship is transformed via the two tables in the lower left part of Figure 2 according to the financial categorization of financial instruments into risky income and fixed income resources. Both tables have a foreign key to the table Derivative_Instrument_Transactional. The missing NOT NULL CONSTRAINT (*) indicates that not all of these resource types have to be part of a hedge portfolio. The inclusion of the two tables promotes the understanding of the different financial natures of the two hedge portfolio constituents, i.e. the stock asset (risky income) and the loan liability (fixed income).

Ad 3) The *Formal* relationship in-/outflows (out-/inflows) between the *Kind* class Economic Resource and the *SubKind* class Debit Event (Credit Event) is transformed via the relationship between the table Derivative_Instrument_Transactional and the reflexive table Event at the bottom of Figure 2. This relationship is needed for triggering events that are – as can be seen in Figure 1 – connected to classes Debit Event and Credit Event. At the pricing dates (timestamp) events are triggered in two cases, i.e. either if the Mark_To_Model mode is active (see table Derivative_Instrument_Master) or if a dynamic replication policy is in place. By triggering the events the changes in the hedge portfolio compositions are recognized via balanced double-entries in the accounting system.

After having presented the construction of the MySQL database PIM its working is addressed. The storage of information starts at the contracting date (initial pricing date). At that date the contracts for the derivative instruments are specified and the corresponding information is stored in the master tables. As time goes by – that is at runtime – additional information is revealed. Of special importance for the calculations of the hedge portfolio compositions are the revealing asset prices (see table Financial_Security_Pricing_Transactional). Furthermore the timestap information in the table Derivative_Instrument_Transactional is important as it allows the calculation of the remaining time to maturity ($T_{t,T}$) together with the attribute Expiration_Date in the table Derivative_Instrument_Master. With this information the hedge portfolio compositions are determined and the corresponding information processing activities are triggered.

Table 2: European Stock Call – Specification and Subsequent Pricing

			PSM Relational Schema	
	Dynamic Hedging		Attribute	Table
Initial Pricing/Static Data	Contracting date:	01.01.	Contracting_Date	Derivative_Information_Master
	Expiration date:	31.12.	Expiration_Date	Derivative_Information_Master
	Exercise price:	100	Exercise_Or_Forward_Price	Derivative_Information_Master
	Initial stock price:	100	Contracting_Security_Price	Derivative_Information_Master
	Volatility:	20%	Volatility	Derivative_Information_Master
	Initial interest rate:	5%	Interest_Rate	Derivative_Information_Master
	Asset weight N(d1):	63,68%	Nd1t	Economic_Resource_Risky_Income
	Liability weight N(d2):	55,96%	-	-
	Stock Asset:	63,68	Value	Economic_Resource_Risky_Income
	Loan Liability:	53,23	Present_Value	Economic_Resource_Fixed_Income
	Fair value: = A - L	10.45 (A)	Fair_Value	Derivative_Instrument_Transactional
Subsequent Pricing/Dynamic Data	Pricing date #1:	31.03.	timestamp	Financial_Security_Pricing_Transactional
	Actual stock price:	100	Price	Financial_Security_Pricing_Transactional
	Actual time to maturity: 9 months		-	-
	Asset weight N(d1):	61,91%	Nd1t	Economic_Resource_Risky_Income
	Stock Asset:	61,91	Value	Economic_Resource_Risky_Income
	Loan Liability:	52,13	Present_Value	Economic_Resource_Fixed_Income
	Fair value: = A - L	9.78 (A)	Fair_Value	Derivative_Instrument_Transactional
	Pricing date #2:	30.06.	timestamp	Financial_Security_Pricing_Transactional
	Actual stock price:	100	Price	Financial_Security_Pricing_Transactional
	Actual time to maturity: 6 months		-	-
	Asset weight N(d1):	59,77%	Nd1t	Economic_Resource_Risky_Income
	Stock Asset:	59,77	Value	Economic_Resource_Risky_Income
	Loan Liability:	50,65	Present_Value	Economic_Resource_Fixed_Income
	Fair value: = A - L	9.12 (A)	Fair_Value	Derivative_Instrument_Transactional
	Pricing date #3:	30.09.	timestamp	Financial_Security_Pricing_Transactional
	Actual stock price:	120	Price	Financial_Security_Pricing_Transactional
	Actual time to maturity: 3 months		-	-
	Asset weight N(d1):	97,72%	Nd1t	Economic_Resource_Risky_Income
	Stock Asset:	117,26	Value	Economic_Resource_Risky_Income
	Loan Liability:	-96,82	Present_Value	Economic_Resource_Fixed_Income
	Fair value: = A - L	20.44 (A)	Fair_Value	Derivative_Instrument_Transactional
	Pricing date #4:	31.12.	timestamp	Financial_Security_Pricing_Transactional
	Actual stock price:	120	Price	Financial_Security_Pricing_Transactional
	Actual time to maturity: 0 months		-	-
	Stock Asset:	117,26	Value	Economic_Resource_Risky_Income
	Loan Liability:	98,03	Present_Value	Economic_Resource_Fixed_Income
	Fair value: = A - L	19.23 (A)	Fair_Value	Derivative_Instrument_Transactional

In Table 2 the calculations for the stock call running example can be seen in the first column. It is interesting to note the last fair value at the end of the year (31.12.) amounting to 19.23 is close to the intrinsic value of the stock call amounting to 20 which is calculated as difference between the stock asset value of 120 and the exercise price of 100. By executing the subsequent pricings more often the difference can theoretically be brought to zero. For completeness it should be mentioned that the resulting fair value is connected to a self-financing policy. According to this policy the changing asset fractions are either used for redemption of the loan if they decrease or financed by increasing the loan if the increase.

Finally, the middle and the right column show with which attribute of which table this information is captured in the MySQL database PIM model.

4 Conclusion

The main contribution of this article is the development of a more advanced MySQL database PSM model compared to [1] that not only covers static hedge portfolio representations for unconditional derivatives (e.g. stock forward) but also dynamic hedge portfolio representations for conditional derivatives (e.g. stock call). The applicability of the proposed database PSM model was demonstrated for a European stock call running example. According to the put/call parity analogue results hold for European stock puts, so that the PSM model covers call and put options. Next to the applicability demonstration the proposed PSM model fulfills all peculiarities that are related to the dynamic hedge portfolio representation specified in the OntoREA© PIM model. In this sense the research objective is reached by delivering the missing proof in [1] in form of a refined database PSM that covers unconditional as well as conditional derivatives.

For further research two considerations seem worthwhile. Firstly, the transformation of the database PSM model into an R/Shiny prototype like in [1] in order to complete also the 2-step in the forward engineering approach in the MDD context. Secondly, the explicit policy level specification seems to be a worthwhile endeavor.

This proposed model can be useful for business analysts in the finance domain as well as for teaching purposes by explaining derivative instruments in form of conceptual PIM and database specific PSM models.

5 References

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