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Flame propagation in arbitrarily shaped vessels

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A new method to capture the flame front in premixed, confined, low Mach number combustion using the flame-sheet approximation is presented. Connected marker points represent the flame front along which a singular volumetric flux source is prescribed. Thus, the jump condition for the normal velocity is fulfilled. Application to two-dimensional combustion in a channel with periodic boundary conditions shows good agreement to linear stability analysis of the perturbed flame front.

1 Physical Model

We consider an ideal gas with constant material parameters and we neglect heat conduction and friction. At some time $\tilde{t}_0$, an explosion starts inside a closed vessel and a flame front develops. It is assumed to be infinitely thin, separating fresh, unburnt gas and combusted, burnt gas. The reaction enthalpy $\Delta\tilde{h}_f$ is assumed to be constant and part of the internal energy of the unburnt gas. The relative normal velocity of the flame front with respect to the unburnt gas, called laminar flame speed, is dependent on the local curvature of the flame front (Markstein model [1]), $\tilde{s}_L = \tilde{s}_{L,0}(1-\tilde{\kappa}\tilde{l}_{\mathrm{Ma}})$, where $\tilde{\kappa}$ denotes the curvature and $\tilde{l}_{\mathrm{Ma}}$ is the Markstein length. $\tilde{s}_{L,0}$ is constant. At the walls of the vessel, a vanishing normal velocity is prescribed and there is no heat loss to the surroundings. Initially, the gas is at rest with constant initial density $\tilde{\varrho}_i$ and pressure $\tilde{p}_i$. A tilde denotes that a quanity has a dimension.

We scale all quantities with intial density and pressure, $\tilde{s}_{L,0}$ and the characteristic length of the vessel $\tilde{L}$. This leads to a non-dimensioanl parameter $\varepsilon = \tilde{\varrho}_i\tilde{s}^2_{L,0}/\tilde{p}_i = \tilde{s}^2_{L,0}/\tilde{c}^2_i\gamma = \mathrm{M}^2_{\mathrm{ref}}/\gamma$ which is proportional to the square of a reference Mach number and $\tilde{c}_i$ is the speed of sound at initial conditions. The governing equations are conservation of mass, momentum and energy, also known as Euler equations. At the interface, jump conditions must be fulfilled [2, 3].

Since $\tilde{s}_{L,0}/\tilde{c}_i \ll 1$, we expand all quantities in terms of small $\varepsilon \ll 1$, e.g. $\varrho = \varrho_0 + \varepsilon\varrho_1 + \dots$. In leading order we get:

$$\frac{1}{\varrho_0}\frac{\mathrm{D}\varrho_0}{\mathrm{D}t} + \nabla\cdot\vec{u}_0, = 0, \qquad \nabla p_0 = \vec{0}, \qquad \frac{\gamma}{\gamma-1}\left(\frac{1}{p_0}\frac{\mathrm{D}p_0}{\mathrm{D}t} - \frac{1}{\varrho_0}\frac{\mathrm{D}\varrho_0}{\mathrm{D}t}\right) = \frac{1}{p_0}\frac{\partial p_0}{\partial t}.$$

Due to the low Mach number assumption and neglecting friction and heat conduction, the density in the unburnt and the leading order pressure, i.e. thermodynamic pressure, are time-dependent only.

We split the velocity field in an irrotational and solenoidal part $\vec{u} = \nabla\times\vec{\psi} + \nabla\varphi$. The scalar potential φ is governed by a Poisson equation with a time-dependent only right-hand side. All quantities are of leading order and the index $_0$ is dropped. W.l.o.g., we normalise the vector potential $\vec{\psi}$, i.e. $\nabla\cdot\vec{\psi} = 0$:

$$\nabla^2\varphi = \nabla\cdot\vec{u} = -\frac{1}{\varrho}\frac{\mathrm{D}\varrho}{\mathrm{D}t} = -\frac{1}{\gamma}\frac{1}{p}\frac{\mathrm{d}p}{\mathrm{d}t}, \qquad \nabla^2\vec{\psi} = -\nabla\times\vec{u} = -\vec{\omega}.$$

Since the flow is isentropic except at the flame front, the material derivative of the density can be expressed with the time-dependent only derivative of the pressure. Here, vorticity $\vec{\omega}$ and vector potential are not considered. This is justified for small dimensionless reaction enthalpies [3]. For the planar flame front, there is a critical wavenumber $k_c = (1-\sigma^{-1})/2\ \mathrm{Ma}$ [4]. Perturbations with a higher wavenumber are stable. k_c depends on the Markstein number $\mathrm{Ma} = \tilde{l}_{\mathrm{Ma}}/\tilde{L}$ and the expansion ratio σ, the ratio of density of unburnt and burnt gas ϱ_u/ϱ_b.

The pressure change in the vessel with volume V_v is due to the release of heat at the flame front (f.f.) and is proporational to the surface area of the interface and the jump of normal velocity $u^n_u - u^n_b$:

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\gamma p}{V_v}\int\limits_{\text{f.f.}} \left(u^n_u - u^n_b\right)\,\mathrm{d}O.$$

2 Numerical Implementation

The flame is captured by marker points which move with the front. They form a polygon along which singular volumetric line-sources are distributed to fulfil the jump conditon for velocity. The vessel is also represented by panels to fulfill the boundary condition of a vanishing normal velocity, see Figure 1a. This corresponds to a panel method [5, 6] for internal flow.

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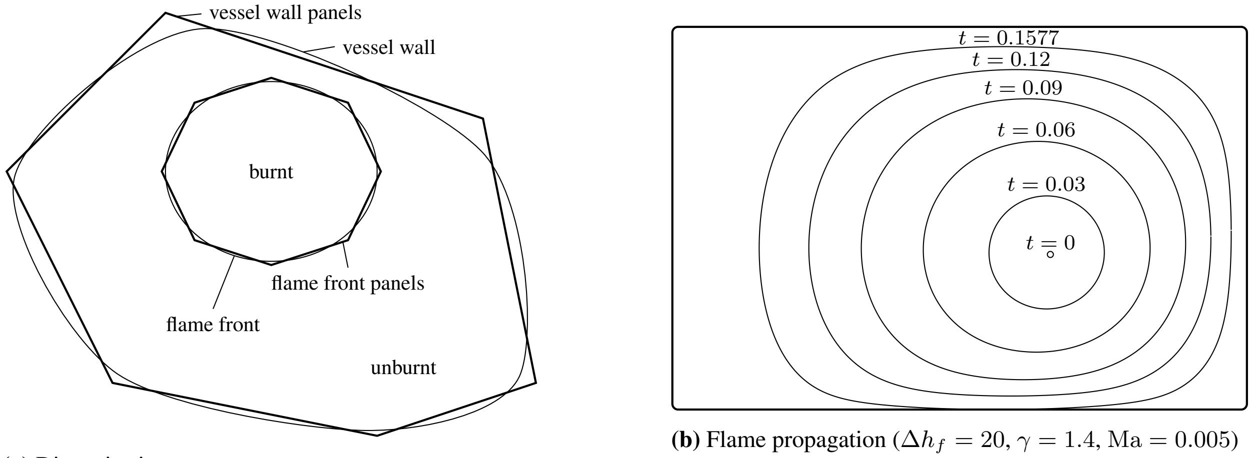


(a) Discretisation

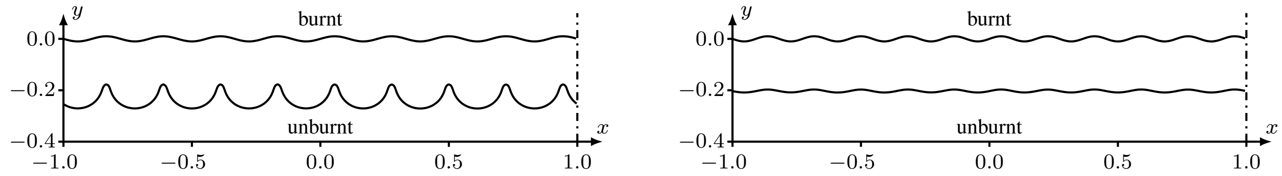
(b) Flame propagation ($\Delta h_f = 20$, $\gamma = 1.4$, $\mathrm{Ma} = 0.005$)

Fig. 1: The flame front inside a vessel

At the flame front, the source strength $q_{f,i}$ of panel i is equal to the jump of normal velocity $u^n_{u,i} - u^n_{b,i}$. The source strength $q_{v,j}$ of the vessel wall panels are chosen that the boundary conditions of vanishing normal velocity at the vessel wall are fulfilled, $u^n_{v,j} = 0$. Since the velocity is singular at the panel edges, i.e. marker points, it is interpolated between two adjacent panel mid-points.

3 Results

To verify the implementation, the critical wavenumber k_c of the linear stability analysis was repoduced inside an infinite channel with periodic boundary conditions, Figure 2.



(a) unstable initial wavenumber $k_i = k_c - 1$

(b) stable initial wavenumber $k_i = k_c + 1$

Fig. 2: Planar flame front in a channel, $k_c = 10$, $\mathrm{Ma} = 0.0416$, $\sigma = 6$, time interval from top to bottom $\Delta t = 0.2$

A flame inside a rectangular vessel, Figure 1b, stays circular at the beginning, $t = 0.03$ and $t = 0.06$. Then, the influence of the vessel walls becomes more and more important and the front adapts to the shape of the vessel, $t = 0.09$ and $t = 0.12$. Once the flame front touches the wall, $t = 0.1577$, the simulation stops, since the assumption of no heat loss becomes invalid.

4 Discussion and Outlook

The presented front-tracking method is faster and simpler than other widely-used techniques such as level-set method or volume of fluid method. Results of the linear stability analysis of planar flames have been reproduced to show the correctness of this implementation.

In the future, we will extend the implemenation to three-dimensional, rotational symmetric geometries. Those kinds of geometries approximate many vessels seen in industry. Furthermore, vorticity and vector potential have to be implemented.

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