Flow of Vapors Through Porous Media

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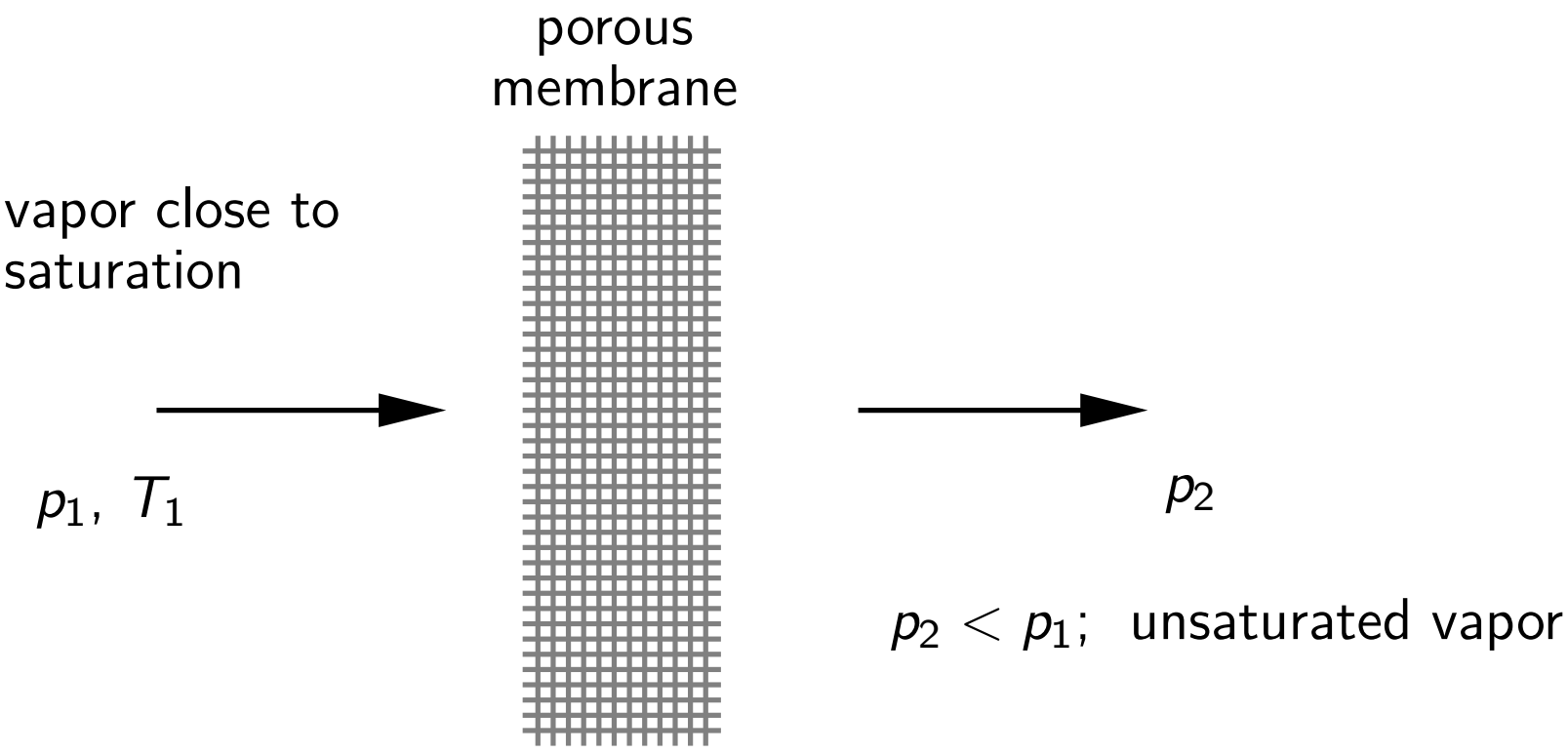
Seminar, 19th June 2017

Outline

Problem statement



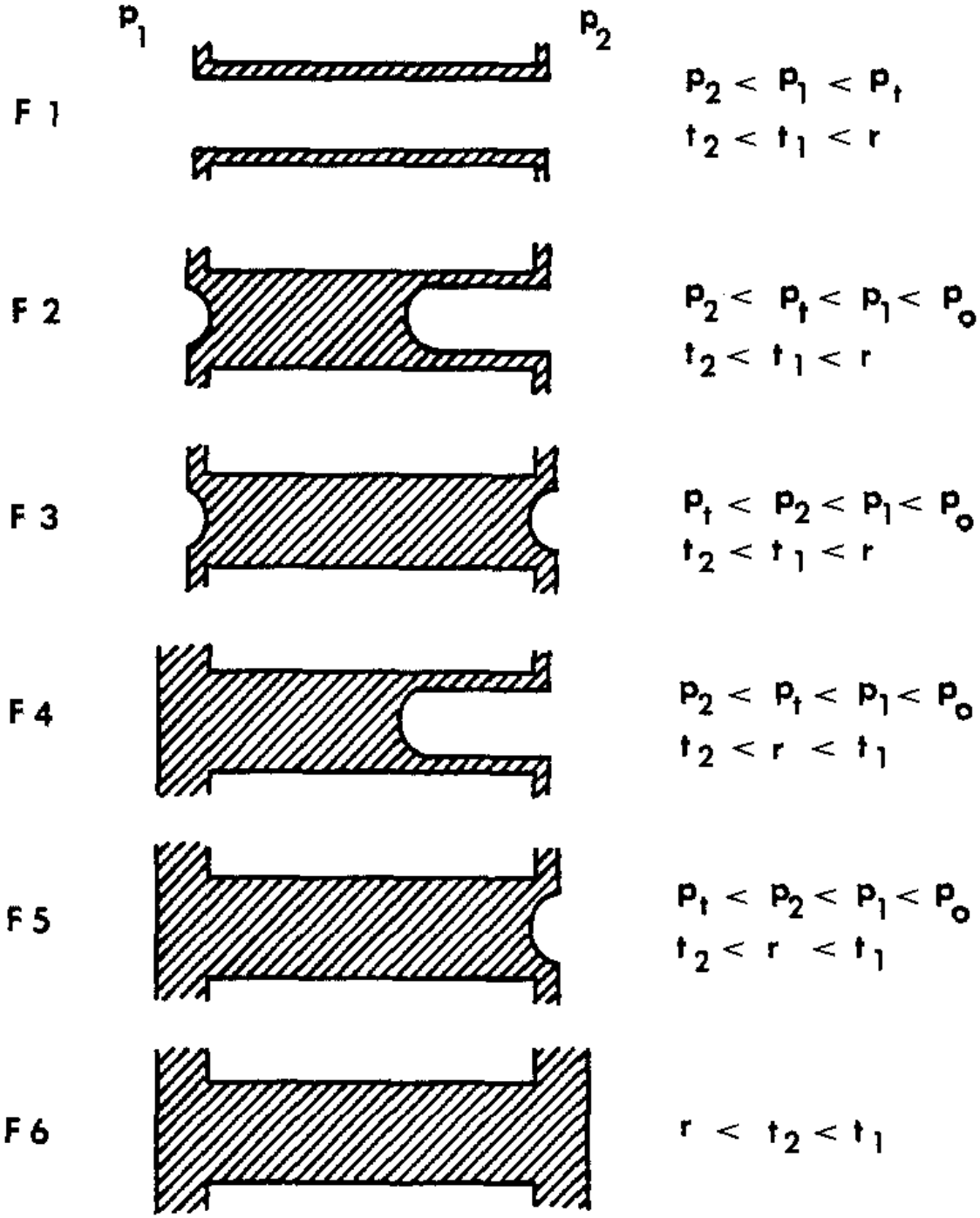
Capillary condensation

Young-Laplace eq.

$$p_k - p_{liq} = 2\sigma/r$$

Kelvin's eq.

$$\ln\left(\frac{p_k}{p_{sat}}\right) = -\frac{2\sigma}{r}\frac{\mathcal{M}}{\rho_{liq}\mathcal{R}T}$$



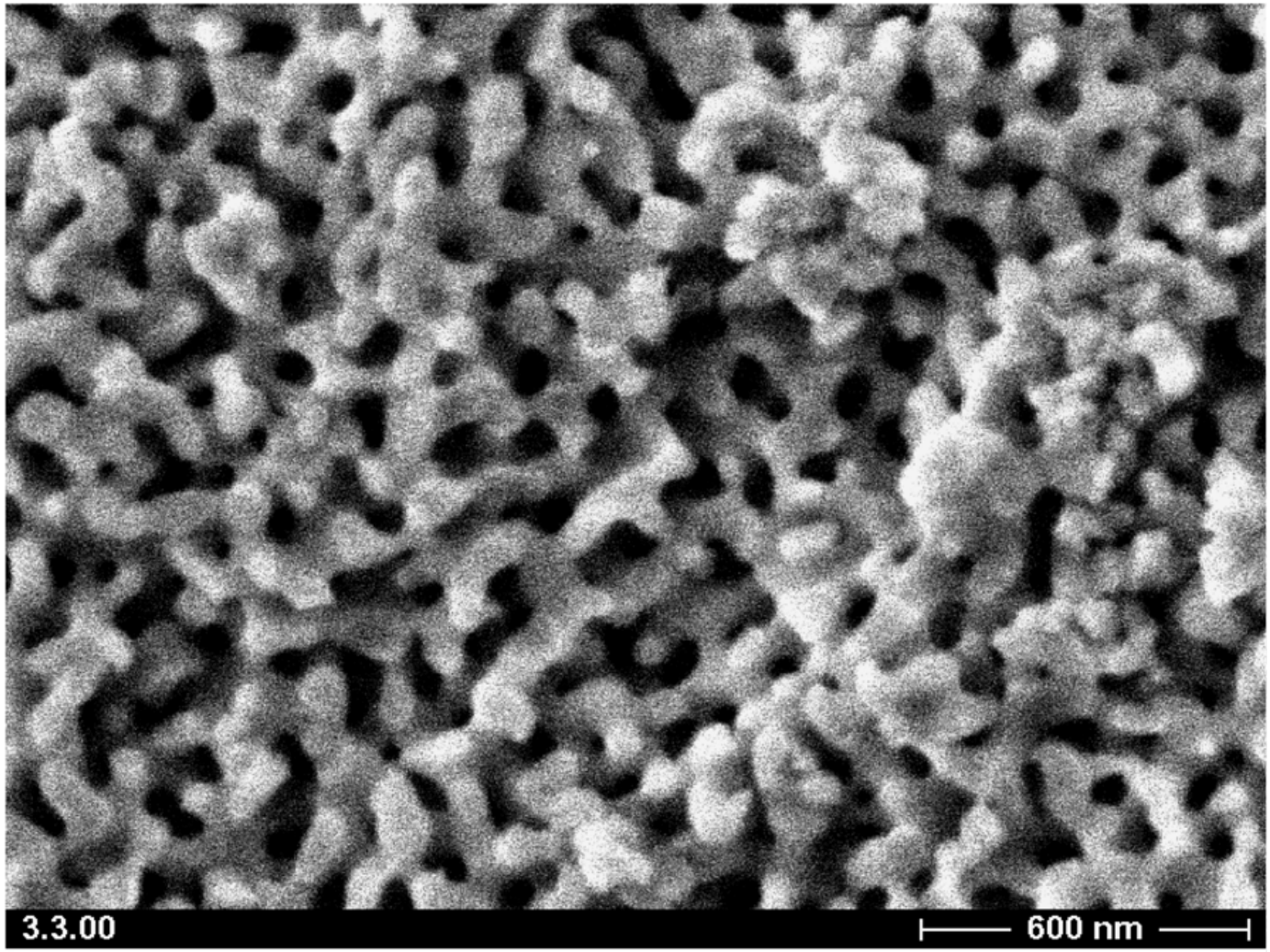
Rhim & Hwang, 1975

Lee & Hwang, 1986

Abeles *et al.*, 1991

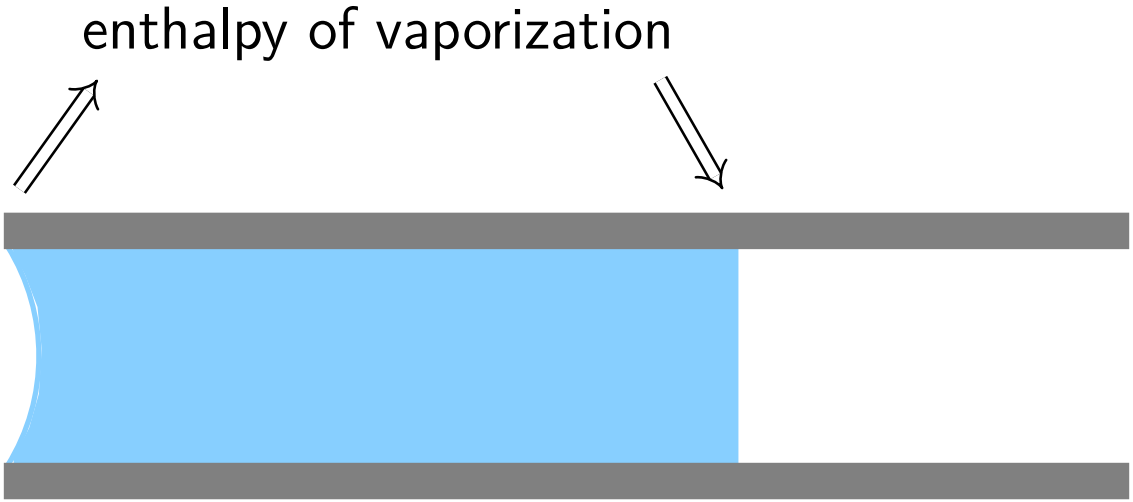
Uhlhorn *et al.*, 1992

Vycor glass



(Image from Gille, Enke & Janowski, 2001)

Enthalpy of vaporization



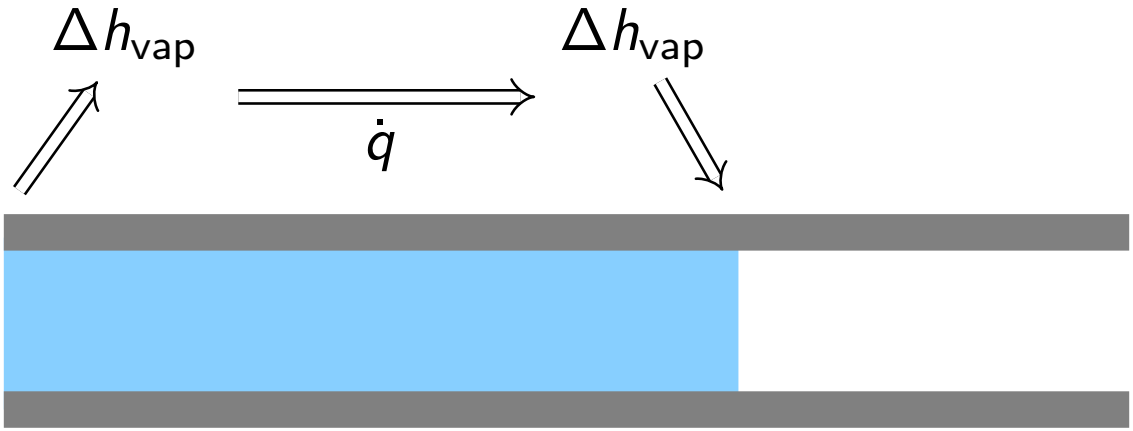
Rhim & Hwang (1975): "... a finite temperature distribution always exists even though the system is kept in an isothermal bath."

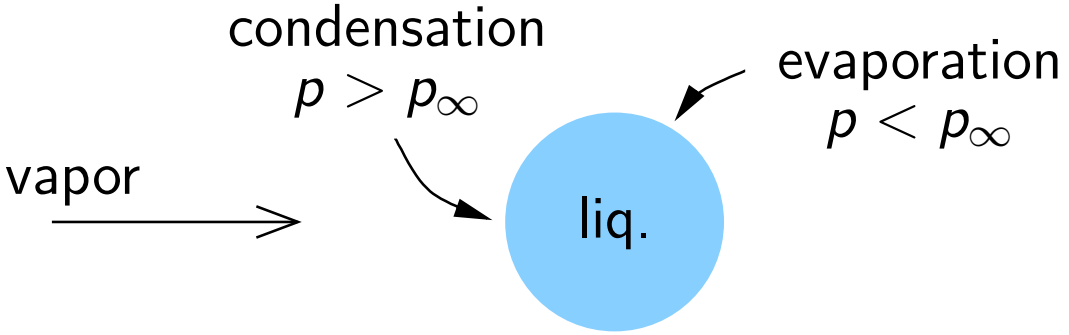
But later:

Isothermal descriptions

(Lee & Hwang, 1986; Abeles *et al.*, 1991; Uhlhorn *et al.*, 1992; ...)

Temperature variation

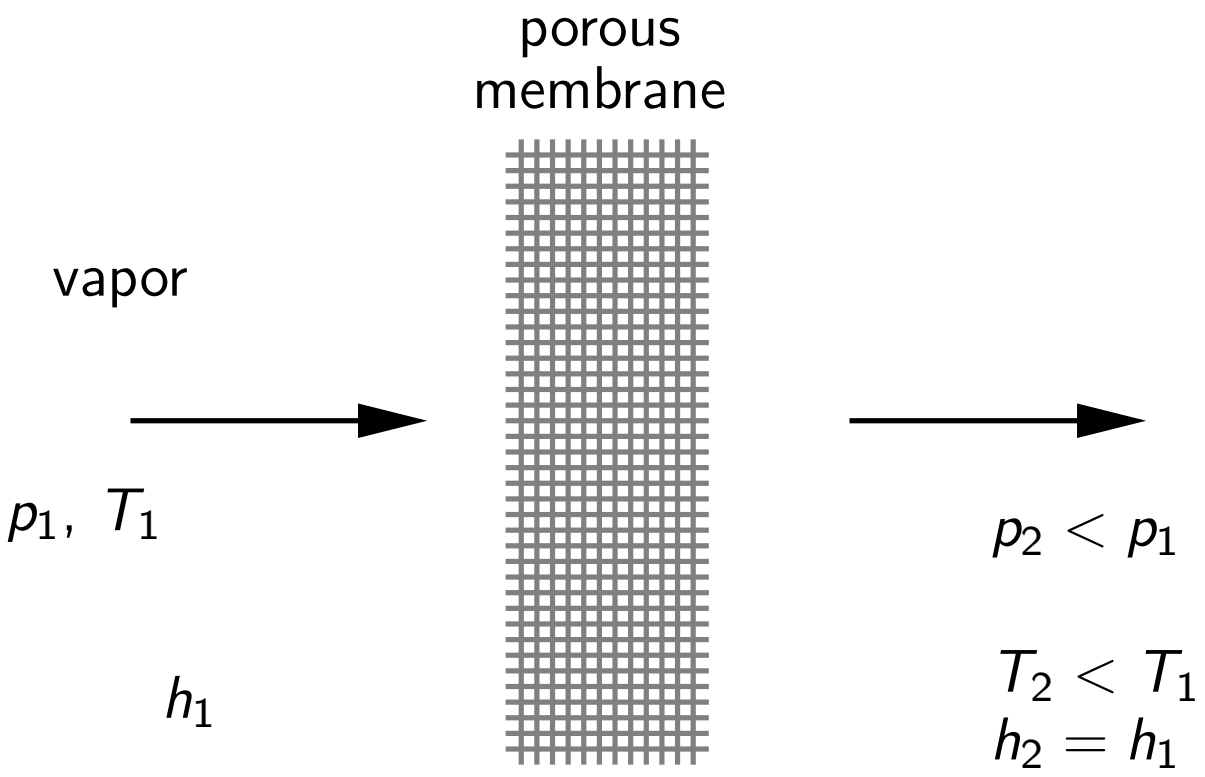




Drag of a droplet moving through its own vapor

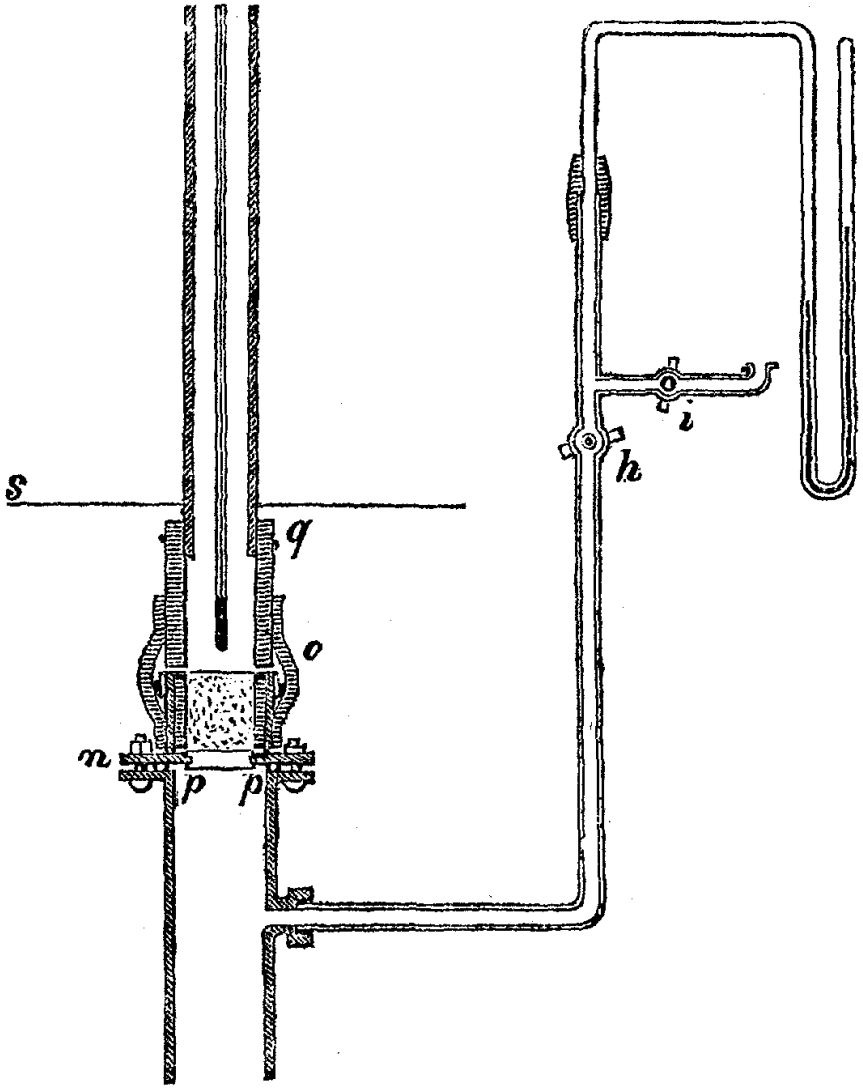
Schneider (1981, 1983)

Joule-Thomson process

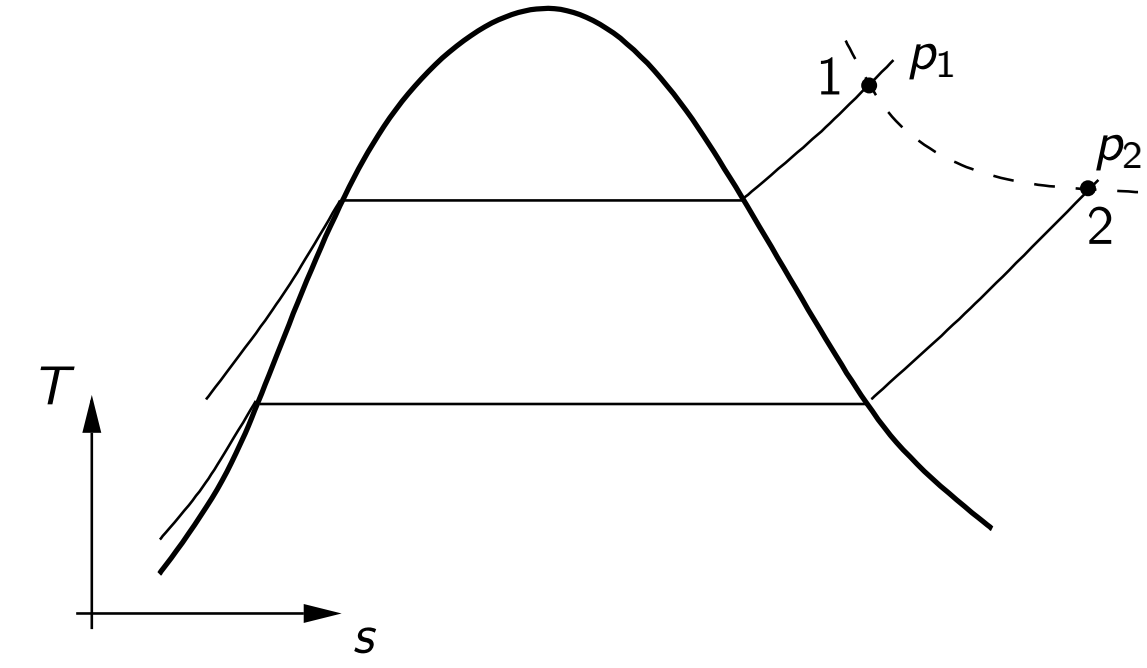


Thomson, 1853

Fig. 4.



Overview temperature-entropy diagram



—— dew line, boiling line

—— isobaric line - - - isenthalpic line

Gaseous flow

$$\dot{m} = const$$

$$\dot{m} = -\frac{\kappa\rho}{\eta}\frac{\mathrm{d}p}{\mathrm{d}x}$$

$$\dot{m}h + \dot{q} = const$$

and

$$\dot{q} = -k\frac{\mathrm{d}T}{\mathrm{d}x}$$

$$\mathrm{d}h = \left(\frac{\partial h}{\partial p}\right)_T \mathrm{d}p + c_p \mathrm{d}T$$

Joule-Thomson coefficient: $$\mu_{\mathrm{JT}} = \left(\frac{\mathrm{d}T}{\mathrm{d}p}\right)_h = -\frac{(\partial h/\partial p)_T}{c_p},$$

here: $(\partial h/\partial p)_T < 0$

Pressure distribution

If $p_1 - p_2 \ll p$ then $\rho \approx const$, $\eta \approx const$,

$$\frac{\mathrm{d}p}{\mathrm{d}x} = -\frac{\dot{m}\eta}{\kappa\rho} = const$$

For $p/\rho = p_1/\rho_1$, $\eta = const$,

$$-\frac{\dot{m}\eta p_1}{\kappa\rho_1}\mathrm{d}x = p\mathrm{d}p$$

$$p^2 = p_2^2 - 2\frac{\dot{m}\eta p_1}{\kappa\rho_1}x, \qquad (x < 0)$$

$$\frac{\mathrm{d}p}{\mathrm{d}x} = -\left(\left(\frac{\kappa\rho_1 p_2}{\dot{m}\eta p_1}\right)^2 - 2\frac{\kappa\rho_1}{\dot{m}\eta p_1}x\right)^{-\frac{1}{2}}$$

Temperature distribution

$$\dot{m}\left(\frac{\partial h}{\partial p}\right)_T \frac{\mathrm{d}p}{\mathrm{d}x} + \dot{m}c_p\frac{\mathrm{d}T}{\mathrm{d}x} + \frac{\mathrm{d}\dot{q}}{\mathrm{d}x} = 0$$

$$\frac{\mathrm{d}\dot{q}}{\mathrm{d}x} - \frac{\dot{m}c_p}{k}\dot{q} = \dot{m}\mu_{\mathrm{JT}}c_p\frac{\mathrm{d}p}{\mathrm{d}x}$$

For $\mathrm{d}p/\mathrm{d}x = const$, $\dot{q}(x=0) = 0$,

$$\dot{q} = -\mu_{\mathrm{JT}}k\frac{\mathrm{d}p}{\mathrm{d}x}\left(1 - e^{\frac{\dot{m}c_p}{k}x}\right) \quad = \dot{q}|_{h=const}\left(1 - e^{\frac{\dot{m}c_p}{k}x}\right)$$

For $\mathrm{d}p/\mathrm{d}x = f(x)$

$$\dot{q} = \int_0^x e^{(x-\xi)\frac{\dot{m}c_p}{k}}\frac{\dot{m}c_p}{k}\dot{q}|_{h=const}(\xi)\,\mathrm{d}\xi$$

Length scale

$$\frac{k}{\dot{m}c_p} = \frac{k}{\rho u c_p} \qquad \left(\frac{k}{\rho u c_p L} \ldots \text{Stanton} \; = \frac{\dot{q}}{\rho u c_p \Delta T} \right)$$

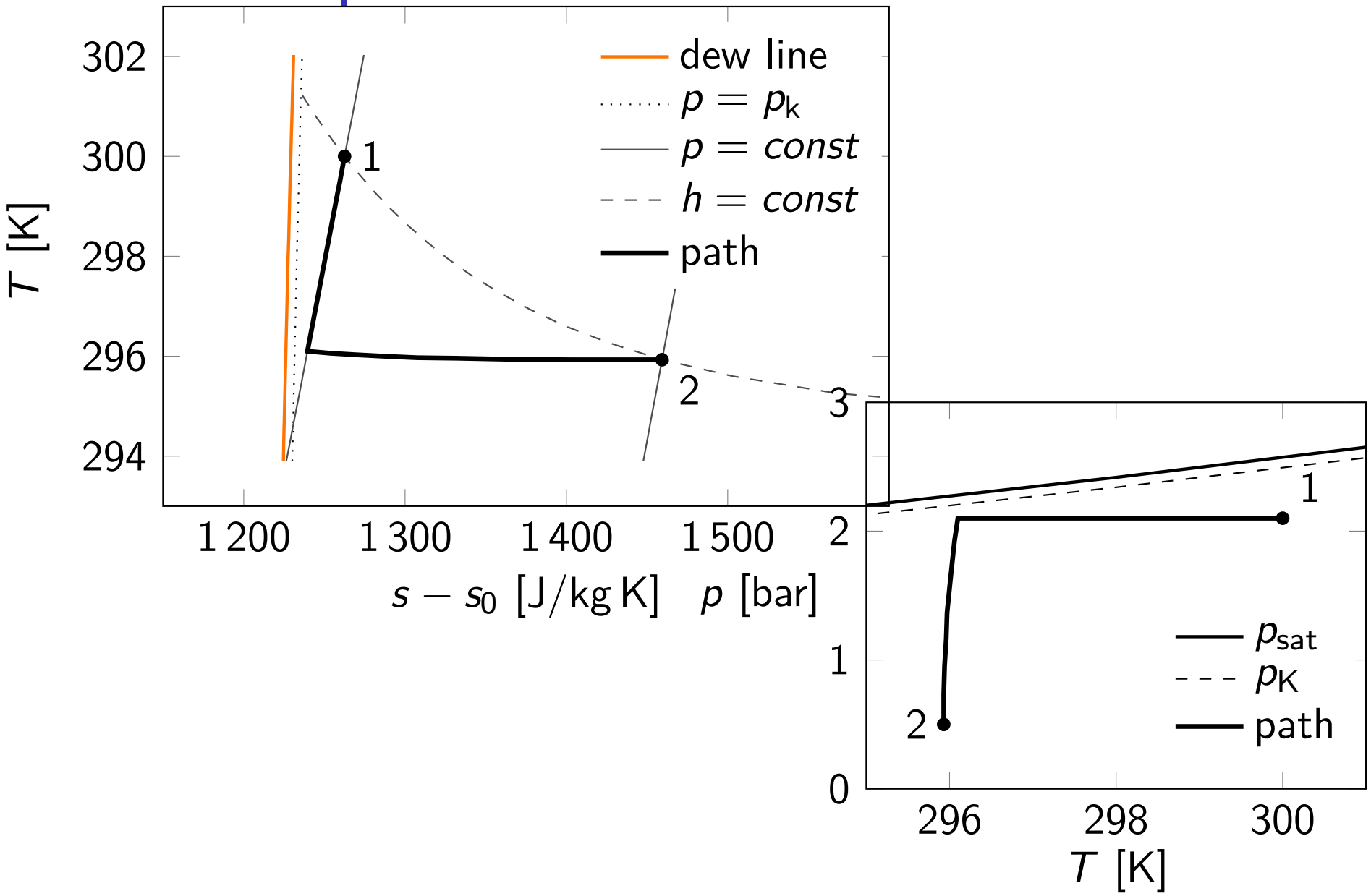
Cotton plug, $L = 1.9$ in, $\dot{m} = 3.36$ kg/m^{2}s, $k = 0.052$ W/mK

$$k/(\dot{m}c_p) = 15 \text{ µm}$$

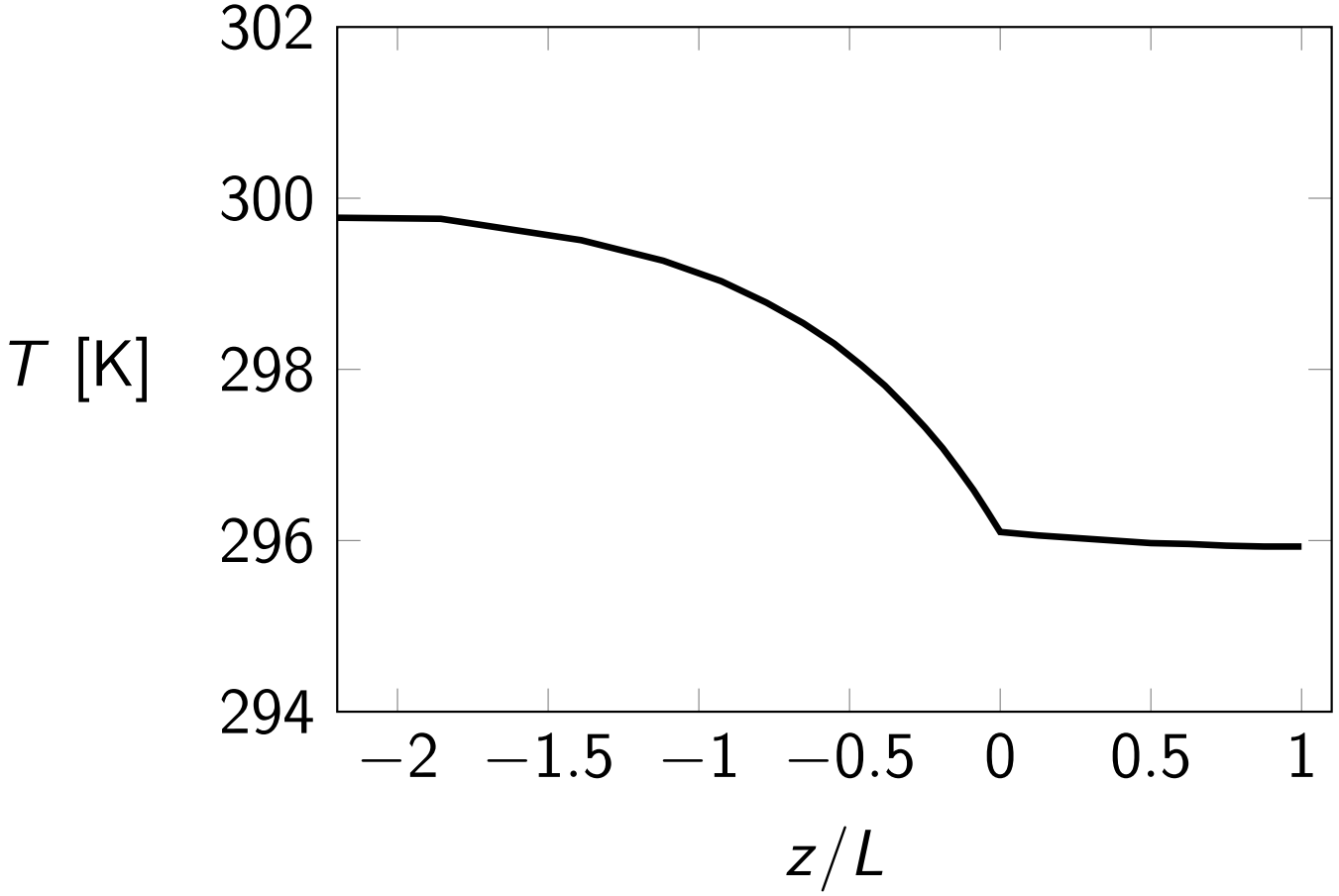
Anodic alumina, $L = 100$ µm, $\dot{m} = 0.0173$ kg/m^{2}s, $k = 1.2$ W/mK,

$$k/(\dot{m}c_p) = 4.2 \text{ cm}$$

Path of the process

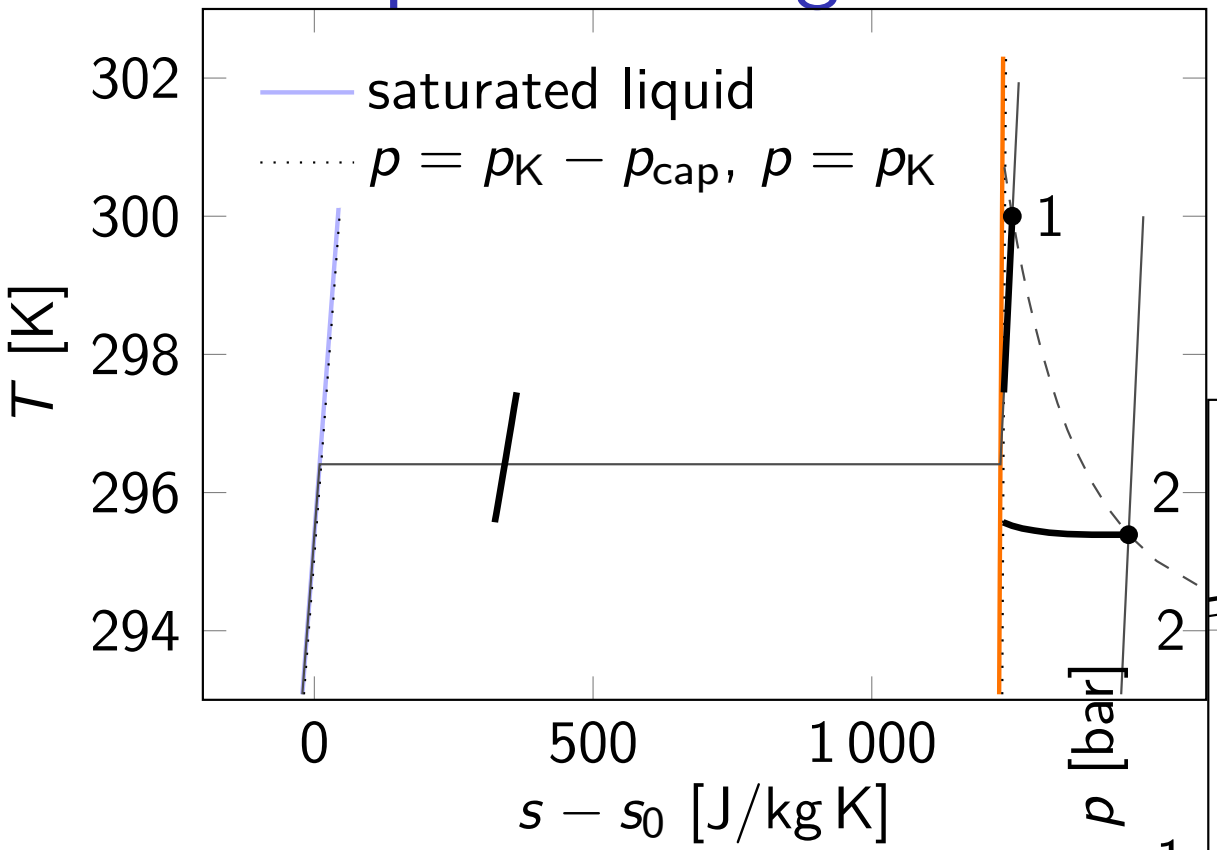


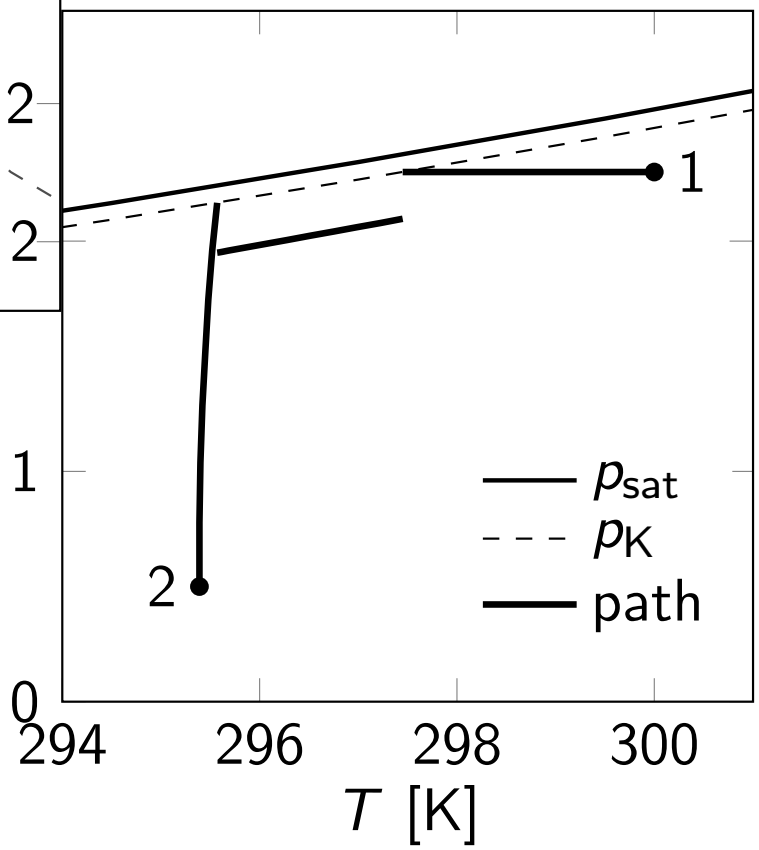
Temperature distribution



flow of butane through a glass capillary, $p_1 = 2.1$ bar, $p_2 = 0.5$ bar

Flow with phase changes





butane, $p_1 = 2.3$ bar, $p_2 = 0.5$ bar

Dimensionless numbers

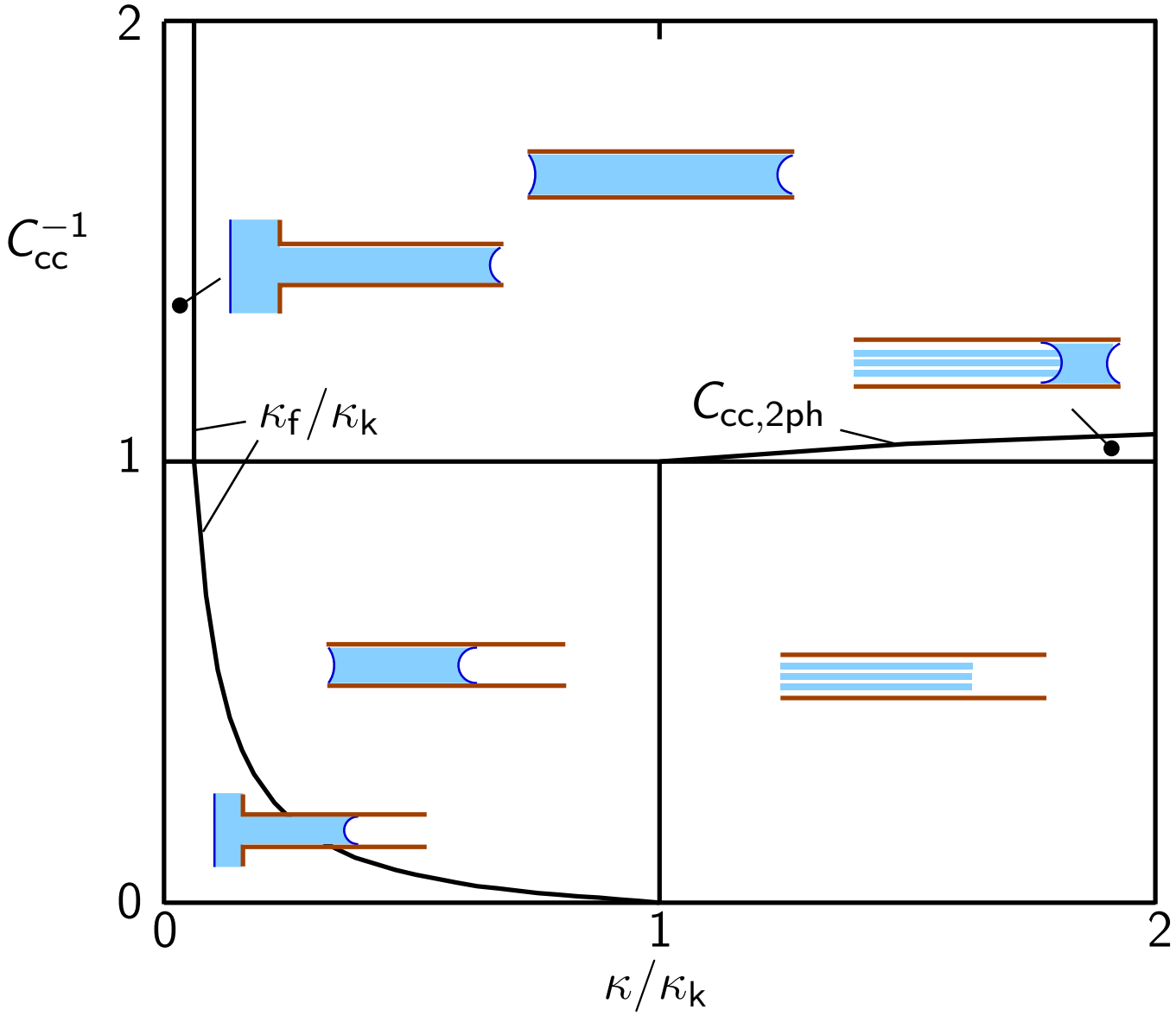
$$\kappa_c = \frac{\nu_{\mathrm{liq}} k_{\mathrm{liq}}}{\Delta h_{\mathrm{vap}} (\mathrm{d}p_{\mathrm{sat}}/\mathrm{d}T)} \qquad \text{(Schneider, 1983)}$$

$$\kappa_l = \frac{v_l k_{\mathrm{m,l}}}{\Delta h_{\mathrm{vap,K}} (\mathrm{d}p_{\mathrm{K}}/\mathrm{d}T - (2\cos\theta/r)\mathrm{d}\sigma/\mathrm{d}T)}$$

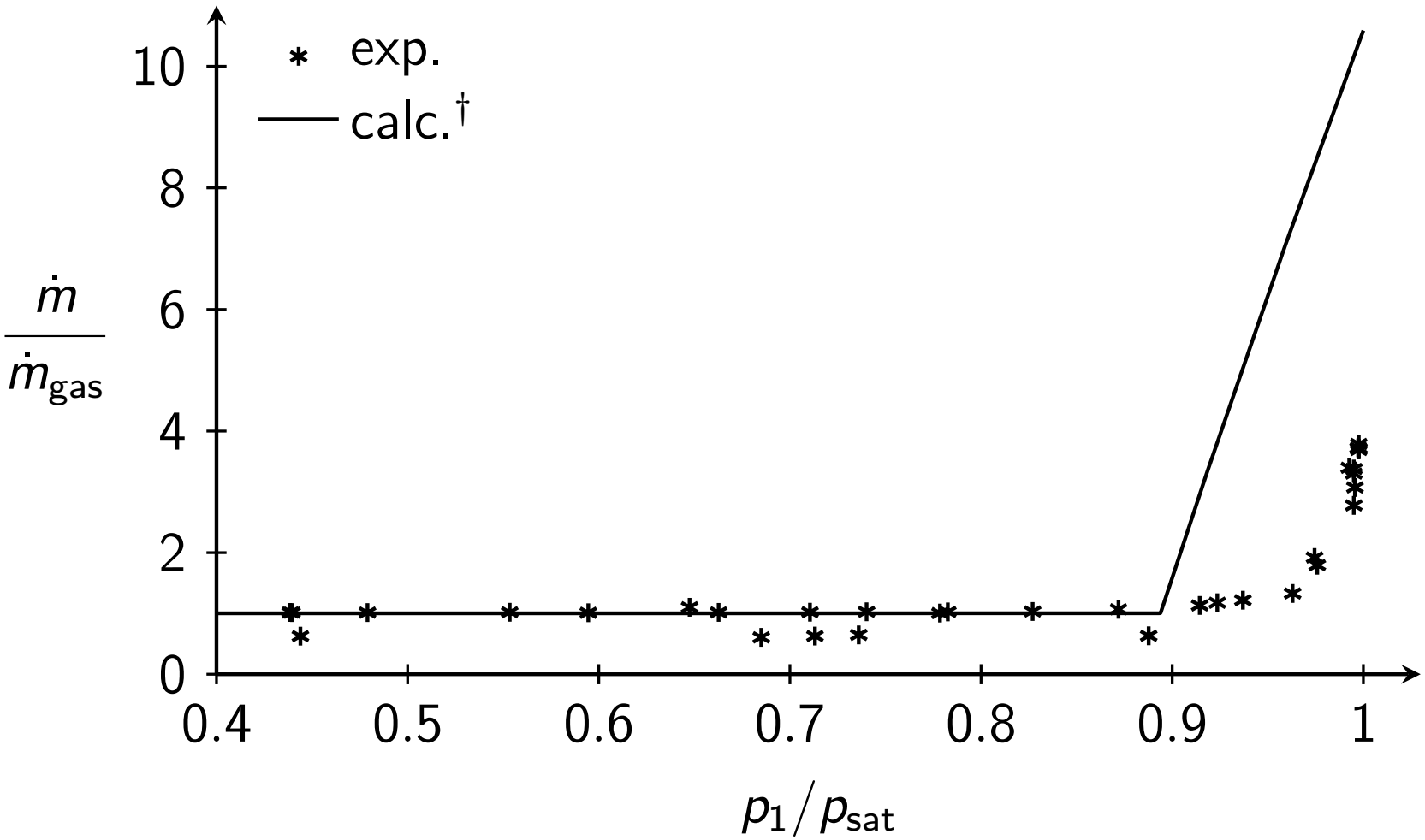
$$n = \left(\frac{\partial T}{\partial p}\right)_h \frac{\mathrm{d}p_{\mathrm{K}}}{\mathrm{d}T}$$

$$C_{\mathrm{cc}} = \frac{(p_1 - p_2)(1 - n)}{p_{\mathrm{sat}} - p_{\mathrm{K}}}$$

Flow map

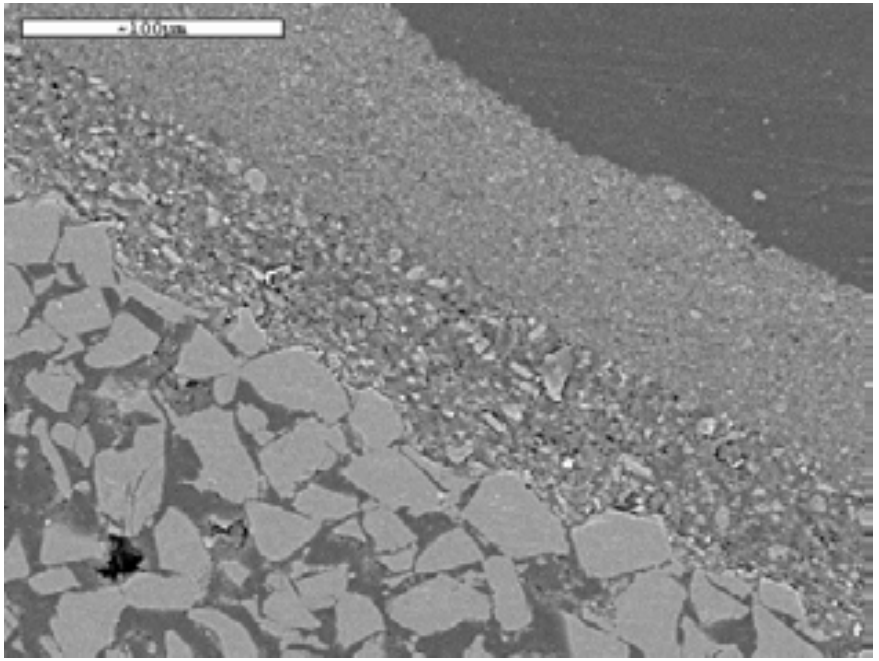


Measurements, Vycor glass & ceramic membranes

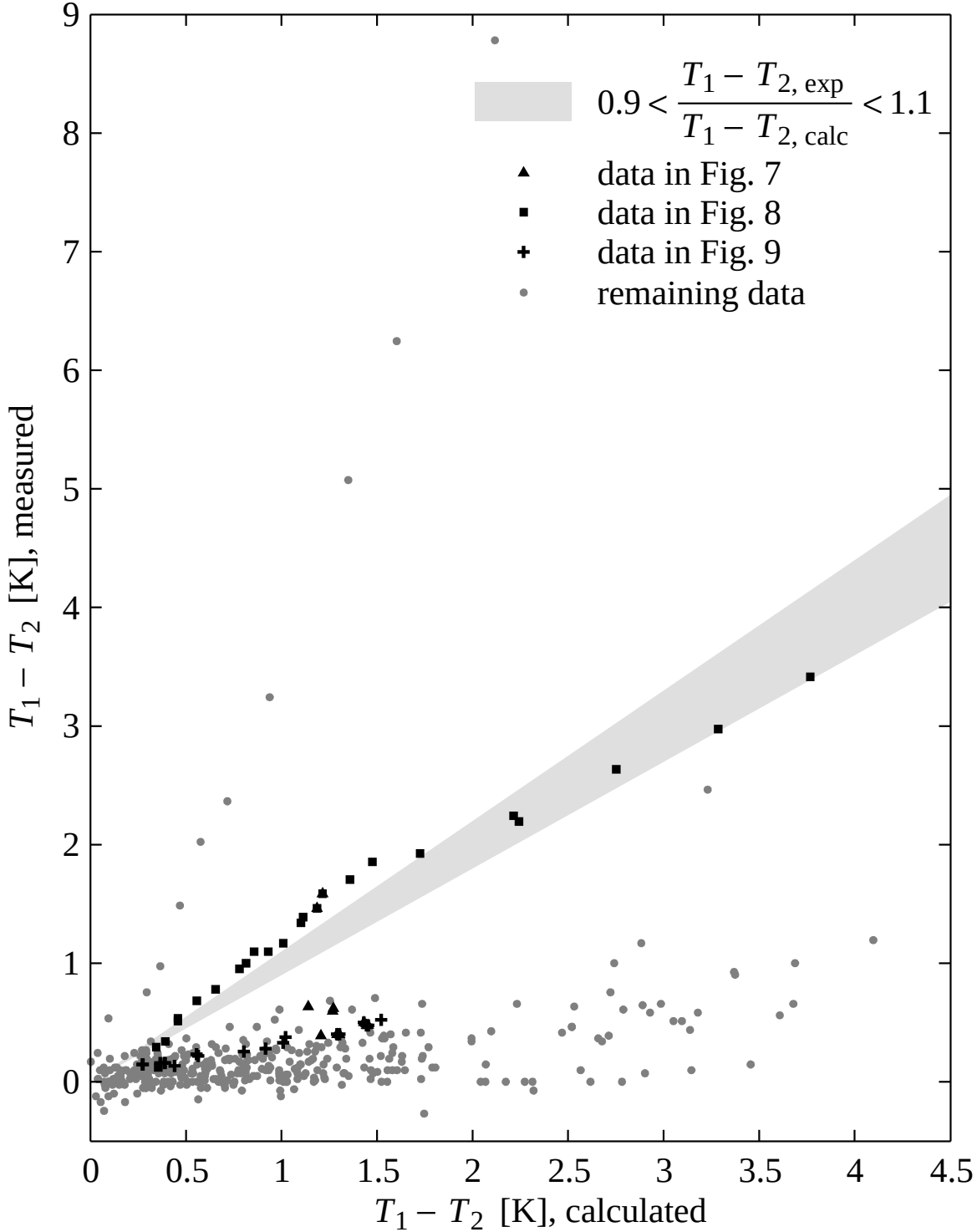


isobutane, Vycor glass, pore diameter 20 nm

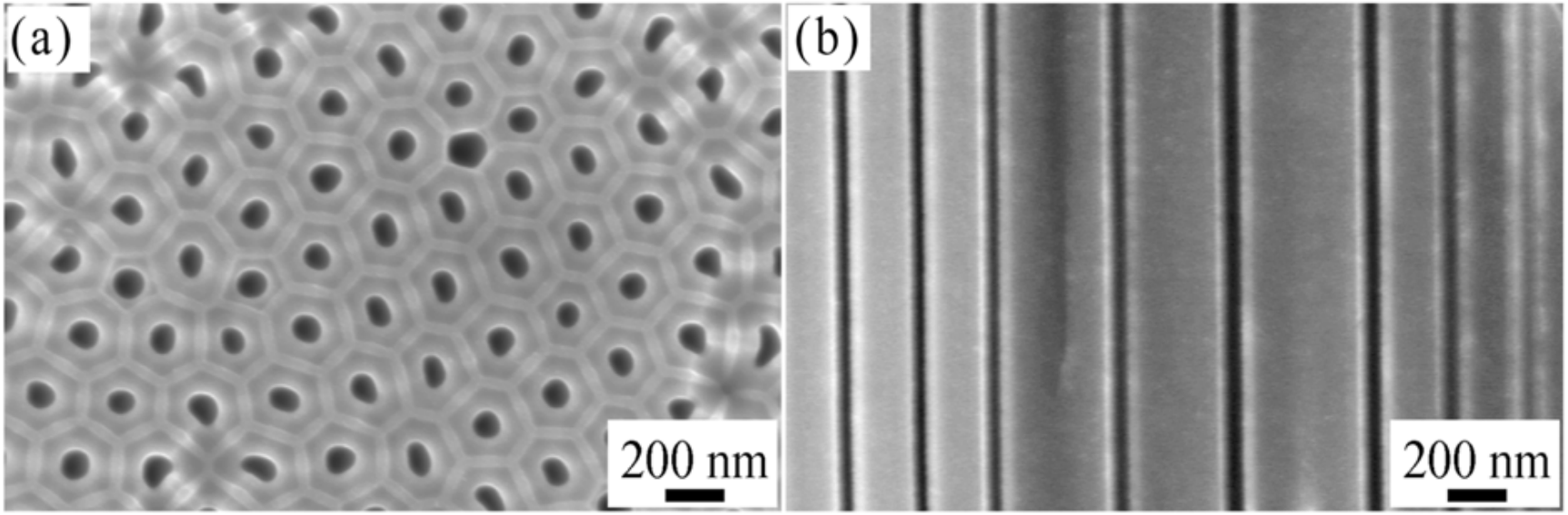
Ceramic membranes



Expected temperature differences

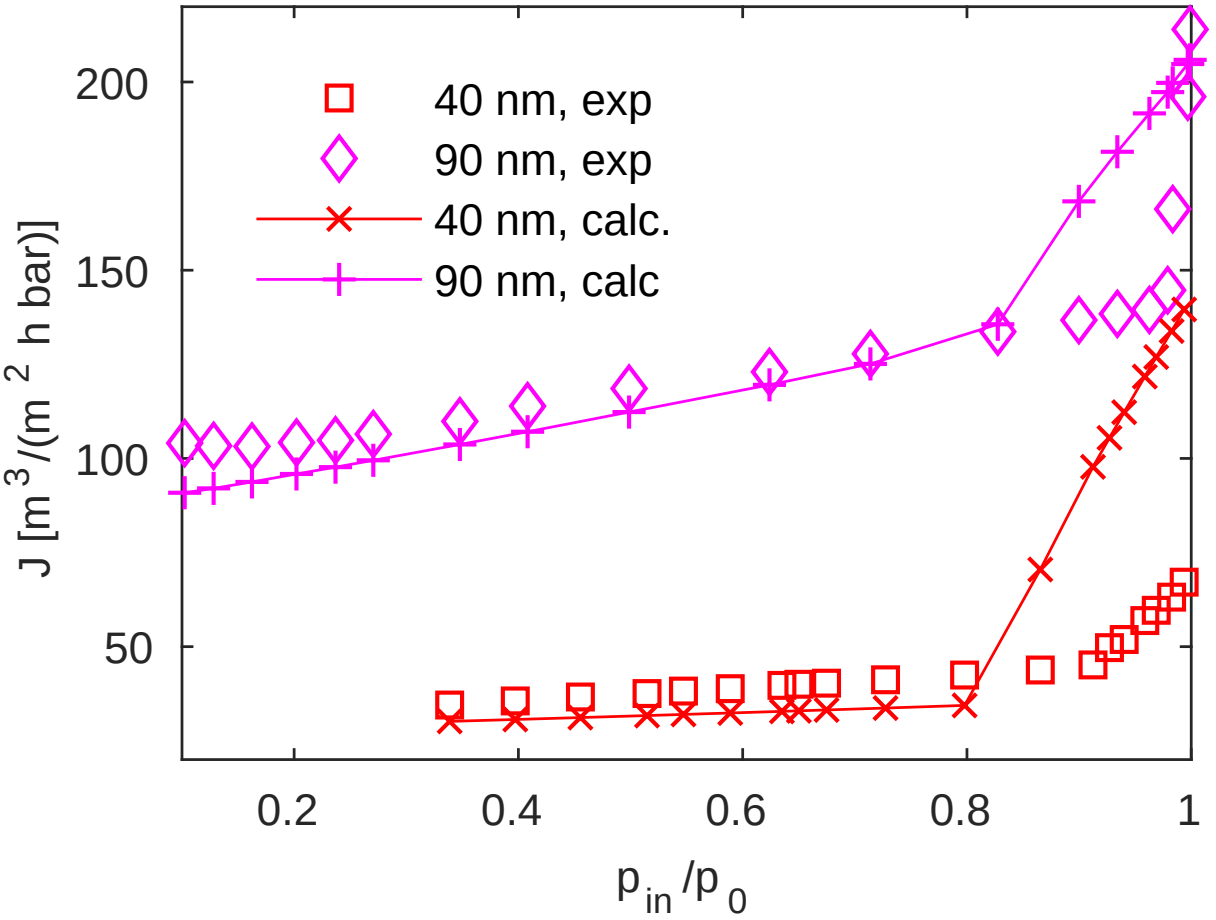


Anodic alumina membranes



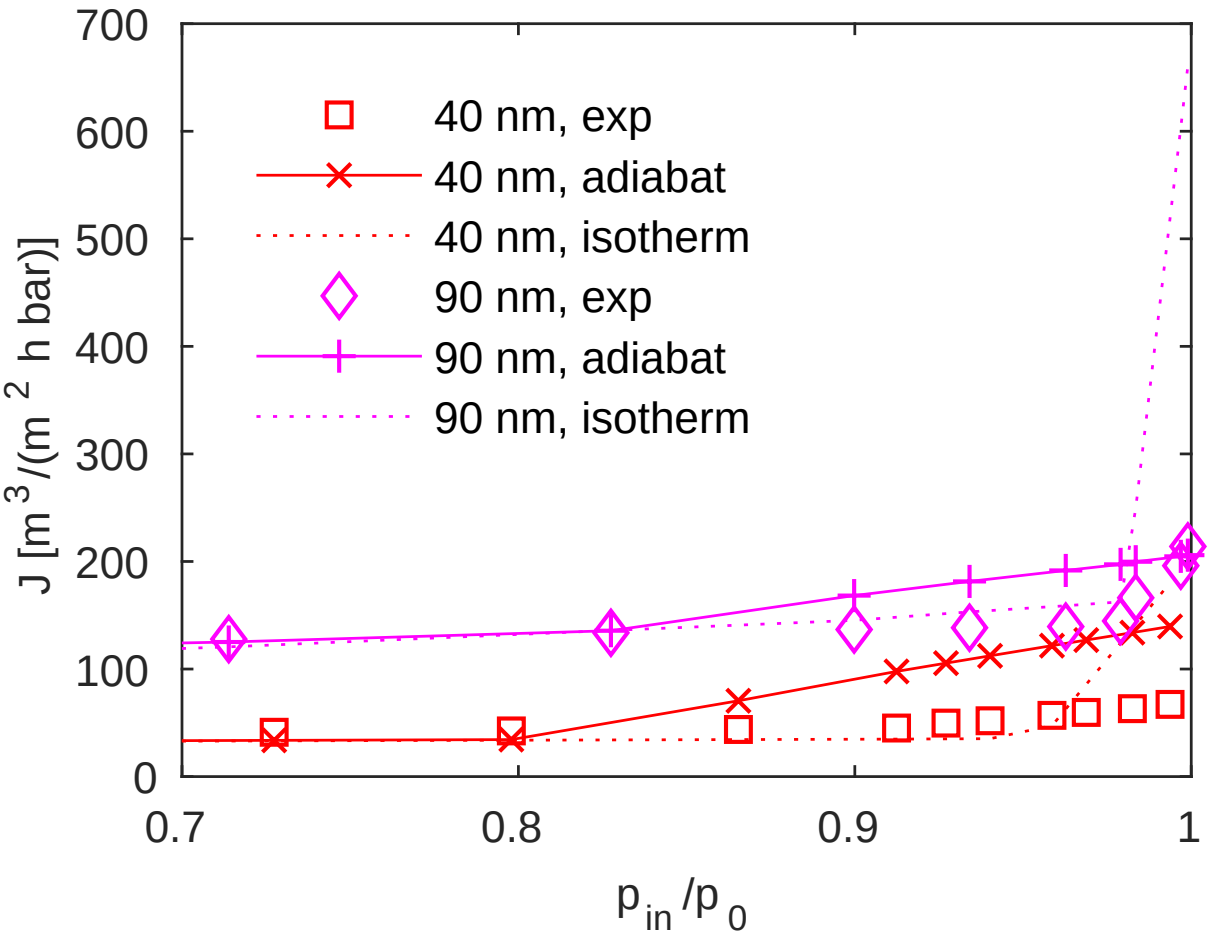
Petukhov *et al.* (2012, 2016)

Measurements, anodic alumina membranes



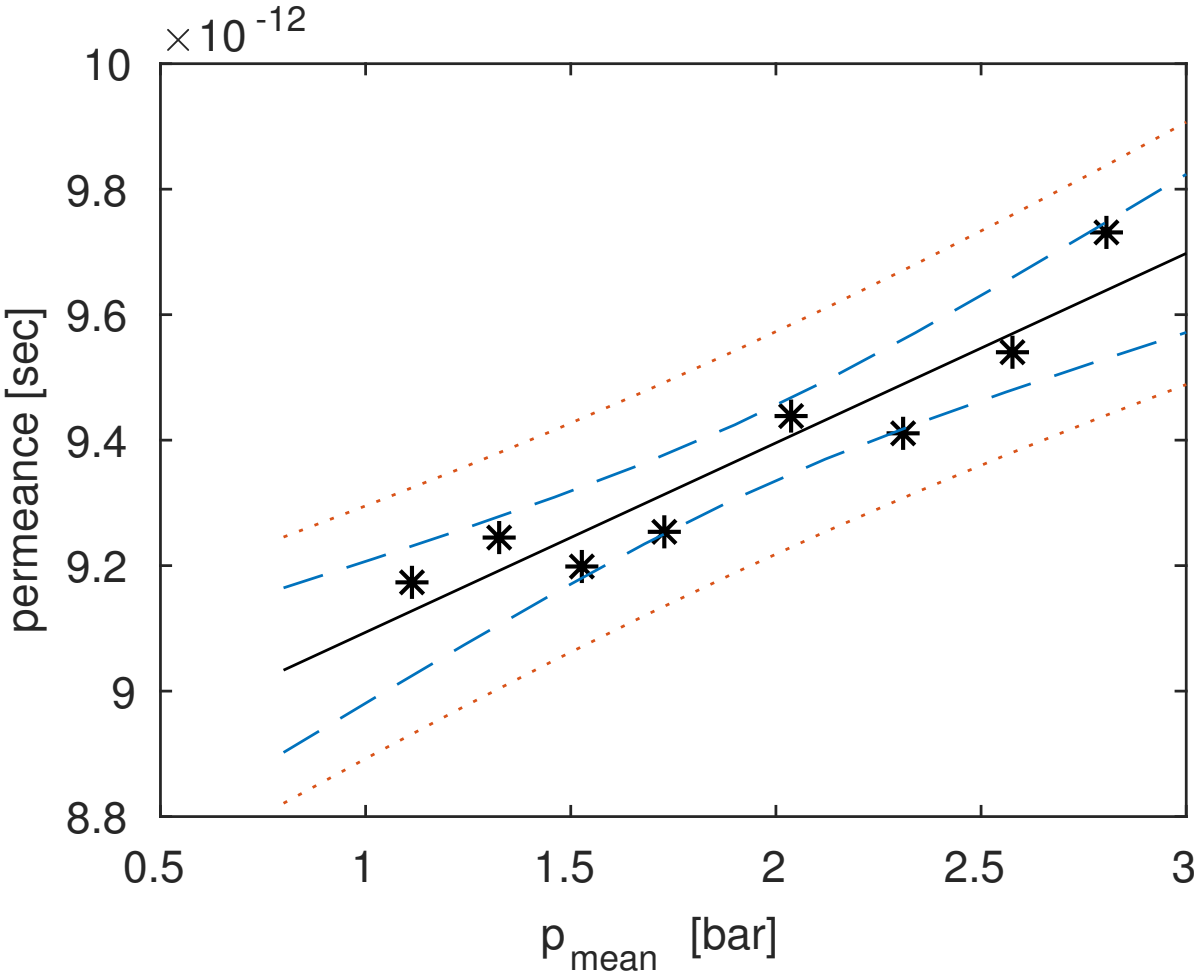
isobutane, pore diameters 40 nm and 90 nm

Measurements, anodic alumina membranes, cont.



isobutane, pore diameters 40 nm and 90 nm

Pore geometry



nitrogen, Vycor glass, pore diameter 20 nm

Summary and conclusions

- The Joule-Thomson process has been analyzed for gaseous flows and for flows with phase changes
- The temperature variation must be considered, to understand properties of the flow:
  - large mass flux
  - complete or partial condensation
- Better agreement with recently published experimental data than using isothermal description