Flow of Vapors Through Porous Media

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Problem statement

Isothermal description
Adiabatic description
Flow field
Experiments
Summary and conclusions

vapor close to saturation
$p_1, T_1$

porous membrane
$p_2 < p_1; \text{ unsaturated vapor}$
Capillary condensation

Young-Laplace eq.

\[ p_k - p_{liq} = \frac{2\sigma}{r} \]

Kelvin’s eq.

\[ \ln \left( \frac{p_k}{p_{sat}} \right) = -\frac{2\sigma}{r} \frac{\mathcal{M}}{\rho_{liq} \mathcal{R} T} \]
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Summary and conclusions

- **F1**
  - $p_2 < p_1 < p_f$
  - $t_2 < t_1 < r$

- **F2**
  - $p_2 < p_f < p_1 < p_o$
  - $t_2 < t_1 < r$

- **F3**
  - $p_f < p_2 < p_1 < p_o$
  - $t_2 < t_1 < r$

- **F4**
  - $p_2 < p_1 < p_f < p_o$
  - $t_2 < r < t_1$

- **F5**
  - $p_1 < p_2 < p_f < p_o$
  - $t_2 < r < t_1$

- **F6**
  - $r < t_2 < t_1$

References:

- Rhim & Hwang, 1975
- Lee & Hwang, 1986
- Abeles et al., 1991
- Uhlhorn et al., 1992
Enthalpy of vaporization

Rhim & Hwang (1975): “... a finite temperature distribution always exists even though the system is kept in an isothermal bath.”

But later:

Isothermal descriptions

(Lee & Hwang, 1986; Abeles et al., 1991; Uhlhorn et al., 1992; ...
Temperature variation

$\Delta h_{\text{vap}} \quad \dot{q} \quad \Delta h_{\text{vap}}$

Drag of a droplet moving through its own vapor

condensation $p > p_\infty$

evaporation $p < p_\infty$

Schneider (1981, 1983)
Joule-Thomson process

\[ p_1, T_1 \] \quad \text{vapor} \quad \text{porous membrane} \quad \text{pore} \quad \text{vapor} \quad \text{vapor} \]

\[ h_1 \] \quad \begin{align*}
 p_2 &< p_1 \\
 T_2 &< T_1 \\
 h_2 &= h_1
\end{align*}
Thomson, 1853
Overview temperature-entropy diagram

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dew line, boiling line

isobaric line

isenthalpic line
Gaseous flow

\[ \dot{m} = \text{const} \]

\[ \dot{m} = -\frac{\kappa \rho \, dp}{\eta \, dx} \]

\[ \dot{m}h + \dot{q} = \text{const} \]

and

\[ \dot{q} = -k \frac{dT}{dx} \]

\[ dh = \left( \frac{\partial h}{\partial p} \right)_T dp + c_p dT \]

Joule-Thomson coefficient:

\[ \mu_{JT} = \left( \frac{dT}{dp} \right)_h = -\frac{(\partial h/\partial p)_T}{c_p} \]

here: \( (\partial h/\partial p)_T < 0 \)
Pressure distribution

If $p_1 - p_2 \ll p$ then $\rho \approx \text{const}$, $\eta \approx \text{const}$,

$$\frac{d\rho}{dx} = - \frac{\dot{m}\eta}{\kappa\rho} = \text{const}$$

For $p/\rho = p_1/\rho_1$, $\eta = \text{const}$,

$$- \frac{\dot{m}\eta p_1}{\kappa \rho_1} dx = \rho dp$$

$$p^2 = p_2^2 - 2 \frac{\dot{m}\eta p_1}{\kappa \rho_1} x, \quad (x < 0)$$

$$\frac{d\rho}{dx} = - \left( \left( \frac{\kappa \rho_1 p_2}{\dot{m}\eta p_1} \right)^2 - 2 \frac{\kappa \rho_1}{\dot{m}\eta p_1} x \right)^{-\frac{1}{2}}$$
Temperature distribution

\[
\dot{m} \left( \frac{\partial h}{\partial p} \right)_T \frac{dp}{dx} + \dot{m} c_p \frac{dT}{dx} + \frac{dq}{dx} = 0
\]

\[
\frac{dq}{dx} - \frac{\dot{m} c_p}{k} q = \dot{m} \mu_J \frac{dp}{dx}
\]

For \( \frac{dp}{dx} = \text{const} \), \( \dot{q}(x = 0) = 0 \),

\[
\dot{q} = -\mu_J \frac{dp}{dx} \left( 1 - e^{\frac{\dot{m} c_p}{k}} \right) = \dot{q}_{\mid h=\text{const}} \left( 1 - e^{\frac{\dot{m} c_p}{k}} \right)
\]

For \( \frac{dp}{dx} = f(x) \)

\[
\dot{q} = \int_0^x e^{(x-\xi)} \frac{\dot{m} c_p}{k} \dot{q}_{\mid h=\text{const}}(\xi) \, d\xi
\]
Length scale

\[ \frac{k}{\dot{m} c_p} = \frac{k}{\rho u c_p} \left( \frac{k}{\rho u c_p L} \right) \ldots \text{Stanton} = \frac{\dot{q}}{\rho u c_p \Delta T} \]

Cotton plug, \( L = 1.9 \text{ in} \), \( \dot{m} = 3.36 \text{ kg/m}^2\text{s} \), \( k = 0.052 \text{ W/mK} \)

\[ k/(\dot{m} c_p) = 15 \mu m \]

Anodic alumina, \( L = 100 \mu m \), \( \dot{m} = 0.0173 \text{ kg/m}^2\text{s} \), \( k = 1.2 \text{ W/mK} \),

\[ k/(\dot{m} c_p) = 4.2 \text{ cm} \]
Path of the process

- **dew line**
- **$p = p_k$**
- **$p = \text{const}$**
- **$h = \text{const}$**
- **path**

Constraints:
- $T \geq 294$
- $T \leq 302$
- $s - s_0 \leq 1$
- $p_k \leq p \leq p_{\text{sat}}$
- $p_{\text{K}}$ (K is constant)

Points:
- 1
- 2
- 3

Parameters:
- $T$ [K]
- $s - s_0$ [J/kg K]
- $p$ [bar]
Temperature distribution

flow of butane through a glass capillary, $p_1 = 2.1$ bar, $p_2 = 0.5$ bar
Flow with phase changes

butane, $p_1 = 2.3$ bar, $p_2 = 0.5$ bar
Dimensionless numbers

\[
\kappa_c = \frac{\nu_{\text{liq}} k_{\text{liq}}}{\Delta h_{\text{vap}} (d p_{\text{sat}}/d T)} \\
\kappa_l = \frac{\nu_{l} k_{m,l}}{\Delta h_{\text{vap},K} (d p_{K}/d T - (2 \cos \theta/r)d \sigma/d T)}
\]

\[
n = \left( \frac{\partial T}{\partial p} \right)_h \frac{d p_{K}}{d T}
\]

\[
C_{cc} = \frac{(p_1 - p_2)(1 - n)}{p_{\text{sat}} - p_{K}}
\]
Flow map

\( C_{cc}^{-1} \)

\( \frac{\kappa_f}{\kappa_k} \)

\( C_{cc,2ph} \)

\( \frac{\kappa}{\kappa_k} \)
Measurements, Vycor glass & ceramic membranes

\[ \frac{\dot{m}}{\dot{m}_{\text{gas}}} \] vs. \[ p_1/p_{\text{sat}} \]

* exp.

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isobutane, Vycor glass, pore diameter 20 nm
Ceramic membranes
Expected temperature differences

$$0.9 < \frac{T_1 - T_2, \text{exp}}{T_1 - T_2, \text{calc}} < 1.1$$

- $\Delta$: data in Fig. 7
- $\blacksquare$: data in Fig. 8
- $+$: data in Fig. 9
- $\ast$: remaining data

Data in Fig. 7, Fig. 8, Fig. 9, and remaining data are plotted on the graph.
Anodic alumina membranes

Petukhov et al. (2012, 2016)
Measurements, anodic alumina membranes

isobutane, pore diameters 40 nm and 90 nm
Measurements, anodic alumina membranes, cont.

isobutane, pore diameters 40 nm and 90 nm
**Pore geometry**

![Graph showing permeance vs. mean pressure](image)

nitrogen, Vycor glass, pore diameter 20 nm
The Joule-Thomson process has been analyzed for gaseous flows and for flows with phase changes.

The temperature variation must be considered, to understand properties of the flow:
- large mass flux
- complete or partial condensation

Better agreement with recently published experimental data than using isothermal description.